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# ALTERNATIVE SOLUTION TO "THE FINITE RESIDUAL MOTION OF A DAMPED FOUR-DEGREE-OF-FREEDOM VIBRATING SYSTEM"

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Very recently in a Letter to the Editor of this journal, Wilms and Cohen [1] introduced a damped four-degree-of-freedom system with two different types of damping matrices. The reader was required to decide which of the systems in Figure 1 oscillates indefinitely, while all oscillations are eventually damped out for the other one. They derived the equations of motion of both systems via Lagrange's procedure by making use of the special nature of the symmetric and asymmetric generalized co-ordinates. They showed that the vibrations of the system (b) damp out totally, whereas those of (a) do not.

As was also done in reference [2] and suggested in reference [3], we would like to give the answer of the posed question in a relatively short and straightforward way. The problem is identical to the task of finding which system is asymptotically stable. It is known that a system with $n$ degrees-of-freedom

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{1}$$

is asymptotically stable if all of the $n \times n$ system matrices $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are symmetric and positive definite [4]. However, as also underlined in reference [3], $\mathbf{D} > 0$, i.e., the fact that the damping matrix is positive definite is only a sufficient, but not a necessary condition for all motions of the system being damped.

In the case that the damping matrix $\mathbf{D}$ is only positive semi-definite, the system can be asymptotically stable if and only if the following rank condition is satisfied [4]:

$$rank(\mathbf{M}^{-1}\mathbf{D} \vdots (\mathbf{M}^{-1}\mathbf{K})(\mathbf{M}^{-1}\mathbf{D}) \vdots \dots \vdots (\mathbf{M}^{-1}\mathbf{K})^{n-1}(\mathbf{M}^{-1}\mathbf{D})) = n, \tag{2}$$

where $n$ denotes the number of the degrees of freedom of the system. If the above rank condition, which comes from control theoretical considerations, is satisfied, then the system is said to be pervasively damped, meaning that the influence of the damping pervades each of the system co-ordinates [4].

The $4 \times 4$ mass and stiffness matrices of both system (a) and (b) for the absolute co-ordinates $x_1$, $x_2$, $x_3$, $x_4$ are the same and read as

$$\mathbf{M} = diag(m), \qquad \mathbf{K} = \begin{bmatrix} 2k & -k & 0 & 0 \\ -k & 2k & -k & 0 \\ 0 & -k & 2k & -k \\ 0 & 0 & -k & 2k \end{bmatrix}, \quad m = k = 1, \tag{3}$$

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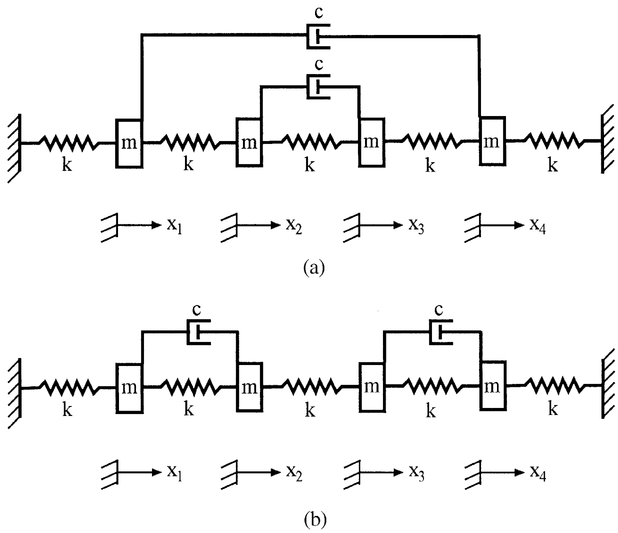


Figure 1. System (a) and system (b); $k = m = 1$.

both being symmetric and positive definite. The damping matrices of the systems are

$$\mathbf{D}_{(a)} = \begin{bmatrix} c & 0 & 0 & -c \\ 0 & c & -c & 0 \\ 0 & -c & c & 0 \\ -c & 0 & 0 & c \end{bmatrix}, \quad \mathbf{D}_{(b)} = \begin{bmatrix} c & -c & 0 & 0 \\ -c & c & 0 & 0 \\ 0 & 0 & c & -c \\ 0 & 0 & -c & c \end{bmatrix}. \tag{4}$$

It is an easy task to show that both of these matrices are only positive semi-definite. Therefore, rank condition (2) has to be used to check the asymptotic stability of the systems (a) and (b). Due to the fact that the mass matrix of the present systems is the $4 \times 4$ unit matrix, rank condition (2) simplifies to

$$rank(\mathbf{D} \vdots \mathbf{KD} \vdots \mathbf{K}^2\mathbf{D} \vdots \mathbf{K}^3\mathbf{D}) = 4. \tag{5}$$

Substitution of $\mathbf{D}_{(a)}$ into the left-hand side of equation (5) yields 2, whereas that of $\mathbf{D}_{(b)}$ yields 4. These results indicate clearly that the system (a) cannot be asymptotically stable, i.e., finite residual motion remains. On the contrary, the system (b) is asymptotically stable, i.e., all motion is damped out for this system. In the terminology of the mechanical vibrations, it can be stated that the system (b) is pervasively damped, whereas the system (a) is not.

## REFERENCES


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