Journal of Sound and Vibration (2002) **255**(5), 899–913
doi:10.1006/jsvi.2001.4193, available online at http://www.idealibrary.com on IDEAL®

OPTIMIZATION OF SIMPLIFIED MODELS MESHED WITH FINITE TRIANGULAR PLATE ELEMENTS

S. Michot, J. Piranda and F. Trivaudey

Laboratoire de Mécanique Appliquée R. Chaléat, U.F.R. Sciences et Techniques,
24 *Chemin de l'Epitaphe*, 25000 *Besançon, France. E-mail: severine_michot@etudes.univ-fcomte.fr*

(*Received* 3 *August* 1998, *and in final form* 20 *September* 2001)

Designers often want to analyze more and more sophisticated structures, thus leading to very large finite element models (typically 10 00 000 degrees of freedom for a body car, for example). These models being too costly for the early stages of design and optimization can be reduced by a substructure analysis or a mesh simplification of the components. A methodology is proposed in this paper for simplifying finite triangular plate element models leading to a dramatic reduction in the number of degrees of freedom while preserving the dynamical properties of the initial system. In particular, the proposed method is developed for models composed of the plate element STIFF63 generated by the software ANSYS. The principle consists in determining the parameters (thickness, Young's modulus, density) of the triangular elements of a coarse model which replaces a large set of elements of the refined model. The simplified mesh must satisfy one of two criteria. The first requires that the mass and stiffness matrices of the simplified model be as close as possible to the Guyan condensed matrices of the refined model on the reduced node set, whilst the second requires that the dynamical properties of the global structure be preserved. The application of these approaches is illustrated on two test structures using the gradient method to solve the resulting optimization problem. The second approach is shown to give the best results. Typically, the size of the models can be reduced by a factor of 20 whilst preserving the dynamical properties of the structure at low frequencies.

1. INTRODUCTION

The recent evolution of computer-aided design (CAD) technology allows the engineer to model very complex and expensive structures which are subjected to increasingly strong constraints. These refined models are needed to optimize structural behaviour and demand very powerful modelling tools.

The finite element method is currently the most popular approach used to model structures and a wide variety of general finite element software coupled with CAD can be found in the domain of solid mechanics, for example NASTRAN and ANSYS.

However, the number of degrees of freedom (d.o.f.) and consequently the computational effort increases dramatically with the precision in the representation of the structure topology. The objective of model simplification is to reduce the number of active d.o.f. while preserving dynamical properties of the initial model in a given frequency range.

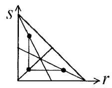
2. ELEMENT STIFFNESS AND MASS MATRICES FOR THE ANSYS PLATE ELEMENT STIFF63

The proposed methods are based on the knowledge of the formal expressions for the stiffness and mass matrices of the plate element STIFF63 of ANSYS. Hence, these matrices

0022-460X/02/$35.00

TABLE 1

Integration points and corresponding weighting used by ANSYS

Integration area	No. of points (n)	Co-ordinates (r_i, s_i)	Weighting (w_i)
	3	(1/6, 1/6) (2/3, 1/6) (1/6, 2/3)	1/6 1/6 1/6

are formulated in order to obtain, using the formal expressions, exactly the same results as those given by ANSYS.

A three-node plate element limited to the bending effect (three d.o.f. per node: w, θ_x, and θ_y, where w is the transversal displacement along Z, and θ_x and θ_y are the rotations around the X- and Y-axis) is used.

For a given element, the stiffness and mass matrices can be expressed as

$$\textit{stiffness matrix:} \qquad \mathbf{K} = \int_V B^{\mathrm{T}} D B \, \mathrm{d}V; \tag{1}$$

$$\textit{mass matrix:} \qquad \mathbf{M} = \int_V \rho N^{\mathrm{T}} N \, \mathrm{d}V, \tag{2}$$

where D is defined from the behaviour law ($\boldsymbol{\sigma} = D\boldsymbol{\varepsilon}$), $\mathbf{B}$ is such that $\boldsymbol{\varepsilon} = B\{\mathbf{u_n}\}$, N is such that $\mathbf{u} = N\{\mathbf{u_n}\}$, $\{\mathbf{u_n}\}$ is the elementary vector of the nodal variables, and $\boldsymbol{\sigma}$ is the stress tensor and $\boldsymbol{\varepsilon}$ the strain tensor.

2.1. STIFFNESS MATRIX

Assuming a plane stress state leads to

$$D = \frac{Eh^3}{12(1-v^2)} \begin{pmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (1-v)/2 \end{pmatrix}. \tag{3}$$

The discrete Kirchhoff triangle (DKT) triangle formulation is used to obtain the expression of $\mathbf{B}$. It is based on the Love–Kirchhoff hypothesis used for thin plates, which consists in neglecting transverse shear strains [1].

The elementary stiffness matrix $\mathbf{K}$ is then calculated with a three-point Hammer integration, which is the numerical integration method used by ANSYS [2].

$\mathbf{K} = \int_{r=0}^{1}\int_{s=0}^{1-r} hB^{\mathrm{T}}DB \det J \, \mathrm{d}r \, \mathrm{d}s$ is then replaced by $\mathbf{K} = \sum_{i=1}^{n} w_i f(r_i, s_i)$ where f represents the function to integrate and h the thickness at any point of the plate ($h = \sum_{i=1}^{3} N_i h_i$).

Table 1 reports the integration points and corresponding weighting used by ANSYS.

2.2. MASS MATRIX

For the mass matrix, the calculation of the N matrix is performed according to the Zienkiewicz formulation [3]. Let ABC denote the triangular plate element to be modelled.

Using the co-ordinate vertices (x_i, y_i),

$$L_1 = \tfrac{1}{2}((y_2 - y_3)(x - x_2) - (x_2 - x_3)(y - y_2))/\text{area}(ABC),$$

$$L_2 = \tfrac{1}{2}((y_3 - y_1)(x - x_3) - (x_3 - x_1)(y - y_3))/\text{area}(ABC),$$

$$L_3 = \tfrac{1}{2}((y_1 - y_2)(x - x_1) - (x_1 - x_2)(y - y_1))/\text{area}(ABC),$$

$$b_1 = y_2 - y_3, \quad c_1 = x_2 - x_3, \quad b_2 = y_3 - y_1,$$

$$c_2 = x_3 - x_1, \quad b_3 = y_1 - y_2, \quad c_3 = x_1 - x_2. \tag{4}$$

The matrix **N** is then given by

$$\mathbf{N} = \begin{bmatrix} L_i(1 - L_j^2 - L_k^2) + L_i^2(L_j + L_k) \\ - b_k L_i L_j (L_i + L_k/2 + b_j L_i L_k (L_i + L_j/2)) \\ - c_k L_i L_j (L_i + L_k/2 + c_j L_i L_k (L_i + L_j/2)) \end{bmatrix}^{\mathrm{T}}, \quad i = 1, 3. \tag{5}$$

The previously defined geometrical transformation is used and, as with the stiffness matrix, the elementary mass matrix **M** is calculated with a three node Hammer integration.

Hence, $\mathbf{M} = \int_{r=0}^{1}\int_{s=0}^{1-r} \rho h N^{\mathrm{T}} N \det J \, \mathrm{d}r \, \mathrm{d}s$ is replaced by $\mathbf{M} = \sum_{i=1}^{n} w_i f(r_i, s_i)$ where f represents the function to integrate and h the thickness at any point of the plate $(h = \sum_{i=1}^{3} N_i h_i)$.

The integration points and corresponding weighting used by ANSYS are the same as those used for the stiffness matrix.

3. PRINCIPLES FOR MODEL SIMPLIFICATION

To simplify a finite element model, it is necessary to choose (1) the parameters of the simplified model to vary in order to approximate the finely meshed model, (2) the cost function that represents the distance between the simplified model and the refined model and (3) the optimization method used to minimize the cost function.

The two principles which have been implemented to optimize simplified finite plate element models use the gradient method.

3.1. THE GRADIENT METHOD

The gradient method consists of minimizing, in a least-squares sense, a cost function of several variables. After convergence, the values of the variables minimizing the function are obtained.

Let **p** be a vector containing the m variables, p^0 the initial value of **p**, **dp** the vector containing the corrections of the parameters, $\mathbf{f}(p)$ the vector of the functions to be minimized.

One can expand **f** in the following manner:

$$\mathbf{f}(p) = \mathbf{f}(p^0) + \sum_{i=1}^{m} \frac{\partial \mathbf{f}}{\partial \mathbf{p}_i}(p^0)\,\mathrm{d}\mathbf{p}_i + \cdots, \tag{6}$$

where $\mathbf{p} = p^0 + \mathbf{dp}$.

Consider the increment **dp** such that $\mathbf{f}(p^0) = -\sum_{i=1}^{m} \partial \mathbf{f}/\partial p_i (p^0)\,\mathbf{d}p_i$.

If $\mathbf{dp}$ is small, then $\mathbf{f}(\mathbf{p} = p^0 + \mathbf{dp})$ will be close to 0. Using a first order approximation from the Taylor expansion, p^0 needs to be chosen close to the solution, which is not usually obvious. So an iterative process must be applied to obtain a solution minimizing $\mathbf{f}$ (it may be a local minimum). Three design parameters corresponding to the thickness at the vertices are used giving

$$\left(\frac{\partial \mathbf{f}}{\partial \mathbf{p}_1}(p^0)\,\frac{\partial \mathbf{f}}{\partial \mathbf{p}_2}(p^0)\,\frac{\partial \mathbf{f}}{\partial \mathbf{p}_3}(p^0)\right)\begin{pmatrix}\mathbf{dp}_1\\ \mathbf{dp}_2\\ \mathbf{dp}_3\end{pmatrix} = -\,(\mathbf{f}(p^0)),$$

$$\mathbf{dF}\,.\,\mathbf{dp} = -\,\mathbf{F}, \qquad \mathbf{dp} = -\,\mathbf{dF}^{\dagger}.\mathbf{F}, \tag{7}$$

where $\mathbf{dF}^{\dagger}$ is the pseudo-inverse of the matrix $\mathbf{dF}$.

The corresponding algorithm is:

(1) initialization: $\mathbf{p} = \mathbf{p}^0$;
(2) computation of $\mathbf{dp} = -\,\mathbf{dF}^{\dagger}(p).\mathbf{F}(p)$, for the current value of $\mathbf{p}$;
(3) replacement of $\mathbf{p}$ by $\mathbf{p} + \mathbf{dp}$;
(4) test of convergence on $\mathbf{f}(p)$:
If $\mathbf{f}(p) < \varepsilon$, the algorithm stops: the optimal solution is p.
If $\mathbf{f}(p) > \varepsilon$, return to step 2.

This method has been used with two different objective functions. The two principles are presented now.

3.1.1. *Principle* 1

The first idea consists in determining the values of h which minimize the functions

$$(\mathbf{K}(h)_{ij} - \mathbf{K}_{G_{ij}})/\mathbf{K}_{G_{ij}}, \quad i = 1, \ldots, n, j = 1, \ldots, n,$$

where h represents all the parameters of the simplified model. The parameters are the thicknesses at each node of the simplified model; $\mathbf{K}(h)$ is the formal stiffness matrix of the simplified model established using MAPLE based on the ANSYS formulation (cf., section 2), and calculated with MATLAB. $\mathbf{K}_G$ is the Guyan condensed stiffness matrix [4] of the refined model; n is the number of d.o.f. of the simplified model and consequently the size of the matrices $\mathbf{K}(h)$ and $\mathbf{K}_G$.

The real structure is assumed to be finely meshed using a large number of STIFF63 elements from ANSYS. Then, the operations for identifying the parameters are performed using a MATLAB program:

(1) The simplified model is defined, selecting among the nodes of the refined model the nodes defining the super-elements of the simplified model.

(2) The matrices $\mathbf{K}_G$ (Guyan condensation of the global stiffness matrix on the selected master nodes) and $\mathbf{K}(h)$ (formal expression of the reduced matrix) are then calculated. The objective function is obtained using the norm of the vector $\mathbf{F}$ whose components are the functions previously defined. The optimal values of h are then obtained based on the gradient method. Constraints can be taken into account in order to ensure physically meaningful parameters.

(3) Once the optimal thicknesses are obtained, the mass density of the simplified model is corrected in order to respect the total mass of the refined model. The simplified model now has both stiffness and mass matrices which are considered to be “equivalent” to those of the

refined model. Therefore, its dynamic behaviour is expected to be as close as possible to that of the refined model.

(4) Finally, the simplified model is computed with ANSYS in order to verify its validity.

3.1.2. *Principle* 2

The second principle consists in determining the values of E (Young's modulus) and ρ (mass density) of each triangular element minimizing a cost function by considering the distances between the eigenfrequencies $(n((\lambda_v(p) - \lambda_v)/\lambda_v)100)$ and the eigenvectors $((\mathbf{y}_{vi}(\mathbf{p}) - \mathbf{y}_{vi})/\|\mathbf{y}_v\|100)$ of the refined and simplified models $(v = 1, \ldots, m,\ i = 1, \ldots, n)$, where $\mathbf{p}$ is the vector of the parameters (E and ρ) of each plate of the simplified model; λ_v and $\mathbf{y}_v$ are the eigenvalues and eigenvectors of the refined model, verifying the equilibrium equation: $(\mathbf{K}_{ANSYS} - \lambda_v \mathbf{M}_{ANSYS})\ \mathbf{z}_v = 0$, where $\mathbf{K}_{ANSYS}$ and $\mathbf{M}_{ANSYS}$ are the global stiffness and mass matrices of the ANSYS refined model leading to the eigenvalues λ_v and the associated eigenvectors $\mathbf{z}_v$. For comparison, $\mathbf{y}_v$ is the subvector of $\mathbf{z}_v$ restricted to the n d.o.f. of the simplified model; $\lambda_v(p)$ and $\mathbf{y}_v(p)$ are the eigenvalues and corresponding eigenvectors of the simplified model, such that $(\mathbf{K}(p) - \lambda_v(p)\mathbf{M}(p))\ \mathbf{y}_v(p) = 0$, $\mathbf{K}(p)$ and $\mathbf{M}(p)$ being calculated using the formal elementary matrices of the simplified model, calculated with MATLAB according to the ANSYS formulation defined in section 2; n is the size of $\mathbf{y}_v(p)$, corresponding to the number of d.o.f. of the simplified model; m is the number of modes taken into account in the cost function.

Moreover, to respect the rigid body mass properties of the structure, one adds a cost function which minimizes the distance between the mass and the moments of inertia of the refined and simplified models: $(\mathbf{v}(p) - \mathbf{v}/\|\mathbf{v}\|)\ 100$, with $\mathbf{v}(p) = \mathrm{diag}(\mathbf{R}_S^{\mathrm{T}}.\mathbf{M}(p)\mathbf{R}_S)$ and $\mathbf{v} = \mathrm{diag}(\mathbf{R}^{\mathrm{T}}\mathbf{M}_{ANSYS}\mathrm{R})$, where $\mathbf{M}_{ANSYS}$ is the global mass matrix of the ANSYS refined model; $\mathbf{M}(p)$ is the mass matrix of the simplified model; $\mathbf{R}$ is the rigid body matrix of the ANSYS refined model; $\mathbf{R}_S$ is the rigid body matrix of the simplified model.

As before, the structure is finely meshed by using ANSYS. The parameters are identified using the gradient minimization method. The procedure is:

(1) Define the simplified model by selecting among the nodes of the refined model those nodes of the triangular macroelements constituting the simplified model.

(2) Compute the first eigensolutions of the refined model in order to choose the number of modes to be taken into account in the identification procedure (choice of m). It is possible to take into account the dynamic behaviour obtained with different limit conditions: free, clamped, supported.

(3) Determine the parameters E and ρ of each element of the simplified model leading to a dynamic behaviour as close as possible to that of the refined model. This is obtained using the gradient method and previously defined cost function. In order to compare the true modes, one uses criteria which ensure the pairing of the modes even if the frequencies cross themselves.

(4) Then the eigensolutions of the simplified model are computed with ANSYS in order to verify its validity.

4. TEST MODELS

The material of the studied structures is steel, with $E = 2{\cdot}1\mathrm{e}11\ \mathrm{N/m^2}$, $v = 0{\cdot}3$, $\rho = 7800\ \mathrm{kg/m^3}$.

In order to illustrate the two simplification principles previously explained, two refined models have been created:

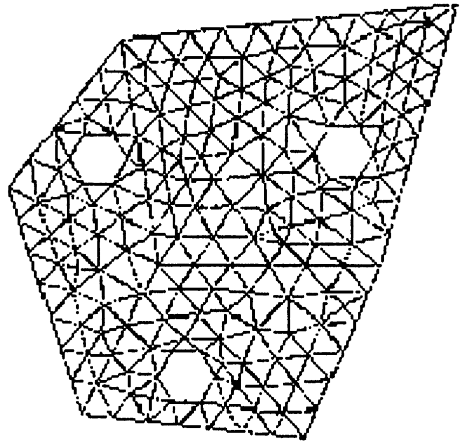


Figure 1. Refined shell 1.

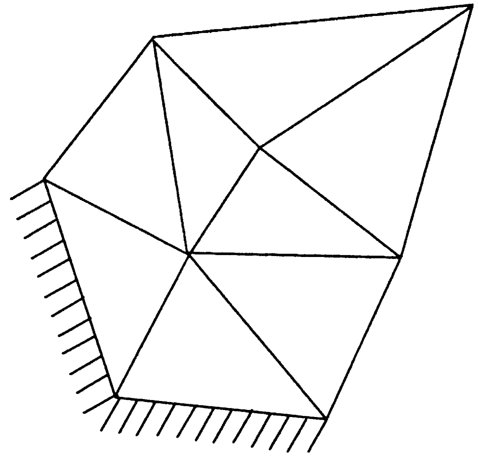


Figure 2. Simplified shell 1.

- a 195 node plate (Figure 1) simplified by a eight-node model (Figure 2)
- a circular plate meshed with 1597 nodes (Figure 3) simplified by an 81-node model (Figure 4).

5. RESULTS

5.1. PRINCIPLE 1

5.1.1. *Plate* 1

Using the original thickness (5 cm) for the simplified model leads to a dynamical behaviour which is far from that of the refined model. For example, in the case of free-boundary conditions, it leads to the relative errors on the eigenfrequencies reported in Table 2. Hence the nodal thickness are used as design parameters in order to reduce these errors.

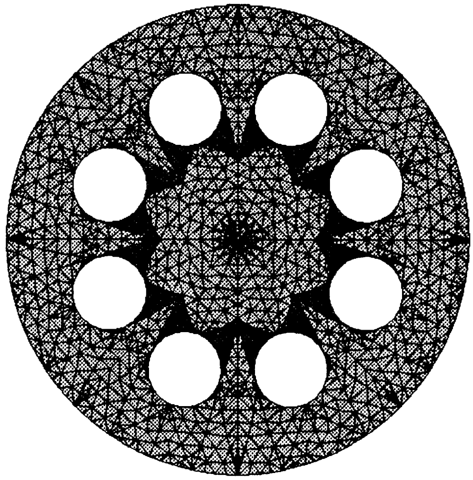


Figure 3. Refined perforated circular plate.

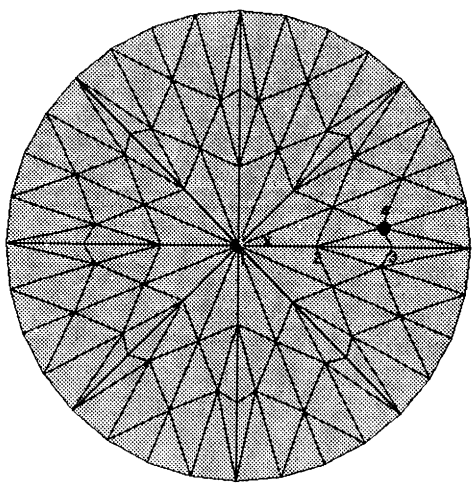


Figure 4. Simplified circular plate.

TABLE 2

Relative percentage error in the eigenfrequencies

Mode no.	1	2	3	4	5	6
Free plate 1	1·4	− 0·2	3·6	− 0·4	16·5	5·4
Clamped plate 1	4·3	5·7	− 2·2	− 9·0	10·9	25·2

The simplification of this 195-node model by an eight-node model leads to the following results.

The 8 nodal thicknesses of the simplified model rapidly converge (Figure 5); The average value of the minimizing functions also converges but remains rather large (Figure 6); The equivalent density of the simplified model with the thicknesses calculated at the twelth

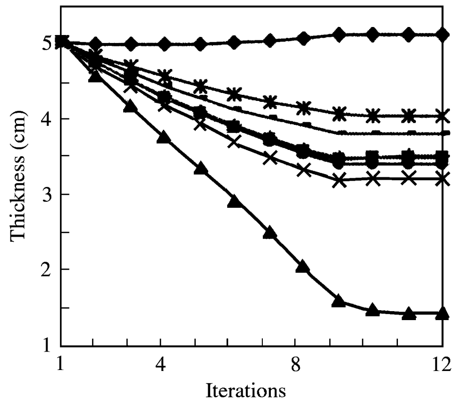


Figure 5. Evolution of the eight nodal thicknesses of the simplified model.

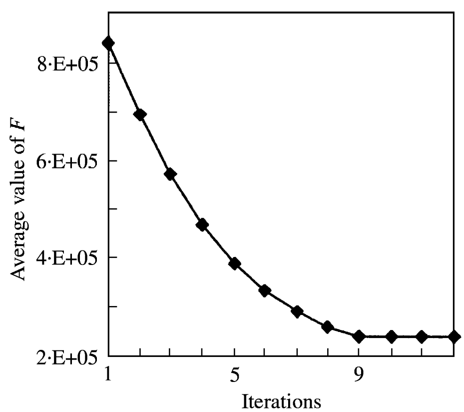


Figure 6. Evolution of the average value of the cost function.

TABLE 3

Comparison of the eigenfrequencies obtained with principle 1

Mode no.	Refined model eigenfrequencies (Hz)	Simplified model eigenfrequencies (Hz)	Relative error (%)
1	6·86	5·14	− 25
2	9·04	6·41	− 29·1
3	12·92	9·35	− 27·6
4	18·24	12·28	− 32·6
5	20·21	15·78	− 21·9

iteration is 10 063 kg/m^3; The eigenfrequency errors of the free simplified model reconstructed with ANSYS are large (Table 3). Note that only two frequencies can be compared with the ones of the refined model due to the cut-off frequency of the Guyan condensed model (8 Hz).

TABLE 4

Relative percentage in the eigenfrequencies of the circular plate

Free mode	1	2	3	4	5	6	7	8	Average
Model 1	9·96	9·80	24·29	13·37	12·69	27·55	19·48		16·73
Model 2	4·16	1·03	13·11	5·86	5·37	15·13	11·21		8·41
Clamped mode									
Model 1	1·07	9·27	– 1·30	9·52	– 3·24	– 0·54	0·75	– 3·59	3·66
Model 2	2·51	7·85	1·79	8·23	1·47	0·45	3·74	2·64	3·59
Supported mode									
Model 1	16·55	19·88	11·48	11·62	2·46	11·13	5·78	0·40	9·91
Mode 2	10·85	12·38	8·32	9·28	4·55	5·38	6·62	3·21	7·57

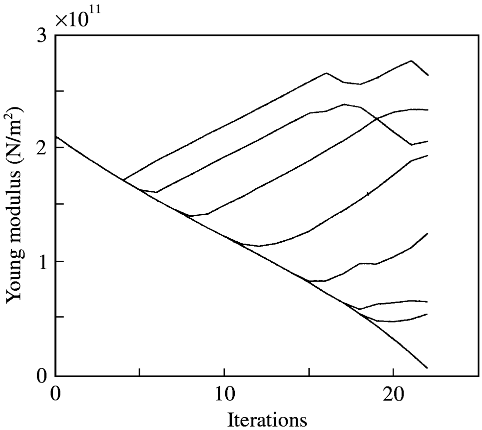


Figure 7. Evolution of Young's modulus of each element of the simplified model.

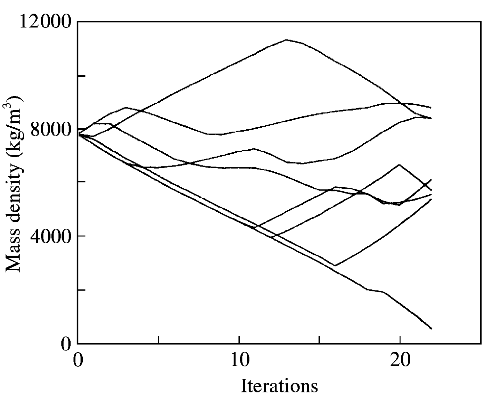


Figure 8. Evolution of the mass density of each element of the simplified model.

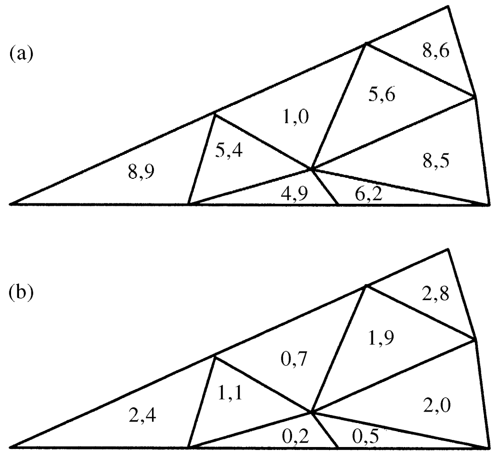


Figure 9. Values of the parameters in a sector of the circular plate: (a) mass density ($\times 10^3$ kg/m^3); (b) Young's, modulus ($\times 10^{11}$ N/m^2).

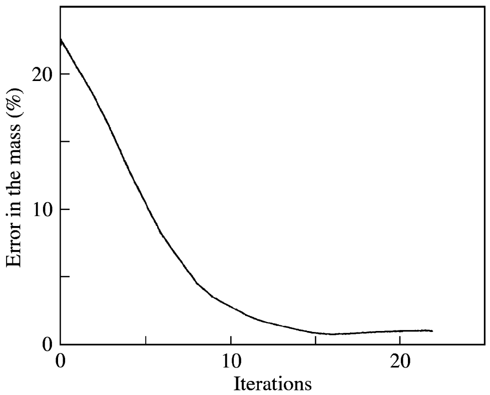


Figure 10. Evolution of the error in the mass of the simplified model.

5.1.2. *Conclusion*

This first principle, used elsewhere to simplify beam-like structures [5], leads to unacceptable results in the case of plate structures, even though the difference between the stiffness matrices was minimized. It can be explained by the fact that the formulation of beams gives exact results in static analysis contrary to the formulation of plate elements.

Indeed, it seems difficult to obtain a simplified model whose mass and stiffness matrices (and consequently the eigensolutions) are exactly the same as those of the refined model. Moreover, even if it were possible, there are some limitations since the parameters of the simplified structure are identified with respect to the Guyan condensed model of the refined structure. The available frequency domain is limited by the cut-off frequency corresponding roughly to a third of the first frequency of the refined model in which the master nodes have been clamped. To avoid this drawback, a second principle using a cost function based on the distance between the first eigensolutions of the refined and simplified models has been formulated.

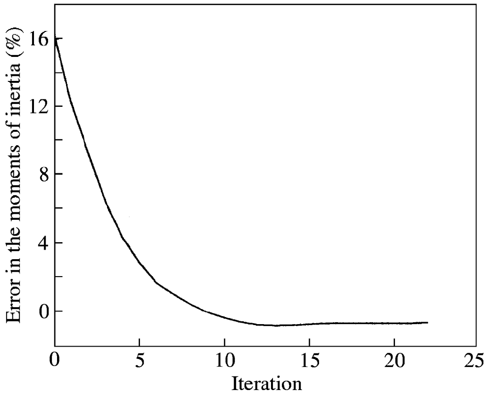


Figure 11. Evolution of the error in the moments of inertia of the simplified model.

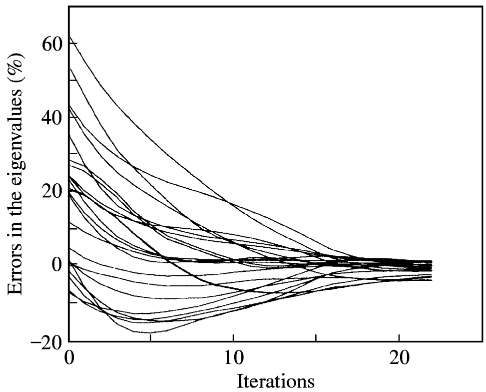


Figure 12. Evolution of the eigenvalue errors of the simplified model.

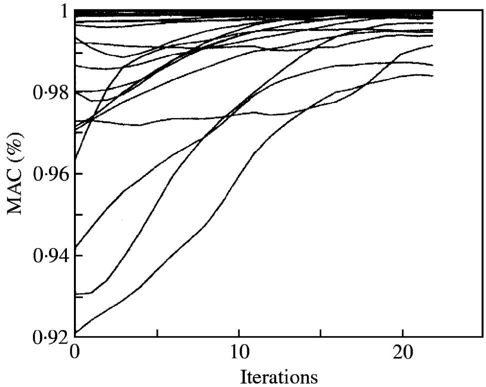


Figure 13. Evolution of the MAC between the eigenvectors of the refined and simplified models.

TABLE 5

Comparison between the refined and simplified models obtained with principle 2

Mode	No.	Refined model (Hz)	Simplified model (Hz)	Relative error (%)	MAC (%)
Free	1	18·97	18·69	1·48	100
	2	19·59	19·31	1·43	100
	3	29·06	29·16	0·34	99·9
	4	43·31	43·28	0·07	99·8
	5	44·54	44·59	0·11	99·8
	6	60·36	60·16	0·33	99·8
	7	68·13	67·04	1·60	99·9
Clamped	1	39·43	39·67	0·61	100
	2	68·75	69·26	0·74	99·9
	3	85·24	84·26	1·15	100
	4	107·92	108·81	0·82	99·7
	5	142·80	142·4	0·79	99·8
	6	154·73	151·9	1·83	98·7
	7	179·18	178·17	0·56	98·4
	8	205·3	207·16	0·91	99·1
Supported	1	16·31	16·37	0·37	100
	2	41·04	41·16	0·29	100
	3	49·29	48·98	0·63	100
	4	80·09	80·79	0·87	99·8
	5	97·8	98·48	0·70	99·8
	6	105·32	105·67	0·33	99·5
	7	135·35	134·77	0·44	99·5
	8	154·26	151·49	1·80	99·4

5.2. PRINCIPLE 2

5.2.1. *Circular plate*

In order to test this principle, an attempt was made to simplify a finely meshed axisymmetric perforated structure having roughly 5000 d.o.f. by a full modal having 20 times fewer d.o.f.

The axisymmetric property of the model (Figure 3) resulted in some problems. The first calculation led to an instability of the modal assurance criterion (MAC). In fact for the antisymmetric modes, the nodal diameters of the simplified and refined models can be moved by an angle of 45°. A solution to this problem is to add a single mass in order to remove the symmetric property of the structure. The position of the added mass of 0·1 kg is represented in Figure 4.

The use of the original parameters with the simplified model leads to a dynamical behaviour which is far from the one of the refined model and leads to large errors on the eigenvalues reported in Table 4 (model 1). Removing the elements corresponding to the holes leads to better results but still quite far from the refined model as can be seen in Table 4 (model 2). So we will attempt to reduce the errors using Young's modulus and the mass density of each element as design parameters.

The simplification of this model by an 81-node model gives the following results when seven free modes, eight clamped modes and eight supported modes are included as targets

TABLE 6

Relative percentage errors in the eigenfrequencies obtained in identifying only seven free modes

Mode no.	1	2	3	4	5	6	7	8	Average
Free mode	0·05	0·26	0·31	0·02	0·29	0·13	0·26		0·19
Clamped mode	0·43	2·21	0·47	4·53	7·17	2·82	6·81	12·60	4·63
Supported mode	3·62	5·31	5·64	9·34	13·21	6·86	13·18	14·46	8·95

TABLE 7

Comparison between the refined and simplified models with a single mass of 0·5 *kg*

Mode	No.	Refined model (Hz)	Simplified model (Hz)	Relative error (%)
Free	1	17·69	17·45	– 1·37
	2	19·59	19·31	– 1·44
	3	28·63	28·81	0·61
	4	39·83	39·74	– 0·24
	5	44·54	44·59	0·12
	6	52·17	52·67	0·97
	7	68·13	67·04	– 1·61
Clamped	1	30·92	31·73	2·61
	2	47·18	47·60	0·88
	3	85·24	84·26	– 1·15
	4	98·76	98·48	– 0·29
	5	141·28	142·40	0·79
	6	153·00	151·05	– 1·27
	7	176·56	174·15	– 1·36
	8	205·30	207·16	0·91
Supported	1	13·27	13·45	1·37
	2	29·79	29·97	0·60
	3	49·28	48·98	– 0·61
	4	70·64	70·83	0·27
	5	97·80	98·47	0·69
	6	104·96	105·30	0·32
	7	129·87	129·74	– 0·10
	8	154·26	151·49	– 1·80

in the cost function.

- Young's modulus (Figure 7) and the mass density (Figure 8) of each element converge to the values represented in Figure 9 on a sector of the axisymmetric structure. One notes that they are consistent and very small for the element corresponding to the hole in the refined model.
- The errors on the mass (Figure 10) and on the moments of inertia (Figure 11) tend toward zero.
- The errors on the eigenvalues (Figure 12) and the MAC values (Figure 13) are minimized.
- The eigenfrequencies and the MAC of the simplified model reconstructed with ANSYS

TABLE 8

Comparison between the refined and simplified models without a mass

Mode	No.	Refined model (Hz)	Simplified model (Hz)	Relative error (%)
Free	1	19·59	19·31	− 1·44
	2	19·59	19·31	− 1·44
	3	29·26	29·32	0·18
	4	44·54	44·59	0·12
	5	44·54	44·59	0·12
	6	68·13	67·04	− 1·61
	7	68·13	67·04	− 1·61
Clamped	1	40·51	40·66	0·38
	2	85·24	84·26	− 1·15
	3	85·24	84·26	− 1·15
	4	141·28	142·40	0·79
	5	141·28	142·40	0·79
	6	165·01	157·98	− 4·26
	7	205·30	207·13	0·91
	8	205·30	207·13	0·91
Supported	1	17·17	17·17	0·04
	2	49·28	48·98	− 0·1
	3	49·28	48·98	− 0·61
	4	97·80	98·47	0·69
	5	97·80	98·47	0·69
	6	107·32	107·79	0·44
	7	154·26	151·49	− 1·80
	8	154·26	151·49	− 1·80

are close to those of the refined model (Table 5): an error of around 1% on the eigenfrequencies and MAC above 98%.

The CPU time for the algorithm running under MATLAB 5.3 on an HP J7000 workstation is about 2 h. This time is rather high but the simplified model runs now in 10 s compared to 230 s for the refined one. Hence, one obtains a significant gain of computational cost for the next use of the model (in a process of optimization for example). However, if only the seven free modes are included in the cost function, the errors on the eigenfrequencies of the clamped modes and the supported modes remain rather large (Table 6). Moreover one observes the same phenomenon if only the clamped modes or the supported modes are identified. It can be explained by pointing out that when the plate is free, the kinetic energy on the elements of the border of the plate is high and the strain energy is low. As a result the algorithm can lead to correct values of ρ but an erroneous value of E at the border. On the contrary, when the plate is clamped the kinetic energy is low at the border which leads to erroneous values of ρ. That is why several boundary conditions (and several modes) are included in the cost function in order to ensure the parameters E and ρ of each element are significant.

In order to check the validity of the results, the simplified model with the previously identified parameters (Figure 9) and the refined model have been compared with a single mass of 0·5 kg (instead of 0·1 kg) and without a mass. The eigenfrequency errors given in Tables 7 and 8 are small (average of 1%).

Table 9

Relative percentage error in the displacements of four nodes

		Displacements of the refined model (mm)	Model 1	Model 2	Model 3
Clamped border	w1	24·29	32·70	19·55	6·13
	w2	20·5	36·33	21·80	4·54
	w3	11·55	40·79	25·61	3·55
	w4	13·26	41·53	26·00	5·66
	Average		37·84	23·24	4·97
Supported border	w1	7·56	16·19	8·98	8·86
	w2	5·03	17·42	9·83	6·76
	w3	1·35	5·36	5·32	4·44
	w4	1·76	8·94	7·14	7·95
	Average		11·98	7·81	7·01

One can also compare the static behaviour of the simplified and refined models. A force of 100 N has been applied to the centre of the plate. The responses of several models have been compared: the refined model, the simplified model with the original parameters with (model 1) and without (model 2) the elements corresponding to the holes and the simplified model with the identified parameters (model 3). The displacements along the z-axis of four nodes (Figure 6) are given in Table 9. Even though the static behaviour has not been taken account of in the cost function, the simplified model with the identified parameters leads to acceptable results (errors of around 5%).

6. CONCLUSION

The feasibility of the simplification of finite element models meshed with thin triangular plate elements has been demonstrated with the second principle. Indeed, it provides acceptable results for the proposed example, providing a model has the same eigenfrequencies and eigenvectors for the first modes with parameters that have a physical meaning. Moreover, the inertial characteristics are well preserved allowing the model to be used as a substructure.

REFERENCES

1. J. L. Batoz and G. Dhatt 1990 *Modélisation des structures par éléments finis*, Vol. 2, Poutres et plaques. Paris: Hermes.
2. G. Dhatt and G. Touzot 1984 *Une présentation de la méthode des éléments finis*. Paris: Maloine; second edition.
3. O. C. Zienkiewicz 1977 *The Finite Element Method*. England: McGraw-Hill Book Company, third edition.
4. N. Bouhaddi 1992 *U.F.R des sciences et techniques de Franche-Comté, France. Sous-structuration par condensation dynamique linéarisée*.
5. S. Corn, J. Piranda and N. Bouhaddi 1997 *Actes du Troisième Colloque National en Calcul des Structures Giens*, *France* 20–23 *May*, 259–264. Simplification de modèles éléments finis de structures à comportement de poutre par des éléments de poutres équivalentes.