Journal of Sound and Vibration (2002) **256**(1), 173–178
doi:10.1006/jsvi.2001.4175, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

SURFACE WAVE PROPAGATION IN A MICROPOLAR THERMOELASTIC MEDIUM WITHOUT ENERGY DISSIPATION

R. KUMAR AND S. DESWAL

Department of Mathematics, Kurukshetra University, Kurukshetra-136119, Haryana, India.
E-mail: search@vidya.kuk.ernet.in

1. INTRODUCTION

Surface waves play an important role in the study of earth quakes, seismology and geophysics. The propagation of surface elastic waves along with other geophysical and geothermal data carry information about the structure and distribution of underground magnum. The surface wave propagation as part of exploration seismology helps in various economic activities like tracing of hydrocarbons and other mineral ores which are essential for various developmental activities like construction of dams, huge buildings, roads and bridges, etc.

Thermoelasticity theories which admit a finite speed for thermal signals (second sound) have aroused much interest in the last three decades. Recently, relevant theoretical developments in the theory of thermoelasticity on the subject of finite velocity of heat propagation are due to Green and Naghdi [1–3], which provide basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems. An important feature of this theory which is not present in other thermoelasticity theories is that, this theory does not accomodate dissipation of thermal energy and is also known as generalized theory of thermoelasticity.

"Micropolar elasticity" termed by Eringen [4] is used to describe deformation of elastic media with oriented particles. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. Typical examples of such materials are granular media and multimolecular bodies, whose microstructures act as an evident part in their macroscopic responses. The physical nature of these materials needs on asymmetric description of deformation, while theories for classical continua fail to accurately predict their physical and mechanical behaviour. For this reason, micropolar theories were developed by Eringen [4, 5] for elastic solids and fluids and are now universally accepted.

The theory of micropolar thermoelasticity has been a subject of intensive study and was developed by extending the theory of micropolar continua to include thermal effects by Eringen [6] and Nowacki [7]. Different authors [8–15] discussed different type of problems in generalized thermoelasticity/micropolar elasticity/micropolar generalized thermoelasticity. The present study is concerned with the problem of surface wave propagation in a micropolar generalized thermoelastic half-space without energy dissipation. As such the practical relevance of our problem in the Earth, planetary and engineering sciences is self-evident.

2. PROBLEM FORMULATION AND SOLUTION

A homogeneous, isotropic, micropolar generalized thermoelastic solid occupying the half-space is considered in an undisturbed state and initially at uniform temperature T_0.

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The rectangular Cartesian co-ordinates are introduced having origin on the surface $z = 0$ and z-axis is chosen in the direction of increasing depth. We are discussing a two-dimensional problem (xz-plane) with wave front parallel to the y-axis. We consider the possibility of a type of wave travelling in the direction Ox, so that the disturbance is largely confined to the neighbourhood of the boundary and at any instant all particles in any line parallel to Oy have equal displacements.

Following Eringen [4] and Green and Naghdi [3] the field equations and constitutive relations for a micropolar generalized thermoelastic solid without body forces, body couples and heat sources can be written as

$$(\lambda + \mu)\,\nabla(\nabla\cdot\mathbf{u}) + (\mu + K)\,\nabla^2\mathbf{u} + K\,\nabla\times\boldsymbol{\phi} - v\nabla T = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{1}$$

$$(\alpha + \beta + \gamma)\,\nabla(\nabla\cdot\boldsymbol{\phi}) - \gamma\,\nabla\times(\nabla\times\boldsymbol{\phi}) + K\,\nabla\times\mathbf{u} - 2K\boldsymbol{\phi} = \rho j\frac{\partial^2\boldsymbol{\phi}}{\partial t^2}, \tag{2}$$

$$K^*\,\nabla^2 T = \rho C^*\frac{\partial^2 T}{\partial t^2} + vT_0\frac{\partial^2}{\partial t^2}(\nabla\cdot\mathbf{u}) \tag{3}$$

and

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijr}\phi_r) - v\delta_{ij}T, \tag{4}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \tag{5}$$

where λ, μ, K, α, β γ are material constants, ρ is the density, j is the microinertia, $v = (3\lambda + 2\mu + K)\alpha_t$, α_t is the coefficient of linear thermal expansion, C^* is the specific heat at constant strain and $K^*(= C^*(\lambda + 2\mu)/4)$ is a material constant characteristic of the theory. $T(x, z, t)$ is the temperature change above the uniform reference temperature T_0, $\mathbf{u}$ is the displacement vector, $\boldsymbol{\phi}$ is the microrotation vector, t_{ij} are the components of force stress and m_{ij} are the components of couple stress.

For the two-dimensional problem, we have

$$\mathbf{u} = (u_x, 0, u_z), \quad \boldsymbol{\phi} = (0, \phi_2, 0), \tag{6}$$

where the displacement components u_x and u_z may be written in terms of potential functions $q(x, y, t)$ and $\psi(x, z, t)$ as

$$u_x = \frac{\partial q}{\partial x} + \frac{\partial\psi}{\partial z}, \quad u_z = \frac{\partial q}{\partial z} - \frac{\partial\psi}{\partial x}, \quad \text{where} \quad \psi = (-\mathbf{U})_y \tag{7}$$

and we assume time harmonic variations for the wave propagating in the positive direction of x-axis as

$$[q, \psi, \phi_2, T] \cong [q(z), \psi(z), \phi_2(z), T(z)]\,\mathrm{e}^{ik(x - ct)}, \tag{8}$$

where, $\omega(= kc)$ is the frequency of the wave, k is the wave number and c is the phase velocity. Substitution of equations (6)–(8) in equations (1)–(3) and then elimination of ϕ_2 and T from the resulting equations would give us two differential equations, solution of which, satisfying the radiation conditions can be written as

$$q = [A_1 e^{-\zeta_1 z} + A_2 e^{-\zeta_2 z}] e^{\imath(kx - \omega t)}, \tag{9}$$

$$T = [q_1 A_1 e^{-\zeta_1 z} + q_2 A_2 e^{-\zeta_2 z}] e^{\imath(kx - \omega t)}, \tag{10}$$

$$\psi = [A_3 e^{-\zeta_3 z} + A_4 e^{-\zeta_4 z}] e^{\imath(kx - \omega t)}, \tag{11}$$

$$\phi_2 = [q_3 A_3 e^{-\zeta_3 z} + q_4 A_4 e^{-\zeta_4 z}] e^{\imath(kx - \omega t)}, \tag{12}$$

where

$$\zeta_{1,2}^2 = \frac{-A \pm \sqrt{A^2 - 4B}}{2}, \quad \zeta_{3,4}^2 = \frac{-D \pm \sqrt{D^2 - 4E}}{2},$$

$$q_{1,2} = \frac{1}{a_6}[\zeta_{1,2}^2 - k^2 + a_0\omega^2], \quad q_{3,4} = \frac{1}{a_4}[\zeta_{3,4}^2 - k^2 + a_3\omega^2],$$

$$A = -2k^2 + \omega^2(a_0 + a_5 + a_6\varepsilon),$$

$$B = k^4[1 - c^2(a_0 + a_5 + a_6\varepsilon) + a_0 a_5 c^4],$$

$$D = -2k^2 + \omega^2(a_2 + a_3) - 2a_1 + a_1 a_4,$$

$$E = k^4 + k^2[-\omega^2(a_2 + a_3) + 2a_1(1 - a_3 c^2) + a_2 a_3 k^2 c^4 - a_1 a_4],$$

$$a_1 = \frac{K}{\gamma}, \quad a_2 = \frac{\rho j}{\gamma}, \quad a_3 = \frac{\rho}{\mu + K}, \quad a_4 = \frac{K}{\mu + K},$$

$$a_5 = \frac{\rho C^*}{K^*}, \quad a_6 = \frac{v}{\lambda + 2\mu + K}, \quad a_0 = \frac{\rho}{\lambda + 2\mu + K}, \quad \varepsilon = \frac{vT_0}{K^*}. \tag{13}$$

A_i $(i = 1, \ldots, 4)$ are constants and $\mathrm{Re}(\zeta_i) \geqslant 0$, $(i = 1, \ldots, 4)$.

3. BOUNDARY CONDITIONS

We assume the plane boundary to be isothermal and the boundary conditions are the vanishing of stresses and temperature on the free surface, i.e.,

$$t_{zz} = t_{zx} = m_{zy} = T = 0 \quad \text{at } z = 0. \tag{14}$$

Making use of equations (4)–(8) and (9)–(12) in the above boundary conditions, we obtain four homogeneous equations in four unknowns namely A_1, A_2, A_3 and A_4. Elimination of these four unknowns gives the wave velocity equation for the Rayleigh waves in a micropolar generalized thermoelastic medium as

$$\begin{aligned} &q_1 q_3 \zeta_3[e_2 e_4 - k^2\zeta_2\zeta_4(2\mu + K)^2] - q_1 q_4 \zeta_4[e_2 e_3 - k^2\zeta_2\zeta_3(2\mu + K)^2] \\ &\quad - q_2 q_3 \zeta_3[e_1 e_4 - k^2\zeta_1\zeta_4(2\mu + K)^2] + q_2 q_4 \zeta_4[e_1 e_3 - k^2\zeta_1\zeta_3(2\mu + K)^2] = 0, \end{aligned} \tag{15}$$

where

$$e_i = (\lambda + 2\mu + K)\zeta_i^2 - \lambda k^2 - v q_i, \quad i = 1, 2$$

and

$$e_j = (\mu + K)\zeta_j^2 + \mu k^2 - Kq_j, \quad j = 3, 4 \tag{16}$$

Equation (15) represents the Rayleigh-wave velocity equation relating the phase velocity c to the wave length $2\pi/k$. The wavelength is a multivalued function of phase velocity (each value corresponding to the different mode of propagation) and hence indicates the dispersive nature of the wave.

Particular cases: (i) In the absence of micropolar effect (i.e., when $K = \alpha = \beta = \gamma = 0$), the frequency equation (15) will reduce to

$$q_1[e_2 e_3^* - 4\mu^2 k^2 \zeta_2 \zeta_3^*] - q_2[e_1 e_3^* - 4k^2 \mu^2 \zeta_1 \zeta_3^*] = 0, \tag{17}$$

where

$$\zeta_3^{*2} = k^2(1 - a_3^* c^2), \quad \mathrm{e}_3^* = \mu(\zeta_3^{*2} + k^2), \quad a_3^* = \rho/\mu \tag{18}$$

and $\zeta_{1,2}$ are same as defined by equation (13), with $K = 0$.

(ii) Neglecting the thermal effect, the frequency equation (15) reduces to

$$q_3\zeta_3[e_1^* e_4 - (2\mu + K)^2 k^2 \zeta_1^* \zeta_4] - q_4\zeta_4[e_1^* \mathrm{e}_3 - (2\mu + K)^2 k^2 \zeta_1^* \zeta_3] = 0, \tag{19}$$

where

$$e_1^* = (\lambda + 2\mu + K)\zeta_1^{*2} - \lambda k^2, \quad \zeta_1^{*2} = k^2(1 - a_0 c^2). \tag{20}$$

Equation (19) is the Rayleigh-wave velocity equation in a micropolar elastic half-space. The frequency equation (19) is same as obtained by De Nath and Sengupta [11].

4. NUMERICAL RESULTS AND DISCUSSION

Equation (15) determines the velocity of Rayleigh waves in a micropolar generalized thermoelastic medium. In order to study the problem numerically, frequency equation (15) is solved by calculating the velocity ratio c/c_1 $(c_1^2 = 1/a_0)$ for given values of the dimensionless wave number kH, where H is a parameter of dimension length. We take the case of magnesium crystal [16, 17] like material for numerical calculations. The physical constants used are

$$\lambda = 9{\cdot}4 \times 10^{11}\,\mathrm{dyn/cm^2}, \quad \mu = 4{\cdot}0 \times 10^{11}\,\mathrm{dyn/cm^2},$$

$$K = 1{\cdot}0 \times 10^{11}\,\mathrm{dyn/cm^2}, \quad \gamma = 0{\cdot}779 \times 10^{-4}\,\mathrm{dyn},$$

$$j = 0{\cdot}2 \times 10^{-15}\,\mathrm{cm^2}, \quad \rho = 1{\cdot}74\,\mathrm{gm/cm^3},$$

$$C^* = 0{\cdot}104 \times 10^7\,\mathrm{cal/g^\circ C}, \quad \nu = 0{\cdot}0268 \times 10^9\,\mathrm{dyn/cm^2\,{}^\circ C},$$

$$T_0 = 23^\circ\mathrm{C}.$$

Using the above values of parameters, the dispersion curves showing the variation of velocity ratio with non-dimensional wave number are shown in Figure 1. The variations of

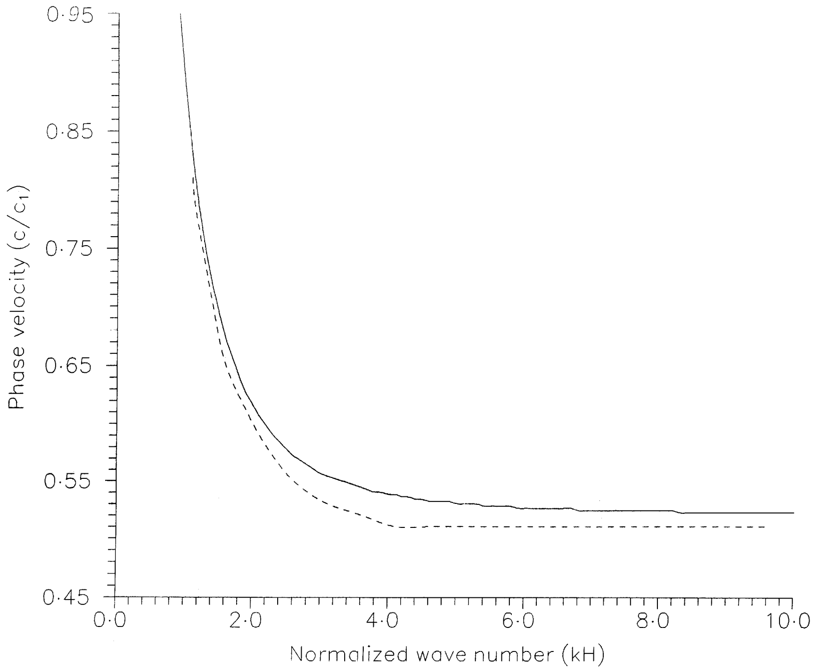

Figure 1. Variation of phase velocity with wave number. ——MGT; - - - - ME.

velocity ratio for micropolar generalized thermoelastic (MGT) medium and micropolar elastic (ME) medium are shown by solid line (——) and dashed line (- - -) respectively. It is found that a number of modes of propagation exist. Only the variations of fundamental modes of propagation for both the cases are presented graphically. From 1, it is evident that due to thermal effect the values of velocity ratio for MGT medium are large in comparison to ME medium. It is seen that the velocity of propagation decreases gradually as the wave number increases for some initial value of kH and then becomes almost constant after a certain range for both the media (MGT and ME). Thus it is observed that the wave velocity equations (15) and (19) for MGT and ME media, respectively, are dispersive, whereas for generalized thermoelastic medium, we obtain the Rayleigh root (0·290025) for the frequency equation (17) and for an elastic medium the Rayleigh root is 0·526022.

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