Journal of Sound and Vibration (2002) **256**(1), 1–15
doi:10.1006/jsvi.2001.4192, available online at http://www.idealibrary.com on IDEAL®

DISTURBANCE DUE TO MECHANICAL SOURCES IN MICROPOLAR ELASTIC MEDIUM WITH VOIDS

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(*Received* 21 *August* 2001, *and in final form* 27 *November* 2001)

The dynamic response of a homogeneous isotropic micropolar half-space with voids subjected to a set of normal point sources is investigated. The integral transforms have been inverted by using a numerical technique to obtain the normal force stress, normal displacement, tangential force stress, tangential couple stress and volume fraction field in the physical domain for the two different sources. The results of these quantities for a magnesium crystal-like material are given and illustrated.

1. INTRODUCTION

Theory of linear elastic materials with voids is one of the generalizations of the classical theory of elasticity. This theory has practical utility of investigating various types of geological, biological and synthetic porous materials for which the elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the void volume is included among the kinematic variables, and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity.

A non-linear theory of elastic materials with voids was developed by Nunziato and Cowin [1]. Later, Cowin and Nunziato [2] developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of a beam and small amplitudes acoustic waves. The problems of quasi-static plane strain and plane stress for a linear elastic material with void was studied by Cowin [3]. Puri and Cowin [4] studied the behavior of plane harmonic waves in a linear elastic material with voids. Iesan [5] developed the basic theories of linear thermoelastic materials with voids, Chandrasekharaiah and Cowin [6], obtained the field equations governing two different continuum theories, namely the theory of thermoelasticity and Biot's theory of poroelasticity. The problem of complete solutions in the theory of isotropic elastic materials with voids was discussed by Chandrasekharaiah [7]. A domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang [8].

The particles of a classical elastic materials have only a translational degree of freedom, and transmission of the load across a differential element of the surface is described by a force vector only. The polycrystalline materials do not confirm this property. These materials are fibrous and composite in nature and exhibit size effects. These materials have additional microdeformational degrees of freedom, i.e., they possess a microstructure whose size cannot be neglected in comparison with length scales of interest. Various degrees of freedom of a microstructure were considered by different authors. Notable among them are Cosserat [9], Eringen and Suhubi [10] and Mindlin [11]. Each one has given an independent set of governing equations. The force at a point of a surface element of bodies of these materials is completely characterized by a stress vector and a couple stress vector at that point. In the classical theory of elasticity, the effect of couple stress is neglected, Eringen [12] has modified his earlier theory and renamed it as the "Linear Theory of Micropolar Elasticity".

Iesan [13] studied shock waves in micropolar elastic material with voids. Recently, Scarpetta [14] worked on the fundamental solutions in micropolar elasticity with voids. Marin [15–17] discussed different problems in micropolar elasticity with voids.

2. FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, micropolar elastic half-space with voids. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface $z = 0$ with the z-axis vertical down into the medium is introduced. A normal delta distribution or continuous point source is assumed to be acting at the origin of the rectangular Cartesian co-ordinates.

Following references [13, 18], the constitutive relations and field equations in micropolar elastic solid with voids without body force and body couple can be written as

$$t_{ij} = \lambda\, u_{r,r}\,\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijr}\,\phi_r) + \delta_{ij}\beta^* q, \tag{1}$$

$$m_{ij} = \alpha\, \phi_{r,r}\,\delta_{ij} + \beta\,\phi_{i,j} + \gamma\,\phi_{j,i} \tag{2}$$

and

$$(\lambda + \mu)\nabla(\nabla\cdot\mathbf{u}) + (\mu + K)\nabla^2\mathbf{u} + K\nabla\times\boldsymbol{\phi} + \beta^*\nabla q = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{3}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla\cdot\boldsymbol{\phi}) - \gamma\nabla\times(\nabla\times\boldsymbol{\phi}) + K\nabla\times\mathbf{u} - 2K\boldsymbol{\phi} = \rho j\frac{\partial^2\boldsymbol{\phi}}{\partial t^2}. \tag{4}$$

$$\alpha^*\nabla^2 q - \zeta^* q - \omega^*\frac{\partial q}{\partial t} - \beta^*\nabla\cdot\mathbf{u} = \rho K^*\frac{\partial^2 q}{\partial t^2}, \tag{5}$$

where λ, μ, K, α, β, γ are the material constants, ρ the density, j the microinertia, $\mathbf{u}$ the displacement vector, $\boldsymbol{\phi}$ the microrotation vector, t_{ij} the component of force stress and m_{ij} the component of couple stress, q the volume fraction field and α^*, β^*, ζ^*, ω^*, K^* are the material constants due to the presence of voids.

We take $\mathbf{u} = (u_x, 0, u_z)$ and $\boldsymbol{\phi} = (0, \phi_2, 0)$ in equations (3)–(5) and then define the non-dimensional quantities as

$$x' = \frac{\bar{\omega}}{c_1}x, \quad z' = \frac{\bar{\omega}}{c_1}z, \quad u_x' = \frac{\bar{\omega}}{c_1}u_x, \quad u_z' = \frac{\bar{\omega}}{c_1}u_z, \quad t' = \bar{\omega}t,$$

$$\phi_2' = \frac{\bar{\omega}^2 j}{c_1^2}\phi_2, \quad t_{ij} = \frac{t_{ij}}{\mu}, \quad m_{ij}' = \frac{\bar{\omega} j}{\gamma c_1}m_{ij}, \quad q' = \frac{\bar{\omega}^2 j}{c_1^2}q, \tag{6}$$

where

$$\bar{\omega}^2 = \frac{K}{\rho j} \qquad \text{and} \qquad c_1^2 = \frac{(\lambda + 2\mu + K)}{\rho}.$$

Equations (3)–(5) may be recast into the dimensionless form after suppressing the dashes as

$$\left[\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_z}{\partial x\,\partial z}\right] + a_1 \nabla^2 u_x - a_2 \frac{\partial \phi_2}{\partial z} + a_3 \frac{\partial q}{\partial x} = a_4 \frac{\partial^2 u_x}{\partial t^2}, \tag{7}$$

$$\left[\frac{\partial^2 u_x}{\partial x\,\partial z} + \frac{\partial^2 u_z}{\partial z^2}\right] + a_1 \nabla^2 u_z + a_2 \frac{\partial \phi_2}{\partial x} + a_3 \frac{\partial q}{\partial z} = a_4 \frac{\partial^2 u_z}{\partial t^2}, \tag{8}$$

$$\nabla^2 \phi_2 + b_1 \left[\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right] - b_2 \phi_2 = b_3 \frac{\partial^2 \phi_2}{\partial t^2}, \tag{9}$$

$$\nabla^2 q - s_1 q - s_2 \frac{\partial q}{\partial t} - s_3 \left[\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right] = s_4 \frac{\partial^2 q}{\partial t^2}, \tag{10}$$

where

$$a_1 = \frac{(\mu + K)}{(\lambda + \mu)}, \quad a_2 = \frac{K c_1^2}{(\lambda + \mu)\bar{\omega}^2 j}, \quad a_3 = \frac{\beta^* c_1^2}{\bar{\omega}^2 j (\lambda + \mu)}, \quad a_4 = \frac{\rho c_1^2}{(\lambda + \mu)},$$

$$b_1 = \frac{jK}{\gamma}, \quad b_2 = \frac{2K c_1^2}{\bar{\omega}^2 \gamma}, \quad b_3 = \frac{\rho c_1^2 j}{\gamma},$$

$$s_1 = \frac{\zeta^* c_1^2}{\alpha^* \bar{\omega}^2}, \quad s_2 = \frac{\omega^* c_1^2}{\alpha^* \bar{\omega}}, \quad s_3 = \frac{\beta^* j}{\alpha^*}, \quad s_4 = \frac{\rho K^* c_1^2}{\alpha^*}. \tag{11}$$

Using the expression relating displacement components $u_x(x, z, t)$, $u_z(x, z, t)$ to the scalar potential functions $\psi_1\ (x, z, t)$ and $\psi_2(x, z, t)$ in dimensionless form

$$u_x = \frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial z}, \qquad u_z = \frac{\partial \psi_1}{\partial z} + \frac{\partial \psi_2}{\partial x} \tag{12}$$

in equations (7)–(10), we obtain

$$\left[(1 + a_1)\nabla^2 - a_4 \frac{\partial^2}{\partial t^2}\right] \psi_1 + a_3 q = 0, \tag{13}$$

$$\left[a_1 \nabla^2 - a_4 \frac{\partial^2}{\partial t^2}\right] \psi_2 + a_2 \phi_2 = 0, \tag{14}$$

$$\left[\nabla^2 - b_2 - b_3 \frac{\partial^2}{\partial t^2}\right] \phi_2 - b_1 \nabla^2 \psi_2 = 0, \tag{15}$$

$$\left[\nabla^2 - s_1 - s_2 \frac{\partial}{\partial t} - s_4 \frac{\partial^2}{\partial t^2}\right] q - s_3 \nabla^2 \psi_1 = 0. \tag{16}$$

Applying the Laplace and Fourier transforms

$$\bar{f}(x, z, p) = \int_0^\infty f(x, z, t)\, e^{-pt}\, dt,$$

$$\tilde{f}(\xi, z, p) = \int_{-\infty}^{\infty} \bar{f}(x, z, p)\, e^{i\xi x}\, dx \tag{17}$$

on equations (13)–(16) and eliminating $\tilde{\phi}_2$ and $\tilde{q}$ from the resulting expressions, we obtain

$$\left[\frac{d^4}{dz^4} + A\frac{d^2}{dz^2} + B\right][\tilde{\psi}_1] = 0 \tag{18}$$

and

$$\left[\frac{d^4}{dz^4} + E\frac{d^2}{dz^2} + F\right][\tilde{\psi}_2] = 0, \tag{19}$$

where

$$A = -[(1 + a_2)(2\xi^2 + s_1 + s_2 p + s_4 p^2) + a_4 p^2 - a_2 s_3]/(1 + a_2),$$

$$B = \xi^4 + [((1 + a_2)(s_1 + s_2 p + s_4 p^2) + a_4 p^2 - a_2 s_3)\,\xi^2$$

$$+ (s_1 + s_2 p + s_4 p^2)\, a_4 p^2]/(1 + a_2),$$

$$E = -[a_1(2\xi^2 + b_2 + b_3 p^2) + a_4 p^2 - a_2 b_1]/a_1,$$

$$F = \xi^4 + [(a_1(b_2 + b_3 p^2) + a_4 p^2 - a_2 b_1)\,\xi^2 + (b_2 + b_3 p^2)\, a_4 p^2]/a_1. \tag{20}$$

Since $\tilde{\psi}_1$, $\tilde{\psi}_2$, $\tilde{\phi}_2$, and $\tilde{q}$ tend to zero as z tends to infinity, the solution of equations (18) and (19) may be written as

$$\tilde{\psi}_1 = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z}, \tag{21}$$

$$\tilde{q} = R_1 A_1 e^{-\lambda_1 z} + R_2 A_2 e^{-\lambda_2 z}, \tag{22}$$

$$\tilde{\psi}_2 = A_3 e^{-\lambda_3 z} + A_4 e^{-\lambda_4 z}, \tag{23}$$

$$\tilde{\phi}_2 = R_3 A_3 e^{-\lambda_3 z} + R_4 A_4 e^{-\lambda_4 z}, \tag{24}$$

where $\lambda_{1,2}^2$ and $\lambda_{3,4}^2$ are the roots of differential equations (18) and (19), respectively, given by

$$\lambda_1^2 = \left[\frac{-A + (-1)^{i+1}\sqrt{A^2 - 4B}}{2}\right], \quad i = 1, 2,$$

$$\lambda_1^2 = \left[\frac{-E + (-1)^{i+1}\sqrt{E^2 - 4F}}{2}\right], \quad i = 3, 4 \tag{25}$$

and

$$R_i = [(1 + a_1)(\xi^2 - \lambda_i^2) + a_4 p^2]/a_3, \quad i = 1, 2,$$

$$R_i = [a_1(\xi^2 - \lambda_i^2) + a_4 p^2]/a_2, \quad i = 3, 4. \tag{26}$$

3. APPLICATION

3.1. CASE I DELTA DISTRIBUTION NORMAL POINT SOURCE

The plane boundary is subjected to a delta distribution normal point force. Therefore, the boundary conditions are

$$t_{zz} = -P\,\delta(x)\delta(t), \quad t_{zx} = 0, \quad m_{zy} = 0 \quad \text{and} \quad \frac{\partial q}{\partial z} = 0 \quad \text{at } z = 0, \tag{27}$$

where P is the magnitude of the force applied and $\delta(\,)$ is Dirac's delta distribution.

Making use of equations (1), (2), (11) and (12) in the boundary conditions (27) and applying the transforms defined by (17) and substituting the values of $\tilde{\psi}_1$, $\tilde{\psi}_2$, $\tilde{q}$ and $\tilde{\phi}_2$ from equations (21)–(24) in the resulting expressions, we obtain the expressions for the displacement components, stresses and volume fraction field.

$$\tilde{u}_x = [-\mathrm{i}\xi(\Delta_1 \mathrm{e}^{-\lambda_1 z} + \Delta_2 \mathrm{e}^{-\lambda_2 z}) + \lambda_3 \Delta_3 \mathrm{e}^{-\lambda_3 z} + \lambda_4 \Delta_4 \mathrm{e}^{-\lambda_4 z}]/\Delta, \tag{28}$$

$$\tilde{u}_z = -[\lambda_1 \Delta_1 \mathrm{e}^{-\lambda_1 z} + \lambda_2 \Delta_2 \mathrm{e}^{-\lambda_2 z}) + \mathrm{i}\xi(\Delta_3 \mathrm{e}^{-\lambda_3 z} + \Delta_4 \mathrm{e}^{-\lambda_4 z}]/\Delta, \tag{29}$$

$$\tilde{t}_{zz} = [H_1 \Delta_1 \mathrm{e}^{-\lambda_1 z} + H_2 \Delta_2 \mathrm{e}^{-\lambda_2 z}) + H_3 \Delta_3 \mathrm{e}^{-\lambda_3 z} + H_4 \Delta_4 \mathrm{e}^{-\lambda_4 z}]/\Delta, \tag{30}$$

$$\tilde{t}_{zx} = [G_1 \Delta_1 \mathrm{e}^{-\lambda_1 z} + G_2 \Delta_2 \mathrm{e}^{-\lambda_2 z} + G_3 \Delta_3 \mathrm{e}^{-\lambda_3 z} + G_4 \Delta_4 \mathrm{e}^{-\lambda_4 z}]/\Delta, \tag{31}$$

$$\tilde{m}_{zy} = -[\lambda_3 R_3 \Delta_1 \mathrm{e}^{-\lambda_3 z} + \lambda_4 R_4 \Delta_4 \mathrm{e}^{-\lambda_4 z}]/\Delta, \tag{32}$$

$$\tilde{q} = [R_1 \Delta_1 \mathrm{e}^{-\lambda_1 z} + R_2 \Delta_2 \mathrm{e}^{-\lambda_2 z}]/\Delta, \tag{33}$$

where

$$\Delta = -\lambda_3 R_3 [G_4(H_1 \lambda_2 R_2 - H_2 \lambda_1 R_1) + H_4(G_2 \lambda_1 R_1 - G_1 \lambda_2 R_2)]$$

$$+ \lambda_4 R_4 [G_3(H_1 \lambda_2 R_2 - H_2 \lambda_1 R_1) + H_3(G_2 \lambda_1 R_1 - G_1 \lambda_2 R_2)],$$

$$\Delta_i = (-1)^{i+1} P \lambda_j R_j [\lambda_3 R_3 G_4 - \lambda_4 R_4 G_3], \; i = 1, j = 2, \; i = 2, j = 1,$$

$$\Delta_i = (-1)^i P \lambda_j R_j [\lambda_1 R_1 G_2 - \lambda_2 R_2 G_1], \; i = 3, j = 4, \; i = 4, j = 3, \tag{34}$$

where

$$H_i = \frac{(\lambda + 2\mu + K)\lambda_i^2}{\mu} - \frac{\lambda\xi^2}{\mu} + \frac{\beta^* c_1^2 R_i}{\bar{\omega}^2 j\mu}, \quad i = 1, 2,$$

$$H_i = \frac{-\imath\,\xi(2\lambda + 2\mu + K)\lambda_i}{\mu} \quad i = 3, 4,$$

$$G_i = \frac{\imath\,\xi\,(2\mu + K)\lambda_i}{\mu} - \frac{(\mu + K)\lambda_i^2}{\mu} - p^2, \quad i = 1, 2,$$

$$G_i = \frac{K\,c_1^2\,R_i}{\bar{\omega}^2 j\mu}, \quad i = 3, 4.$$

Particular case: If we neglect the influence of the voids, i.e., ($\alpha^* = \beta^* = \zeta^* = \omega^* = K^* = 0$) in equations (28)–(33), the expressions for the displacement components and force stresses are obtained in a micropolar elastic medium as

$$\tilde{u}_x = [-\mathrm{i}\xi \Delta_1^* \mathrm{e}^{-\lambda_1^* z} + \lambda_2^* \Delta_2^* \mathrm{e}^{-\lambda_2^* z} + \lambda_3^* \Delta_3^* \mathrm{e}^{-\lambda_3^* z}]/\Delta^*, \tag{35}$$

$$\tilde{u}_z = -[\lambda_1^* \Delta_1^* \mathrm{e}^{-\lambda_1^* z} + \mathrm{i}\xi(\Delta_2^* \mathrm{e}^{-\lambda_2^* z} + \Delta_3^* \mathrm{e}^{-\lambda_3^* z})]/\Delta^*, \tag{36}$$

$$\tilde{t}_{zz} = [H_1^* \Delta_1^* \mathrm{e}^{-\lambda_1^* z} + H_2^* \Delta_2^* \mathrm{e}^{-\lambda_2^* z} + H_3^* \Delta_3^* \mathrm{e}^{-\lambda_3^* z}]/\Delta^*, \tag{37}$$

$$\tilde{t}_{zx} = [G_1^* \Delta_1^* \mathrm{e}^{-\lambda_1^* z} + G_2^* \Delta_2^* \mathrm{e}^{-\lambda_2^* z} + G_3^* \Delta_3^* \mathrm{e}^{-\lambda_3^* z}]/\Delta^*, \tag{38}$$

$$\tilde{m}_{zy} = -[\lambda_2^* R_2^* \Delta_2^* \mathrm{e}^{-\lambda_2^* z} + \lambda_3^* R_3^* \Delta_3^* \mathrm{e}^{-\lambda_3^* z}]/\Delta^*, \tag{39}$$

where

$$\Delta^* = [G_1^*(H_2^* \lambda_3^* R_3^* - H_3^* \lambda_2^* R_2^*) - H_1^*(G_2^* \lambda_3^* R_3^* - G_3^* \lambda_2^* R_2^*)],$$

$$\Delta_1^* = P[\lambda_3^* R_3^* G_2^* - \lambda_2^* R_2^* G_3^*], \qquad \Delta_2^* = -P\lambda_3^* R_3^* G_1^*,$$

$$\Delta_3^* = -P\lambda_2^* R_2^* G_1^*, \qquad H_1^* = \frac{(\lambda + 2\mu + K)\lambda_1^{*2}}{\mu} - \frac{\lambda\xi^2}{\mu},$$

$$H_2^* = \frac{-\imath\,\xi(2\lambda + 2\mu + K)\lambda_2^*}{\mu}, \qquad H_3^* = \frac{-\imath\,\xi(2\lambda + 2\mu + K)\lambda_3^*}{\mu},$$

$$G_1^* = \frac{\imath\,\xi(2\mu + K)\lambda_1^*}{\mu} - \frac{(\mu + K)\lambda_1^{*2}}{\mu} - p^2,$$

$$G_2^* = \frac{K c_1^2 R_2^*}{\bar{\omega}^2 j\mu}, \qquad G_3^* = \frac{K c_1^2 R_3^*}{\bar{\omega}^2 j\mu},$$

$$R_2^* = [a_1(\xi^2 - \lambda_2^*) + a_4 p^2]/a_2,$$

$$R_3^* = [a_1(\xi^2 - \lambda_3^*) + a_4 p^2]/a_2,$$

$$\lambda_1^* = \xi^2 + \frac{a_4 p^2}{(1 + a_1)}, \quad \lambda_2^* = \lambda_2^2 \quad \text{and} \quad \lambda_3^* = \lambda_3^2. \tag{40}$$

3.2. CASE II CONTINUOUS NORMAL POINT SOURCE

When the plane boundary is subjected to a continuous point source, the boundary conditions are

$$t_{zz} = -P\delta(x)H(t), \quad t_{zx} = 0, \quad m_{zy} = 0 \quad \text{and} \quad \frac{\partial q}{\partial z} = 0 \quad \text{at } z = 0, \tag{41}$$

where P is the magnitude of the continuous force applied and $H(\ )$ the Heaviside distribution.

With the help of these boundary conditions (41), the expressions for the displacement components, force stresses, couple stress and volume fraction are obtained by equations (28)–(33) replacing Δ_i with Δ_i' ($i = 1, \ldots, 4$), where

$$\Delta_i' = \Delta_i/p. \tag{42}$$

Particular case: If we neglect effect of voids, the analytical expressions for the displacement components, force stress in a micropolar elastic medium are given by equations (35)–(39) with $\Delta_i^*(i = 1, 2, 3)$ replaced by $\Delta_i^{*\prime}$ ($i = 1, 2, 3$), respectively, where

$$\Delta_i^{*'} = \Delta_i^*/p. \tag{43}$$

4. INVERSION OF TRANSFORM

We get expressions for displacement, microrotation, stress solution in equations (28)–(33) and (35)–(39). These expressions are functions of z, the parameters of Laplace and Fourier transforms p and ξ, respectively, and hence of the form $\tilde{f}(\xi, z, p)$. To get the function $f(x, z, t)$ in the physical domain, first we invert the Fourier transform using

$$\bar{f}(x, z, p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\xi x} \tilde{f}(\xi, z, p)\,\mathrm{d}\xi = \frac{1}{\pi}\int_0^{\infty} [\cos(\xi x)\tilde{f}_e - \mathrm{i}\sin(\xi x)\tilde{f}_o]\,\mathrm{d}\xi, \tag{44}$$

where $\tilde{f}_e$ and $\tilde{f}_o$ are, respectively, even and odd parts of the function $\tilde{f}(\xi, z, p)$. Thus, expression (44) gives us the Laplace transform $\bar{f}(x, z, p)$ of function $f(x, z, t)$.

Then, for the fixed values of ξ, x and z, the function $\bar{f}(x, z, p)$ in expression (44) can be considered as the Laplace transform $\tilde{g}(p)$ of $g(t)$. Following Honig and Hirdes [11], the Laplace transformed function $\bar{g}(p)$ can be inverted as follows: The function $g(t)$ can be obtained by using

$$g(t) = \frac{1}{2\pi\imath}\int_{C-\mathrm{i}\infty}^{C+\mathrm{i}\infty} \mathrm{e}^{pt}\tilde{g}(p)\,\mathrm{d}p, \tag{45}$$

where C is an arbitrary real number greater than all the real part of singularities of $\bar{g}(p)$. Taking $p = C + \mathrm{i}y$, we get

$$g(t) = \frac{\mathrm{e}^{Ct}}{2\pi \imath} \int_{C-\mathrm{i}\infty}^{C+\mathrm{i}\infty} \mathrm{e}^{\mathrm{i}ty}\, \tilde{g}(C + \mathrm{i}y)\, \mathrm{d}y. \tag{46}$$

Now, taking $\mathrm{e}^{-Ct}\, g(t)$ as $h(t)$ and expanding it as Fourier series in $[0, 2L]$, we obtain approximately the formula

$$g(t) \cong g_{\infty}(t) + E_D, \tag{47}$$

where

$$g_{\infty}(t) = \frac{C_o}{2} + \sum_{k=1}^{\infty} C_k, \quad 0 \leqslant t \leqslant 2L \tag{48}$$

and

$$C_k = \frac{\mathrm{e}^{Ct}}{L} \operatorname{Re}\left[\mathrm{e}^{\mathrm{i}k\pi t/L}\ \bar{g}\left(C + \frac{\mathrm{i}k\pi}{L}\right)\right].$$

E_D is the discretization error and can be made arbitrarily small by choosing C large enough.

Since the infinite series in equation (48) can be summed up only to a finite number of terms N, the approximate value of $g(t)$ becomes

$$g_N(t) = \frac{C_o}{2} + \sum_{k=1}^{N} C_k, \quad 0 \leqslant t \leqslant 2L. \tag{49}$$

Now we introduce a truncation error E_T, that must be added to the discretization error to produce the total approximate error in evaluating $g(t)$ using the above formula. The discretization error is reduced by using the "Korrecktur method" and then the "ε-algorithm" is used to reduced the truncation error and hence to accelerate the convergence.

The Korrecktur method formula to evaluate the function $g(t)$ is

$$g(t) = g_{\infty}(t) - \mathrm{e}^{-2CL} g_{\infty}(2L + t) + E'_D,$$

where

$$|E'_D| \ll |E_D|.$$

Thus the approximate value of $g(t)$ becomes

$$g_{N_k}(t) = g_N(t) - \mathrm{e}^{-2CL} g_{N'}(2L + t), \tag{50}$$

where N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm, which is used to accelerate the convergence of the series in equation (49). Let N be an odd natural number and $S_n = \sum_{k=1}^{n} C_k$ be the sequence of partial sums of equation (49). We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \qquad \varepsilon_{1,m} = S_m,$$

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,\,m+1} + \frac{1}{\varepsilon_{n,\,m+1} - \varepsilon_{n,m}}; \quad n, m = 1, 2, 3, \dots .$$

The sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots, \varepsilon_{N,1}$ converges to $g(t) + E_D - C_o/2$ faster than the sequence of partial sums S_m, $m = 1, 2, 3, \ldots$. The actual procedure to invert the Laplace transform consists of equation (50) together with the ε-algorithm. The values of C and L are chosen according to the criteria outlined by Honig and Hirdes [19].

The last step is to calculate the integral in equation (44). The method for evaluating this integral is described in reference [20], and involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

5. NUMERICAL DISCUSSION

The analysis is conducted for a magnesium crystal-like material. Following reference [21], the values of physical constants are

$\lambda = 9{\cdot}4 \times 10^{11}$ dyn/cm^2, $\mu = 4{\cdot}0 \times 10^{11}$ dyn/cm^2,

$K = 1{\cdot}0 \times 10^{11}$ dyn/cm^2, $\rho = 1{\cdot}74$ g/cm^3,

$\gamma = 0{\cdot}779 \times 10^{-4}$ dyn $j = 0{\cdot}2 \times 10^{-15}$ cm^2

and the void parameters are

$\alpha^* = 3{\cdot}688 \times 10^{-4}$ dyn, $\beta^* = 1{\cdot}13849 \times 10^{11}$ dyn/cm^2,

$\xi^* = 1{\cdot}475 \times 10^{11}$ dyn/cm^2, $\omega^* = 0{\cdot}0787 \times 10^{-2}$ dyn s/cm^2,

$K^* = 1{\cdot}753 \times 10^{-15}$ cm^2.

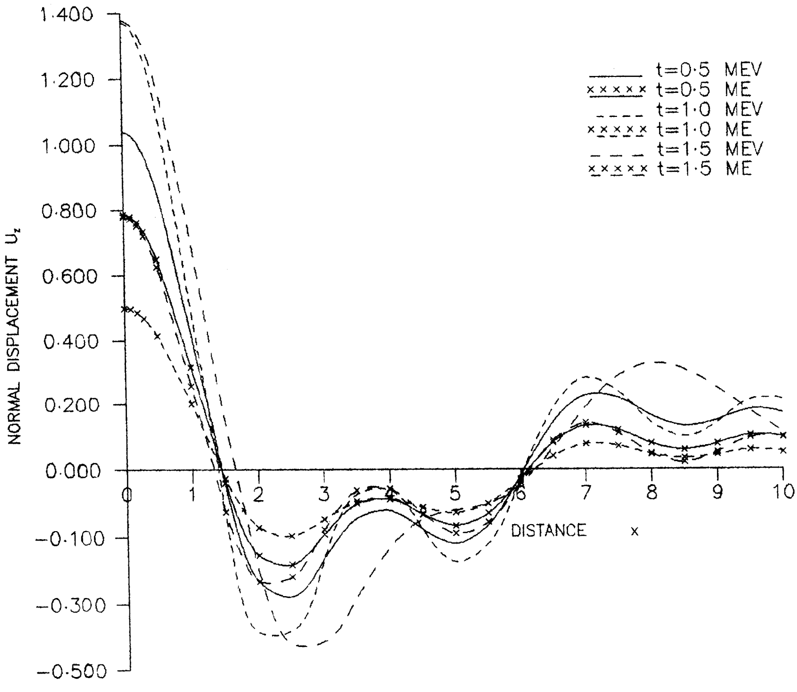

Figure 1. Variations of normal displacement $U_z(= u_z/P)$ with distance x.

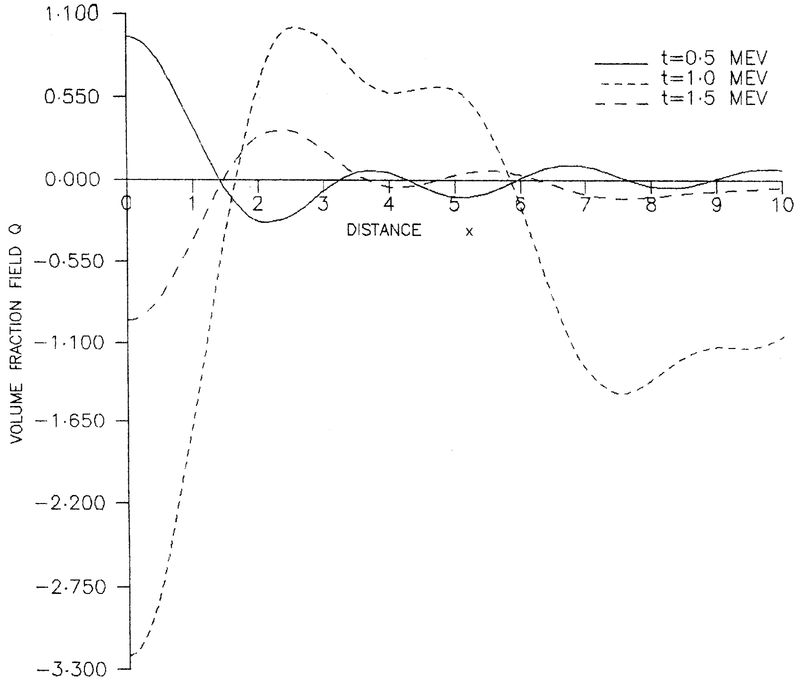

Figure 2. Variations of volume fraction field $Q(=q/P)$ with distance x.

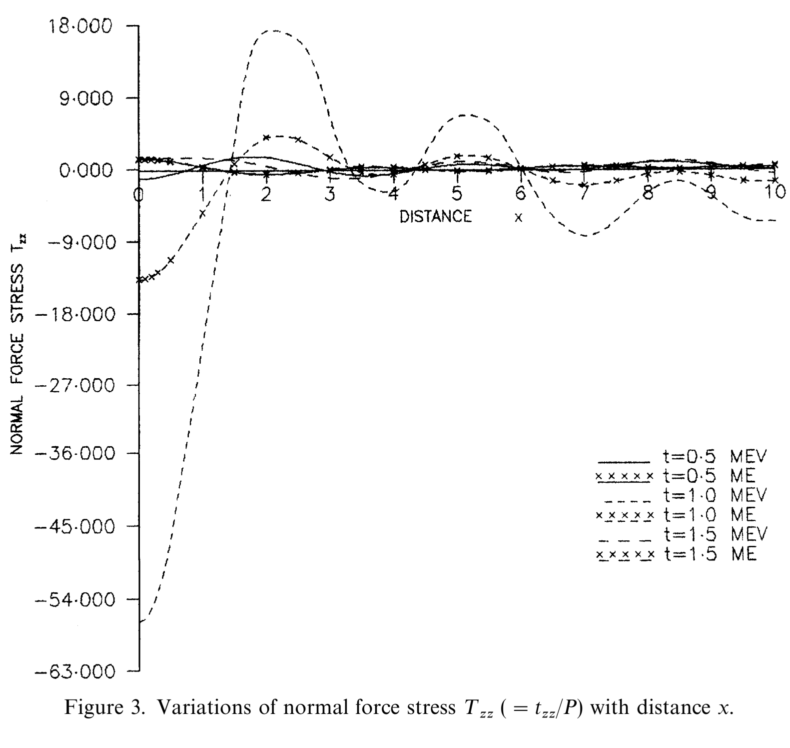

Figure 3. Variations of normal force stress T_{zz} $(=t_{zz}/P)$ with distance x.

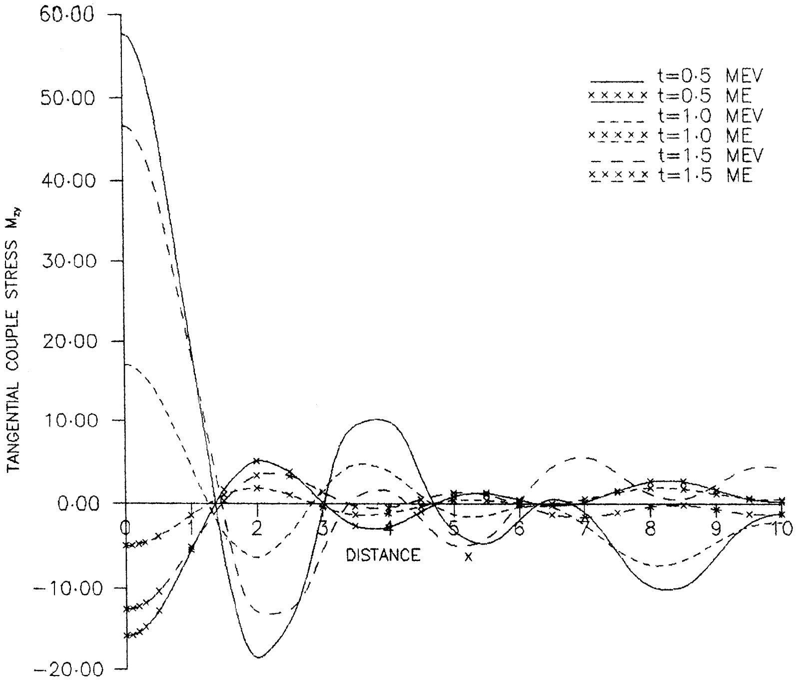

Figure 4. Variations of tangential couple stress M_{zy} ($= m_{zy}/P$) with distance x.

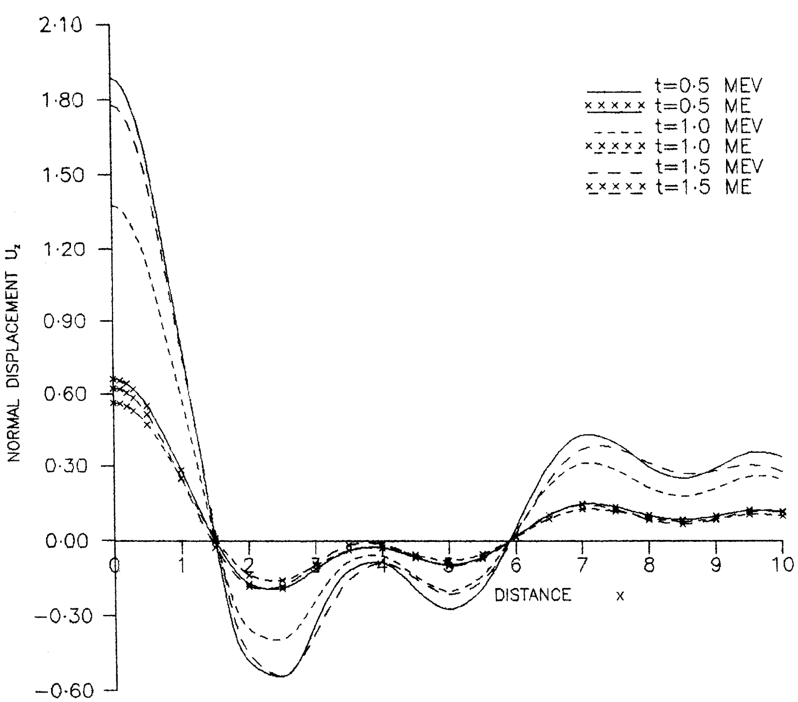

Figure 5. Variations of normal displacement U_z($= u_z/P$) with distance x.

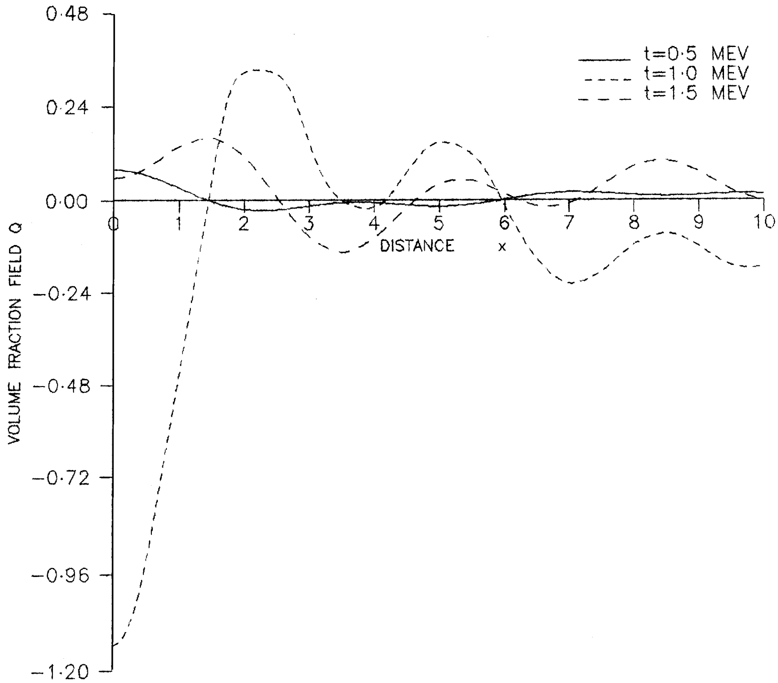

Figure 6. Variations of volume fraction field $Q(=q/P)$ with distance x.

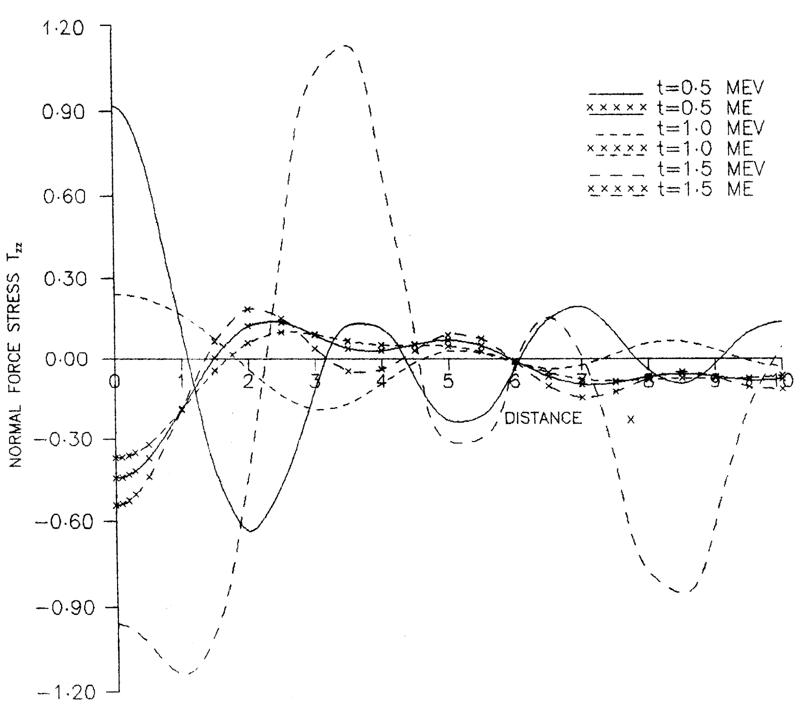

Figure 7. Variations of normal force stress $T_{zz}(=t_{zz}/P)$ with distance x.

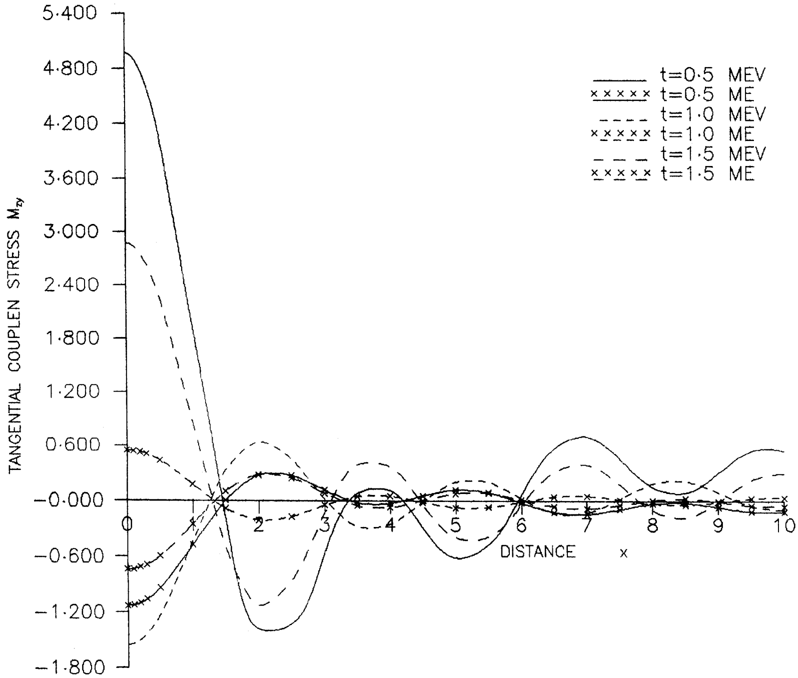

Figure 8. Variations of tangential couple stress M_{zy} ($= m_{zy}/P$) with distance x.

The computations were carried out for three values of non-dimensional time $t = 0{\cdot}5, 1{\cdot}0, 1{\cdot}5$ at $z = 1$ in the range $0 \leqslant x \leqslant 10$. The distribution of non-dimensional tangential couple stress $M_{zy}(= m_{zy}/p)$, non-dimensional normal displacement $U_z(= u_z/p)$, non-dimensional normal force stress $T_{zz} = (= t_{zz}/P)$ and non-dimensional volume fraction field $Q(= q/p)$ with non-dimensional distance 'x' have been shown in Figures 1–8. For all three times, the solid line, small dashed line and large dashed lines without asterisk symbol predicted the variations of components for micropolar elastic medium with void (MEV) whereas the lines with asterisk symbols are for micropolar elastic medium (ME). Six curve predicted by three different times and two (MEV, ME) theories. Solid lines either with asterisk symbol (–*—*) or without asterisk symbol (——) correspond to the case when $t = 0{\cdot}5$, small dashed lines either without asterisk symbol (––––) or with asterisk symbol (-*-------*) corresponds to the case when $t = 1{\cdot}0$, large dashed lines either without asterisk symbol (– – –) or with asterisk symbol (-*---*) corresponds to the case when $t = 1{\cdot}5$.

5.1. CASE I DELTA DISTRIBUTION NORMAL POINT SOURCE

The variation of normal displacement, volume fraction field, normal force stress and tangential couple stress with distance x for MEV and ME when instantaneous normal point source is applied have been shown in Figures 1, 2, 3, 4 respectively.

Figure 1 shows the variations of normal displacement U_z with distance x. The value of U_z for all three times greater for MEV than for ME in the range of $0 \leqslant x \leqslant 1{\cdot}5$ and $6 \leqslant x \leqslant 10$. For MEV the value of U_z for time 0·5 is smaller than that for times 1·0 and 1·5 in the range $0 \leqslant x \leqslant 1{\cdot}5$ and the trend of variation is oscillatory in the whole range.

The variations of volume fraction $Q(= q/P)$ with distance x are shown in Figure 2. The values of Q is smaller for time 1·0, 1·5 than that for times 0·5 in the range $0 \leqslant x \leqslant 1{\cdot}5$ and

$6 \leqslant x \leqslant 10$. For time 1·0 and 1·5, the value of Q initially increases, whereas it decreases for time 0·5.

The variations of normal force stress $T_{zz}(=t_{zz}/P)$ with distance x area shown in Figure 3. The value of T_{ZZ} for both MEV and ME increases sharply in the range $0 \leqslant x \leqslant 2$ and then starts to oscillate. For times 0·5 and 1·5, the values of T_{zz} are very small and the behavior is oscillatory for both ME and MEV.

Figure 4 shows the variation of $M_{zy}(=m_{zy}/P)$ with distance x. For MEV, its value initially decreases sharply for all three times whereas for ME initially the value increases as M_{zy} increases. In the ranges $0 \leqslant x \leqslant 1{\cdot}5$ and $3 \leqslant x \leqslant 4{\cdot}5$, the values for MEV are greater than that for ME. The behavior of variation is oscillatory for both ME and MEV but with opposite signs. For MEV as time increases, the value of M_{zy} decreases in the ranges $0 \leqslant x \leqslant 1{\cdot}5$, $3 \leqslant x \leqslant 5$ and $6 \leqslant x \leqslant 7$.

5.2. CASE II CONTINUOUS NORMAL POINT SOURCE

The variation of normal displacement, volume fraction field, normal force stress and tangential couple stress with distance x for MEV and ME when continuous normal point source is applied have been shown in Figures 5, 6, 7, 8 respectively.

Figure 5 shows the variations of normal displacement U_z with distance x. The value of U_z for all three times are greater for MEV than for ME in the ranges $0 \leqslant x \leqslant 1{\cdot}5$ and $6 \leqslant x \leqslant 10$ and smaller in the rest of the range. For both MEV and ME, the value of U_z increases as time decreases in the ranges $0 \leqslant x \leqslant 1{\cdot}5$ and $6 \leqslant x \leqslant 10$. For both MEV and ME and all three times, the value of U_z decreases sharply in initial range of x. The behavior of variations is oscillatory in the whole range.

The variations of volume fraction $Q(=q/P)$ with distance x are shown in Figure 6. The value of Q is smaller for time 1·0, 1·5 than that for time 0·5 initially. For times 1·0 and 1·5, the value of Q initially increases whereas it decreases for time 0·5. Its value is small for time 0·5 comparatively in the whole range.

The variations of normal force stress $T_{zz}(=t_{zz}/P)$ with distance x are shown in Figure 7. For the case of ME, the value of T_{zz} increases sharply in range $0 \leqslant x \leqslant 2$ and then starts oscillating and the behavior of variation for all three times are similar. For the case of MEV, the values of T_{zz} are very small for time 1·5 and their value decreases as time increases initially.

Figure 8 shows the variation of $M_{zy}(=m_{zy}/P)$ with distance x. For MEV, its value initially decreases sharply for times 0·5 and 1·5 whereas it increases for time 1·0. For ME, its value initially increases sharply for times 0·5 and 1·5 whereas it decreases for time 1·0. For ME initially, the value as M_{zy} increases. In the ranges $0 \leqslant x \leqslant 2$ and $6 \leqslant x \leqslant 10$, the values for MEV are greater than that for ME for times 0·5 and 1·5. The behaviors of the variations are oscillating for both ME and MEV but with opposite signs.

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