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DESIGN SENSITIVITY ANALYSIS AND OPTIMIZATION OF AN ENGINE MOUNT SYSTEM USING AN FRF-BASED SUBSTRUCTURING METHOD

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The FRF-based substructuring method is one of the most powerful methods in analyzing the responses of complex built-up structures with high modal density. In this paper, a general procedure for the design sensitivity analysis of a vibro-acoustic system has been presented using the FRF-based substructuring formulation. For an acoustic response function, the proposed method gives a parametric design sensitivity expression in terms of the partial derivatives of the connection element properties and the transfer functions of the substructures. The derived noise sensitivity formula is combined with a non-linear programming module to obtain the optimal design for the engine mount system of a passenger car. The objective function is defined as the area of the interior noise graph integrated over a concerned r.p.m. range. The interior noise variations with respect to the dynamic characteristics of the engine mounts and bushings have been calculated using the proposed sensitivity formulation and transferred to a non-linear optimization software. To obtain the FRFs, a finite element analysis was used for the engine mount structures and experimental techniques were used for the trimmed body including the cabin cavity. The optimization based on the sensitivity analysis gives the ideal stiffness of the engine mount and bushings. The resultant interior noise in the passenger car shows that the proposed method is efficient and accurate.

1. INTRODUCTION

There are many tools that can be used to get the optimum design of a dynamic system. Among them, a sensitivity analysis combined with a mathematical programming technique has been a practical tool for design engineers when a large and complex system is considered. The design sensitivity analysis is a study of the rate of changes in system characteristics with respect to design parameter variations. Haug *et al.* [1] summarized the general procedures of the design sensitivity analysis for a structural system. For dynamic problems, the design sensitivity analysis has focused on the changes of the natural frequencies and modal vectors [2, 3] where the sensitivity information of a system response was obtained from the derivatives of the natural frequencies and modal vectors using modal superposition. To calculate the sensitivity of the response directly, researchers developed the design sensitivity formula of the frequency response functions (FRF). Akiyama *et al.* [4] introduced a structural modification concept in the transfer function synthesis method. Lin and Lim [5] developed a design sensitivity formula of the frequency response function from the experiments.

Dynamic substructuring technique is a method that predicts the dynamic behavior of a structure based on the dynamic behavior of the composing substructures. A method that calculates the frequency response functions of a structure composed of several substructures from the FRFs of the substructures is called the transfer function synthesis method or the FRF-based substructuring method (FBSM) [6, 7]. The FRF-based substructuring method is one of the most powerful methods available for the analysis of the response of a complex built-up structure with high modal density. Its superiority comes primarily from the ability to incorporate experimental FRFs into the formulation. It can predict the response of the total structure from the FRFs of the substructures, but it cannot give the systematic guides for the structural modification or optimal design. A design sensitivity analysis for the dynamic problem provides valuable information that the designer cannot predict by intuition or experience. However, little attention has been paid to the design sensitivity analysis in the frame of the substructuring method. Heo and Ehmann [8] presented a substructural sensitivity synthesis method by using component modal sensitivities. Santos and Arruda [9] also derived the joint stiffness sensitivity formula of a component mode synthesis model. Recently, Lallemand *et al.* [10] proposed a semi-analytical sensitivity analysis method in a component mode synthesis framework. As a direct approach, Jee [11] derived a response function sensitivity by differentiating the frequency response function formed by the transfer function synthesis method with respect to the substructure receptance function. Chang and Park [12] extended Jee's method and applied it to the structural dynamic modification.

In this paper, a general procedure for the design sensitivity analysis of vibro-acoustic problems is presented using the FRF-based substructuring formulation. By introducing the direct differentiation approach for the reaction forces on the interface elements, we derive a parametric noise sensitivity formula, in which algebraic linear equations can be solved. The present method is very powerful since the system matrix that is inverted for the sensitivity analysis is the same one used for the calculation of the system response by the transfer function synthesis method. In the formula, the additional term to be calculated for the sensitivity analysis is only a vector that consists of partial derivatives of substructure FRFs and connection element properties. To verify the efficiency of the proposed method, it is applied to a realistic problem related to the engine mount system of a passenger car. The sensitivities of a response with respect to the stiffness or damping coefficients of the connecting elements are calculated and compared with those from the conventional finite difference approach. To obtain the ideal stiffness of the engine mounts and bushings, an optimization problem is defined for the engine mount system of the real passenger car, and the solution is obtained by using the derived sensitivity formula in the non-linear optimization software.

2. DESIGN SENSITIVITY ANALYSIS USING THE FBSM

For the analysis of the structure-borne noise in a passenger car, the FRF-based substructuring method is more accurate due to its ability to combine the experimental

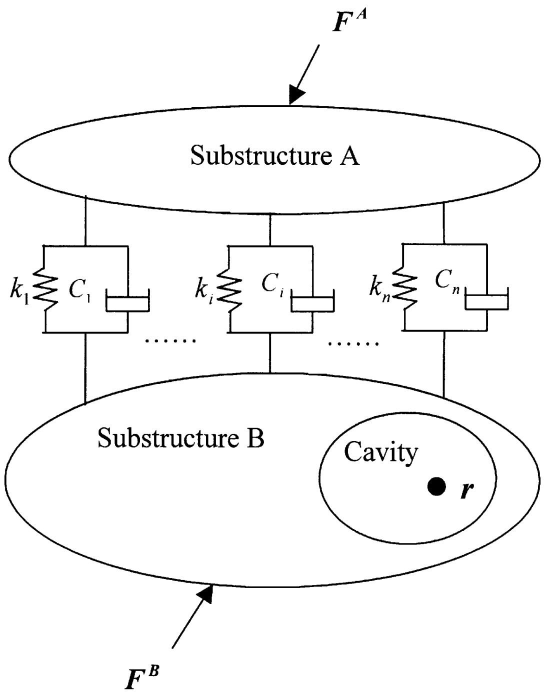

Figure 1. A structural-acoustic system consisting of two substructures.

and numerical FRFs. This section briefly summarizes the FRF-based substructuring method.

Consider the vibro-acoustic system shown in Figure 1, which has two substructures connected by springs and dampers. Substructure B has a closed cavity, in which a point is selected as a response point to be analyzed. When the external forces $\mathbf{F}^A$ and $\mathbf{F}^B$ excite the substructures, the response at point $\mathbf{r}$ on substructure B will be derived. The variables $\mathbf{k}_i$ and $\mathbf{C}_i$ represent the stiffness and damping coefficients of the ith connection element respectively. There are n connection degrees-of-freedom (d.o.f.) along the interface boundary. Hereafter, we adopt the summation convention in the indicial notation. The displacements of the connection d.o.f. on substructure A, $\mathbf{x}_i^A$, can be written as follows:

$$\mathbf{x}_i^A = \mathbf{H}_{ij}^A \mathbf{R}_j + \mathbf{H}_{if}^A \mathbf{F}^A, \quad i = 1, \ldots, n. \tag{1}$$

In this equation, $\mathbf{R}_j$ is the reaction force of the jth connection d.o.f. and $\mathbf{H}_{ij}^A$ the frequency response of the ith d.o.f. when a unit force excites the jth d.o.f. For substructure B, we can write the displacements of the connection d.o.f. as

$$\mathbf{x}_i^B = -\mathbf{H}_{ij}^B \mathbf{R}_j + \mathbf{H}_{if}^B \mathbf{F}^B, \quad i = 1, \ldots, \mathbf{n}. \tag{2}$$

The directions of the reaction forces are reversed to be consistent with equation (1). Neglecting the airborne noise and excitations by the wind, the acoustic response at point $\mathbf{r}$ in substructure B, $\mathbf{p}_r^B$, is

$$\mathbf{p}_r^B = -\mathbf{H}_{ri}^B \mathbf{R}_i + \mathbf{H}_{rf}^B \mathbf{F}^B. \tag{3}$$

$\mathbf{H}^B_{ri}$ and $\mathbf{H}^B_{rf}$ are the noise transfer functions of the response d.o.f. when a unit force is exerted at the ith connection d.o.f. and the external excitation point respectively. Since the substructures are connected by a number of elastic springs and viscous dampers, the compatibility conditions between the substructures along the interface give us the following relations:

$$\mathbf{H}^I_{ij}\mathbf{R}_j = \mathbf{x}^B_i - \mathbf{x}^A_i, \quad i = 1, \ldots, n, \tag{4}$$

where

$$\mathbf{H}^I_{ij} = \begin{cases} \dfrac{1}{(k_i + \sqrt{-1}\omega C_i)} & \text{if } i = j, \\ 0 & \text{if } i \neq j \end{cases}$$

and ω is the angular frequency.

The reaction forces are derived by the substitution of equations (1) and (2) into equation (4):

$$\mathbf{R}_i = \mathbf{D}^{-1}_{ij}(\mathbf{H}^B_{jf}\mathbf{F}^B - \mathbf{H}^A_{jf}\mathbf{F}^A), \quad i = 1, \ldots, n, \tag{5}$$

where

$$\mathbf{D}_{ij} = \mathbf{H}^I_{ij} + \mathbf{H}^A_{ij} + \mathbf{H}^B_{ij}. \tag{6}$$

Finally, the acoustic response in substructure B can be obtained by substituting equation (5) into equation (3) as follows:

$$\mathbf{p}^B_r = \mathbf{H}^B_{ri}\mathbf{D}^{-1}_{ij}(\mathbf{H}^A_{jf}\mathbf{F}^A - \mathbf{H}^B_{jf}\mathbf{F}^B) + \mathbf{H}^B_{rf}\mathbf{F}^B. \tag{7}$$

Using equation (7), we can predict the interior noise level in a passenger car from the frequency response functions of the subsystems and the interface conditions.

To obtain the variation of the acoustic response function due to a design change, we need the gradient information with respect to the design variable, which comes from the design sensitivity analysis. The first step of design sensitivity analysis is to express the variation of response as a function of the design change. Differentiating equation (3) with respect to the design variable gives

$$\frac{\mathrm{d}\mathbf{p}^B_r}{\mathrm{d}b} = -\frac{\partial \mathbf{H}^B_{ri}}{\partial b}\cdot\mathbf{R}_i - \mathbf{H}^B_{ri}\cdot\frac{\partial \mathbf{R}_i}{\partial b} + \frac{\partial \mathbf{H}^B_{rf}}{\partial b}\cdot\mathbf{F}^B + \mathbf{H}^B_{rf}\cdot\frac{\partial \mathbf{F}^B}{\partial b}. \tag{8}$$

Here, b is the design variable such as the stiffness or the damping coefficient of the connection element. In equation (8), the first and the third terms on the right-hand side can be computed by the conventional formulations [13–17]. However, the second term, $\partial\mathbf{R}_i/\partial b$, is not explicit to the design change because the reaction forces are determined by the dynamic characteristics of the assembled system. To obtain an explicit expression of $\partial\mathbf{R}_i/\partial b$, we start from equation (5). Multiplication of $\mathbf{D}_{ij}$ on each side of equation (5) results in

$$\mathbf{D}_{ij}\mathbf{R}_j = -\mathbf{H}^A_{if}\mathbf{F}^A + \mathbf{H}^B_{if}\mathbf{F}^B. \tag{9}$$

By differentiating the above equation with respect to the design variable, equation (9) can be written as

$$\mathbf{D}_{ij}\cdot\frac{\partial \mathbf{R}_j}{\partial b} = -\frac{\partial \mathbf{D}_{ij}}{\partial b}\cdot\mathbf{R}_j - \frac{\partial \mathbf{H}^A_{if}}{\partial b}\cdot\mathbf{F}^A - \mathbf{H}^A_{if}\cdot\frac{\partial \mathbf{F}^A}{\partial b} + \frac{\partial \mathbf{H}^B_{if}}{\partial b}\cdot\mathbf{F}^B + \mathbf{H}^B_{if}\cdot\frac{\partial \mathbf{F}^B}{\partial b}. \tag{10}$$

Since all terms on the right-hand side of equation (10) are known functions, we can solve linear algebra equations to obtain the variation of the reaction forces with respect to the design variable. Note that there is no additional cost to obtain the inverse system matrix, $\mathbf{D}_{ij}^{-1}$, since we already obtained it during the calculation of responses in equation (5). Assuming that the external force is independent of the design variable, we can rewrite the noise sensitivity formula by replacing $\partial\mathbf{R}_i/\partial b$ in equation (8) with equation (10):

$$\frac{\mathrm{d}\mathbf{p}^B_r}{\mathrm{d}b} = -\frac{\partial \mathbf{H}^B_{ri}}{\partial b}\,\mathbf{R}_i + \mathbf{H}^B_{ri}\mathbf{D}_{ij}^{-1}\left\{\frac{\partial \mathbf{D}_{jk}}{\partial b}\,\mathbf{R}_k + \frac{\partial \mathbf{H}^A_{jf}}{\partial b}\,\mathbf{F}^A - \frac{\partial \mathbf{H}^B_{jf}}{\partial b}\,\mathbf{F}^B\right\} + \frac{\partial \mathbf{H}^B_{rf}}{\partial b}\,\mathbf{F}^B. \tag{11}$$

To obtain the sensitivity information, the terms $\partial\mathbf{D}_{jk}/\partial b$, $\partial\mathbf{H}^A_{jf}/\partial b$, $\partial\mathbf{H}^B_{jf}/\partial b$, $\partial\mathbf{H}^B_{ri}/\partial b$, $\partial\mathbf{H}^B_{rf}/\partial b$ need to be calculated and several matrix multiplications need to be performed. Note that the change of a design variable in one subsystem will not affect the dynamic characteristics of the others in the substructuring approach, which makes many terms zero in the noise sensitivity formula. This feature makes the design sensitivity formulation efficient.

Frequently, the properties of the connection element, i.e. the stiffnesses and damping coefficients of the engine mounts and bushings, have a significant influence on the noise and vibration problems of a passenger car. In this case, $\partial\mathbf{D}_{ij}/\partial b$ is equal to $\partial\mathbf{H}^I_{ij}/\partial b$, which can be evaluated by the differentiation of the analytic expression of $\mathbf{H}^I_{ij}$, equation (4). All of the other derivative terms in equation (11) are equal to zero. A typical example of this case is the optimization of an automotive engine mount system, which will be treated in the example that follows.

Note that it is not difficult to expand the proposed formulation to general complex systems. We can obtain the sensitivity information for a system with multiple substructures, excitations or responses, even though we considered a relatively simple structure in the derivation.

3. DESIGN SENSITIVITY ANALYSIS OF AN ENGINE MOUNT SYSTEM

The interior noise level in a passenger car can be predicted by the FRF-based substructuring method. To analyze the structure-borne noise in the cavity of a car due to engine excitation, the car is divided into two substructures. Substructure A contains the powertrain and the sub-frame, and substructure B is a trimmed body structure including the cabin cavity. The excitation forces come from the engine due to explosion and unbalance of rotary parts. The response concerned is the sound pressure level at the passenger's ear position. Two engine mounts and five rubber elements connect two substructures as shown in Figure 2. The sensitivities of the interior noise level with respect to the properties of engine mounts and bushings are very important when considering the interior noise level.

The FRF-based substructuring method can use the experimental or the analytical data depending on the characteristics of the substructures. To obtain the FRFs of substructure A, we used a finite element analysis. Through the frequency response analysis in MSC/NASTRAN, the FRFs are calculated. For substructure B, the noise transfer functions

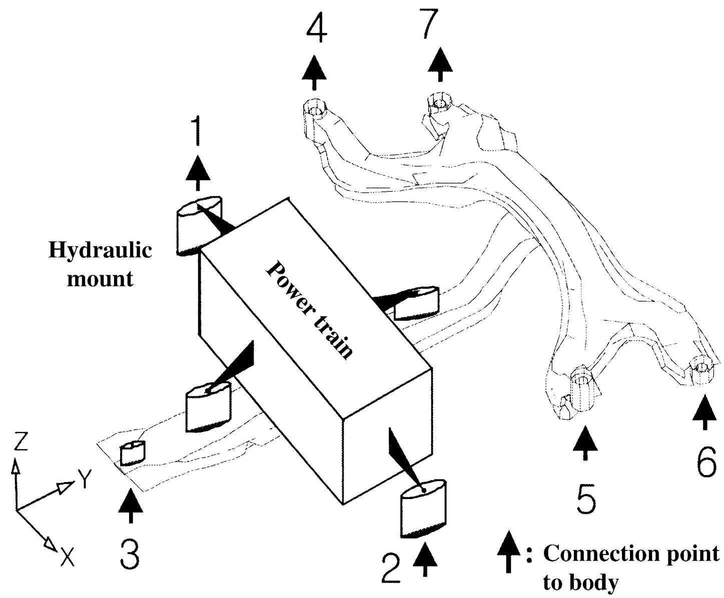

Figure 2. The engine mount system of a passenger car.

from the connection point to the passenger's ear position are measured experimentally by the impact hammer test. In general, rubber elements show non-linear characteristics according to load and preload, which should be considered in the interior noise analysis. Especially, the stiffness and damping coefficients of the hydraulic engine mount are highly dependent on these factors. This feature causes the engine mount problem to be a non-linear modal analysis problem, which is one of the barriers for the modal superposition approach. In this research, we use the stiffnesses and damping coefficients of the connection elements experimentally measured by the elastomer tester. To identify the engine forces, combustion pressures with respect to angle of crank–shaft in a cylinder are measured and converted to external force acting on the engine block.

Figure 10 shows the sound pressure level at the ear position calculated by the FRF-based substructuring method. The interior sound pressure level shows a high peak around 1800 r.p.m., which is identified as a structure-borne noise due to the engine excitation. To investigate the influences of the engine mounts and sub-frame bushings on the interior sound pressure level systematically, the proposed noise sensitivity analysis method is applied. A target response is the interior sound pressure at the passenger's ear position. The design variables are the stiffness and the damping coefficients of each engine mount and bushing between substructures A and B. There are six design variables at each connection point because the three orthogonal directions must be counted separately. There are 42 total design variables since there are seven connecting points between the two substructures as shown in Figure 2.

Figures 3–5 show the sensitivities of the interior noise level with respect to the stiffnesses of each connection element. Note here that the relative phase of the noise sensitivity to the interior noise as well as the magnitude of the noise sensitivity are important because the overall change of the acoustic response is the vector sum of the sound pressure level and the noise sensitivity multiplied by the design change. As seen in Figures 3–5, the interior sound pressure level is very sensitive to the stiffness changes at connection point 3 in all directions. The next most sensitive design variables are the stiffnesses of connection points 6 and 7.

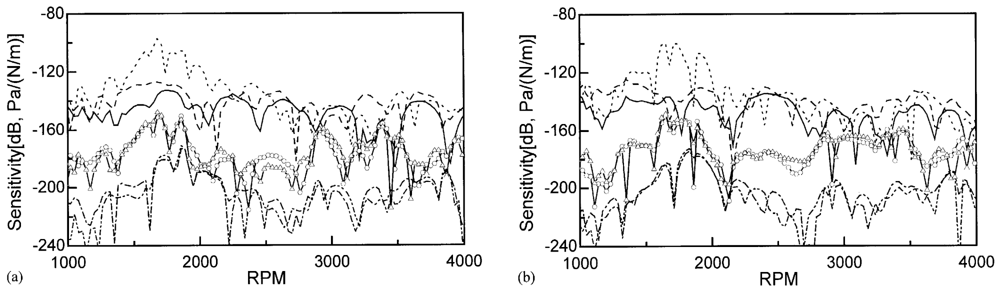

Figure 3. Noise sensitivity results w.r.t. the stiffness in the x direction: ——, no. 1; - - - - - -, no. 2; ········, no. 3; –·–·–·–, no. 4; –··–··–, no. 5; —○—, no. 6; —△—, no. 7. (a) Real part, (b) imaginary part.

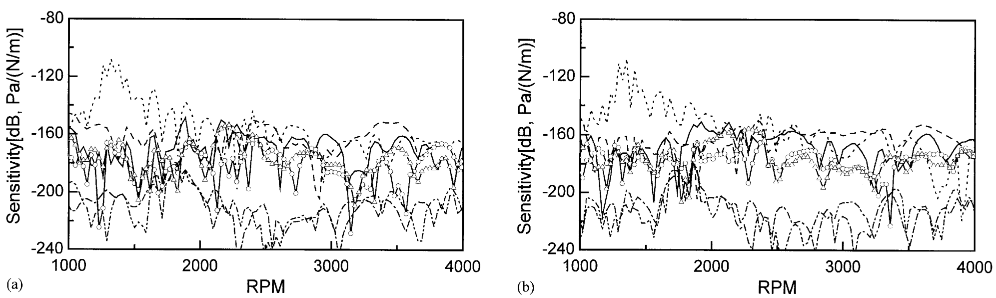

Figure 4. Noise sensitivity results w.r.t. the stiffness in the y direction: ——, no. 1; - - - - - -, no. 2; ········, no. 3; –·–·–·–, no. 4; –··–··–, no. 5; —○—, no. 6; —△—, no. 7. (a) Real part, (b) imaginary part.

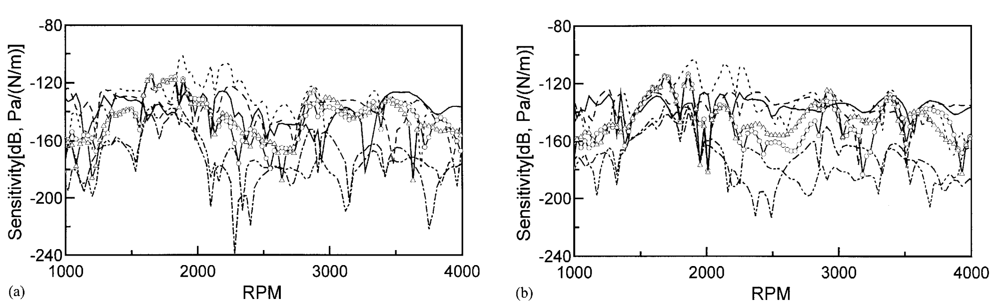

Figure 5. Noise sensitivity results w.r.t. the stiffness in the z direction: ——, no. 1; - - - - - -, no. 2; ········, no. 3; –·–·–·–, no. 4; –··–··–, no. 5; —○—, no. 6; —△—, no. 7. (a) Real part, (b) imaginary part.

Figure 6 compares the calculated design sensitivities to those found by the forward finite difference method for all seven points. In the finite difference method, the accuracy of the result depends on perturbation step size and a procedure to choose the finite difference interval is recommended by Gill *et al.* [18]. For the finite difference method, each design variable is perturbed by 0·01% of the present value. Although the design sensitivities are complex values, only their magnitudes are compared for brevity. The comparison shows excellent agreement between the two methods and the maximum errors are less than 0·1%. In Figures 6(b) and 6(c), the errors of connection point three are relatively large compared with the other points over some frequency range. This is due to the fact that the amount of perturbation is constant with respect to the current value, while the relative magnitude of

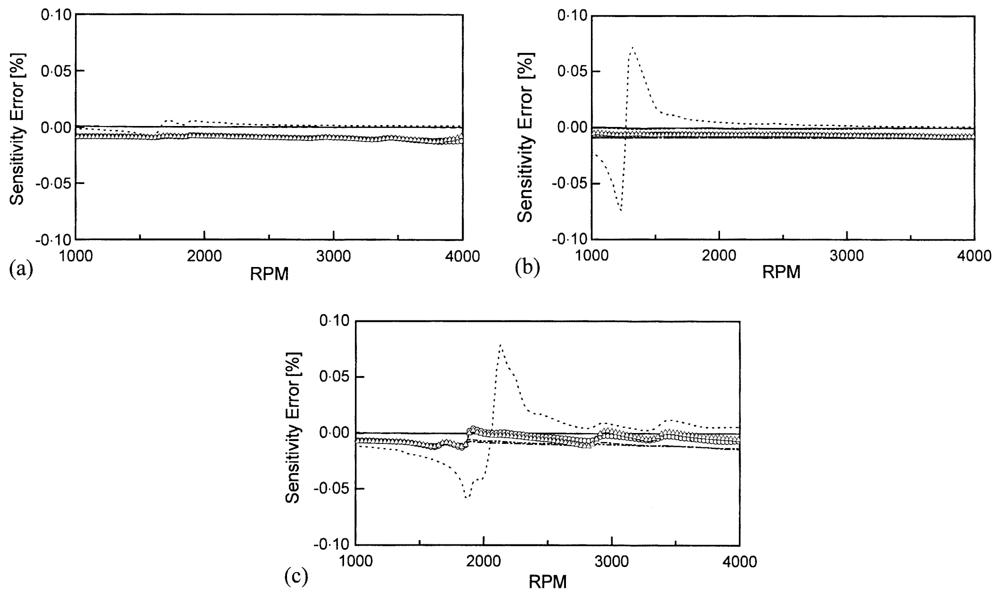

Figure 6. Errors of noise sensitivity w.r.t. the stiffness compared to those of finite difference method: ——, no. 1; - - - - - -, no. 2; ········, no. 3; – · – · – · –, no. 4; – · · – · · –, no. 5; —O—, no. 6; —△—, no. 7. (a) x direction, (b) y direction, (c) z direction.

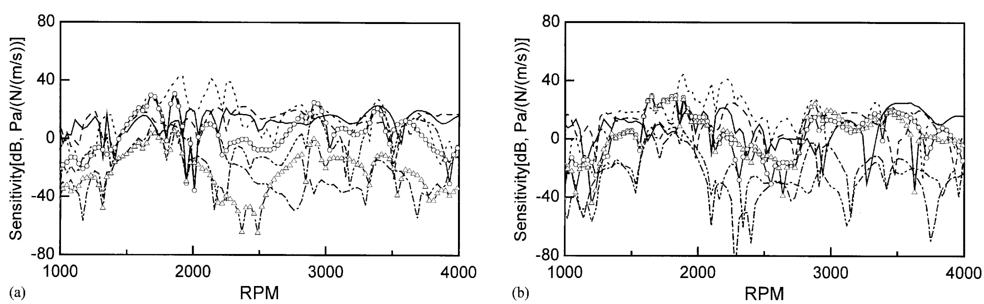

Figure 7. Noise sensitivity results w.r.t. the damping coefficients in the z direction: ——, no. 1; - - - - - -, no. 2; ········, no. 3; – · – · – · –, no. 4; – · · – · · –, no. 5; —O—, no. 6; —△—, no. 7. (a) Real part, (b) imaginary part.

the noise sensitivity is very large compared with the others over that frequency range as shown in Figures 4 and 5.

Figure 7 shows the sensitivity results with respect to the damping coefficients of the connection elements. The sensitivity results of the damping coefficients look very similar to those of the stiffnesses except the absolute magnitude and the phase of the sensitivity vector. Equation (4) predicts that this would occur. In Figure 8, the sensitivities from the proposed method and the finite difference method are compared. The sensitivities calculated by the proposed method show good accuracy and the maximum error of the damping coefficient sensitivities is less than 0·01%. This shows that the proposed method can calculate the sensitivities in a simple and easy way even for a complicated real problem.

Note also that the presented sensitivity formulation has advantages in accuracy and computational speed whereas the finite difference method is simple and straightforward in

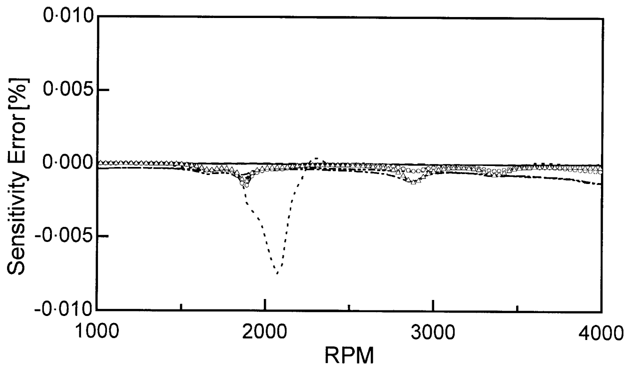

Figure 8. Errors of noise sensitivity w.r.t. the damping coefficients compared to those of the finite difference method: ——, no. 1; -----, no. 2; ·······, no. 3; –·–·–·–, no. 4; –··–··–, no. 5; —○—, no. 6; —△—, no. 7.

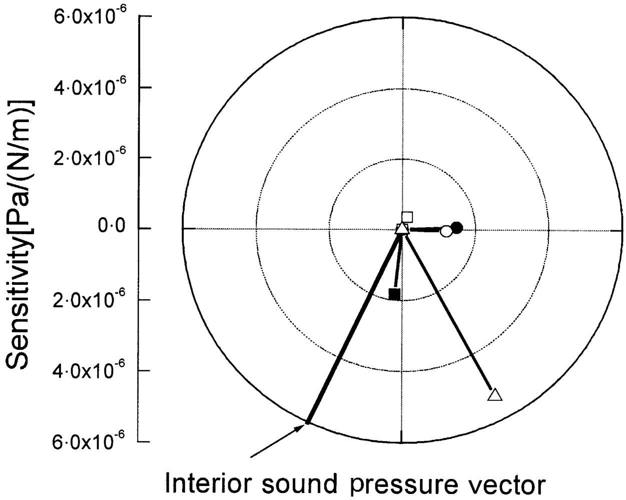

Figure 9. Polar plot of the sensitivity w.r.t. stiffness at 1800 r.p.m.: —□—, no. 2(x); —△—, no. 3(x); —■—, no. 3(z); —●—, no. 6(z); —○—, no. 7(z).

implementation. Accuracy in the finite difference method depends on the perturbation size and this can cause a slow convergence problem near optimum [18]. However, the sensitivity formula in closed form always gives accurate results. Furthermore, the present formulation is faster than the finite difference schemes because in the present formulation the system matrix for the sensitivities is the same matrix used for response calculation. Therefore, the additional cost for the sensitivities is to calculate the right-hand-side vectors. Obtaining the right-hand-side vectors, the sensitivities can be computed by backward substitutions to the system matrix LU decomposed in response calculation step. This feature makes the sensitivity calculation very efficient. As an example, cpu times to calculate the stiffness sensitivities by the present formulation and the forward finite difference method were 29·9 and 57·8 s on a Pentium III PC, respectively, which shows 48·3% decrease of computation time.

In order to identify the design variables that have a large influence on the interior noise for the peak around 1800 r.p.m., the sensitivity vectors at 1800 r.p.m. are plotted as shown in Figure 9. In the sensitivity polar plot, the direction as well as the magnitude of the sensitivity

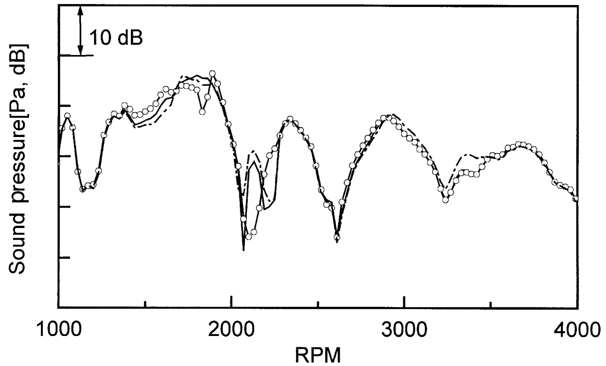

Figure 10. Interior sound pressure levels synthesized by the FRF-based substructuring method: ——, baseline; —O—, no. 3 + 15%; –·–·–, no. 6, 7 -- 15%.

is very important since the expected amount of variation due to the design change is proportional to the projected magnitude of the sensitivity vector to the direction of the response vector. As shown in Figure 5, the sensitivities with respect to the stiffness of elements 6 and 7 are large in magnitude, but the directions of the vectors are nearly perpendicular to the response vector. This means that the stiffness changes of these elements will have a small effect on the response. On the contrary, the sensitivities with respect to the stiffness of element 3 in z direction has a similar magnitude but the direction is close to the response vector, which means that the design change at this element can modify the response effectively. To verify these results, the interior noise levels are calculated when the stiffness of element 3 is increased by 15%, and also when the stiffness of elements 6 and 7 are decreased by 15% in all directions. Figure 10 shows the interior noise level when the modifications are applied. As expected, the noise level does not decrease very much by reducing the stiffness of points 6 and 7. However, the increase in stiffness at point 3 has a more significant effect on the noise level, just as the sensitivity analysis predicted.

4. OPTIMIZATION OF AN ENGINE MOUNT SYSTEM

The following section applies the design sensitivity formulation presented in the previous section to systematically optimize the engine mount system of a passenger car.

The characteristics of the connection elements connecting two substructures are the design variables of the present optimization problem. Two engine mounts and five bushings connect two substructures as shown in Figure 2. In this problem, the external forces such as the road-induced forces on substructure B are not considered because the problem is focused on the structure-borne noise due to powertrain vibration.

Figure 11 shows the interior noise of the initial design calculated by the FRF-based substructuring method. The interior noise level from the initial design has several peaks as shown in Figure 11. In the optimization problem, the objective function should represent the design purpose correctly. The design purpose in this problem is to minimize the interior noise level over a fixed r.p.m. range. We define the objective function as the area between the interior noise level in the decibel scale and the x-axis. To emphasize the high-level peaks in the objective function, only the area between the noise level and the target level is considered as shown in Figure 11. Therefore the objective function is written as

$$f(\mathbf{b}) = \int_{\omega_l}^{\omega_u} h(\omega)\langle p^* \rangle p^* \, \mathrm{d}\omega, \tag{12}$$

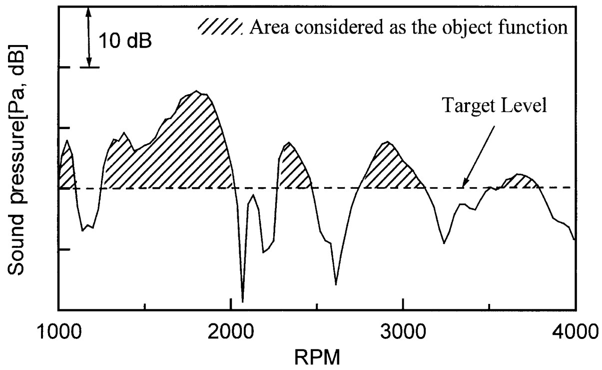

Figure 11. Sound pressure level of the cabin cavity at the initial design and the definition of the objective function.

where

$$p^{*} = 20\log_{10}\frac{\sqrt{(\mathrm{Re}(\mathbf{p}(\omega)))^2 + (\mathrm{Imag}(\mathbf{p}(\omega)))^2}}{P_{ref}} - P_{target}.$$

In these equations, $\mathbf{b}$ is the design vector and f is the objective function. $\langle\ \rangle$ is a step function, which is one for the positive argument and zero for the negative argument. P_{target} is the target value of the interior noise level in the decibel scale, P_{ref} 2·0E-5 Pa, ω the frequency and h a user-defined weighting function. Re and Imag refer to the real and imaginary values respectively.

In the design stage of the engine mount system, the stiffness coefficients of the engine mounts and bushings are the main design variables. The control of the stiffness coefficients of the rubber parts is relatively easy, since their size and shape determine their stiffness. On the contrary, the damping coefficients are not good candidates for the design variables since they are only dependent on the material itself. In this example, the design variables are the coefficients of the stiffness at the connection elements. Since the connection elements can have different stiffness in three orthogonal directions, 21 design variables are defined for the engine mount system. In addition, bushings 4, 5, 6 and 7 are assumed to have axisymmetric properties for ease of manufacturing so that the individual stiffnesses in the x direction are equal to those in the y direction.

In summary, the design objective of the engine mount optimization problem is to determine the stiffnesses of the engine mounts and bushings such that the objective function is minimized under the constraints of axisymmetry and the spatial limits of the design variables. Finally, a design optimization problem is defined as follows:

$$\begin{aligned}
&\text{Find } \mathbf{b} \text{ such that} \\
&\text{minimize} && f(\mathbf{b}) \\
&\text{subjected to} && b(\mathbf{I}(k)) = b(\mathbf{J}(k)), \quad k = 1, \ldots, n, \\
& && b_L \leqslant b(i) \leqslant b_U, \quad i = 1, \ldots, m.
\end{aligned} \tag{13}$$

In this equation, b_L and b_U are the lower and upper bounds of the design variables respectively. Here $\mathbf{I} = \{10\ \ 13\ \ 16\ \ 19\}$, $\mathbf{J} = \{11\ \ 14\ \ 17\ \ 20\}$, and m and n are 21 and 4 respectively. The gradient of the objective function is obtained from equation (12) analytically as

$$\frac{\partial f}{\partial \mathbf{b}} = C^* \int_{\omega_L}^{\omega_U} h(\omega)\langle \mathbf{p}^* \rangle \left[\frac{\mathrm{Re}(\mathbf{p}(\omega))\mathrm{Re}(\partial \mathbf{p}/\partial \mathbf{b}) + \mathrm{Imag}(\mathbf{p}(\omega))\,\mathrm{Imag}(\partial \mathbf{p}/\partial \mathbf{b})}{\|\mathbf{p}\|^2} \right] \mathrm{d}\omega. \tag{14}$$

Here, $C^* = 20\log_{10} e$ and $\| \; \|$ refers to the magnitude of the vector. The design sensitivities of the interior noise with respect to the design variables, $\partial \mathbf{p}/\partial \mathbf{b}$ in the above equation are calculated by the presented design sensitivity formula, equation (11). The sensitivity information of the other constraints is obtained by direct differentiation of the analytic equation.

To obtain the optimum design of the engine mount system in equation (13), the proposed design sensitivity analysis method is implemented with the commercial optimization software, MATLAB and its optimization toolbox function, constr [19]. The current stiffness values normalize the design variables and the upper and lower bounds are 1·3 and 0·7 respectively. To compute the interior noise of the passenger car during iterations, the model from the previous section is used.

The optimum design for the reduction of noise is calculated over the speed range of 1000–4000 r.p.m. Two different optimization cases are introduced. For the first case (CASE I), the weighting function is 1·0 over the entire concerned speed range. For the second case (CASE II), the weighting function is 1·0 for speeds between 1000 and 1100 r.p.m., 100·0 for the range from 1100 to 2200 r.p.m. and 1·0 from 2200 to 4000 r.p.m. This method was used to concentrate on reduction of the greatest peak around 1800 r.p.m. To verify the sensitivity formulation in section 2, the sensitivity results of the objective

TABLE 1

Design sensitivity result of the objective function for the initial design parameter values ($f = 385{\cdot}80$)

Design variable no	Present method f'	FDM $\Delta f/\Delta \mathbf{b}$	Ratio (%) $(f'/\Delta f/\Delta \mathbf{b}) \times 100$
1	3·9759E + 00	3·9760E + 00	100·00
2	6·8000E − 01	6·8010E − 01	99·99
3	4·5367E + 01	4·5361E + 01	100·01
4	2·5028E + 01	2·5029E + 01	100·00
5	4·6973E − 01	4·6976E − 01	99·99
6	7·9448E + 01	7·9434E + 01	100·02
7	1·2608E + 01	1·2598E + 01	100·08
8	1·0092E + 01	1·0011E + 01	100·82
9	5·7427E + 01	5·7181E + 01	100·43
10	− 2·5340E − 01	− 2·5400E − 01	99·76
11	− 3·1899E − 01	− 3·1881E − 01	100·06
12	− 3·0087E + 00	− 3·0135E + 00	99·84
13	− 6·4258E − 01	− 6·4281E − 01	99·96
14	− 4·4085E − 01	− 4·4059E − 01	100·06
15	− 1·1003E + 01	− 1·1001E + 01	100·02
16	4·0691E − 01	4·0391E − 01	100·74
17	3·6530E + 00	3·6496E + 00	100·09
18	− 5·0292E + 01	− 5·0304E + 01	99·98
19	9·4720E − 01	9·4261E − 01	100·49
20	− 8·5042E + 00	− 8·4993E + 00	100·06
21	− 8·7769E + 00	− 8·8101E + 00	99·62

function for the first case are compared with those from the forward finite difference method. For the finite difference method, the amount of perturbation is 0·1% of the current design value. Cpu times to calculate the sensitivities using the present formulation and the finite difference method were 28·7 and 57·1 s, respectively. The two results, compared in Table 1, show very good agreement, which proves that the presented design sensitivity formula is correct and accurate as well as efficient. Figure 12 shows the interior noise levels of the optimum design. The effect of the weight function is shown well in Figure 12 although the noise levels minimized with different weight functions around 1800 r.p.m. are almost identical due to the upper and lower bounds of the design variables. Note also that the highest peak around 1800 r.p.m. at the optimum design has decreased by more than 3 dB compared to the initial design. The cost function for the first case has been reduced from 385·8 to 300·2 in 29 iterations and that for the second case from 2637 to 2001 in 34 iterations. The objective functions decrease by 22·2 and 24·10% respectively.

5. CONCLUSION

The sensitivity analysis is very useful in the design or trouble shooting of vibro-acoustic phenomena. For large and complex structures such as full vehicles, the FRF-based substructuring approach is known as one of the most powerful tools in analyzing the system response. However, little attention has been paid to the design sensitivity analysis in the framework of substructuring methods until now.

An efficient formulation for the design sensitivity analysis of the vibro-acoustic problems using an FRF-based substructuring formulation has been presented. The present methods can guide a systematic design when analyzing structure-borne noise by the FRF-based substructuring method. For an acoustic response function, the proposed method gives a parametric design sensitivity formula in terms of the partial derivatives of the connection element properties and the transfer matrices of the subsystems. The design sensitivities of the interior noise with respect to the engine mounts and bushings in an engine mount system are calculated by the present method and compared with those obtained using the finite difference method. The comparison shows that the proposed method can calculate accurate design sensitivities efficiently.

The derived noise sensitivity formula combined with non-linear programming software gives the optimal design for the engine mount system of a passenger car. The objective function is the area of the interior noise graph integrated over the concerned r.p.m. range.

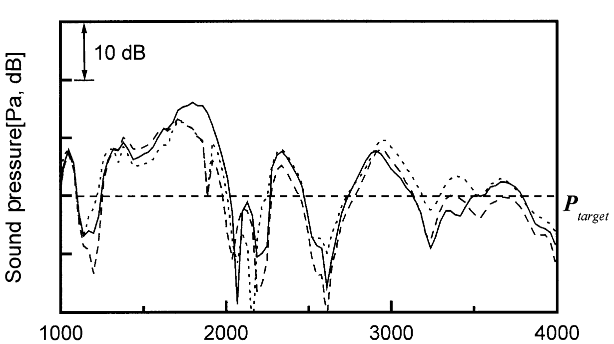

Figure 12. Sound pressure levels of the cabin cavity at the optimum designs: ——, Baseline; - - - -, optimized-CASE I; ··········, optimized-CASE II.

The interior noise variations with respect to the stiffness changes of the engine mounts and bushings have been calculated using the proposed sensitivity formulation and transferred to the non-linear optimization software to obtain the optimal stiffnesses of the engine mounts and bushings. The highest peak around 1800 r.p.m. at the optimum design has lowered by more than 3 dB compared to the initial design. The optimal design based on the proposed sensitivity formulation can be a convenient tool when determining the characteristics of the engine mount systems in the system level design.

In the present analysis, the properties of the connecting elements in a substructure, for example, two engine mounts in substructure A, cannot be used as the design variables since they are parts of a substructure. The property of any mount can become a design variable through the extension of the present formulation to multiple substructure systems although the extension needs complex programming efforts.

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