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# EXCITATION OF HELMHOLTZ RESONATOR BY GRAZING AIR FLOW

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## 1. INTRODUCTION

A problem of flow-excited sound in cavity resonators is important in the design of wind tunnels, gas transport systems and aircraft because the induced sound can cause an intense noise with discrete frequency components and considerable vibrations in surrounding mechanical structures. The details of the flow-acoustic interaction in a resonator opening have been the subject of several theoretical studies. In the model of Elder [1] the shear layer displacement was shaped by a Kelvin–Helmholtz wave, while an acoustic response of the resonant system was modelled by an equivalent impedance circuit of a resonator adopted from organ pipe theory [2]. Taking into account a relation between fluctuation of volume drive flow and an induced acoustic flow in the resonator opening, Elder found the feedback equation for oscillations. A concept of feedback mechanism was also exploited in the theoretical study of Mast and Pierce [3]. In their approach, the resonator-flow system was treated as an autonomous non-linear system and limit cycles of the system were found using describing-function analysis. In the study of Bruggeman [4] flow in the opening was modelled by discrete vortices with a circulation growing linearly in a time. This model showed that the excitation of the oscillation is determined by the position of the vortex during the acoustic cycle and by the distribution of the vorticity.

In this letter, the theory is outlined to describe an excitation of a Helmholtz resonator by a grazing air flow. A one-dimensional, lumped-element model of the resonator is applied and the excitation is assumed to be associated with an aerodynamic force generated by compact vortices shed periodically from a separation edge of the resonator mouth. The theory is used to determine the frequency and the amplitude of acoustic oscillations versus flow speed.

## 2. THEORETICAL MODEL

The problem of the excitation of a Helmholtz resonator by a grazing flow is illustrated in Figure 1. The resonator consists of a cavity with the volume $V_r$ and a narrow, rectangular neck. The mouth of the resonator lies in a smooth rigid surface and occupies the portion $0 \leqslant x \leqslant l$, $0 \leqslant z \leqslant s$. The dimension $s$ of the mouth greatly exceeds the dimension $l$ of the mouth edge which is parallel to the flow direction. The effect of the external excitation is assumed to be equivalent to the application of a uniform time dependent pressure perturbation $p$ over the mouth of the resonator. Thus, the differential equation for the

 

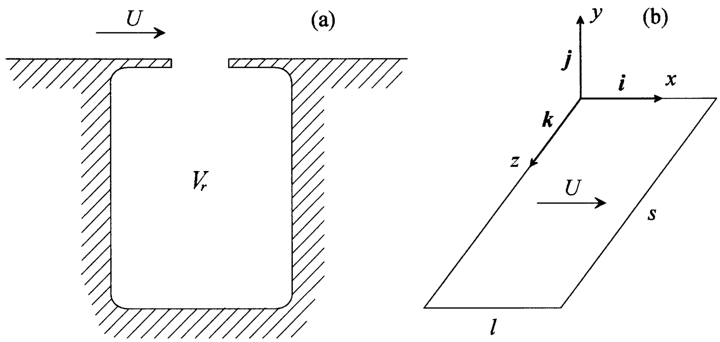


Figure 1. (a) Grazing flow over Helmholtz resonator of volume $V_r$ having rectangular orifice of dimensions $l$ and $s$, (b) location of co-ordinate system in resonator orifice.

volume displacement $\xi$ of the slug of air in the resonator neck can be written as [5]

$$M\frac{d^2\xi}{dt^2} + R\frac{d\xi}{dt} + \frac{\xi}{C} = p, \tag{1}$$

where $M$ is the acoustic inertance of the mass of air in the resonator neck, $C = V_r/\rho c$ is the acoustic compliance of the air in the cavity ($\rho$ is the air density, $c$ is the speed of sound) and $R$ is the acoustic resistance which provides the damping of the motion due to viscous effects in the neck and the energy loss by a radiation.

The excitation of the resonator is caused by a feedback mechanism between disturbances in a shear layer and the acoustic field in the resonator neck. The feedback is provided by the acoustic particle velocity $\mathbf{u}$, which induces disturbances in the shear layer at the separation edge. When the amplitude of $\mathbf{u}$ is moderate, the disturbed shear layer rolls up into a discrete vortex because of non-linear saturation. As was shown by measurements [6, 7], the vortex is shed from the separation edge at the instant when the velocity $\mathbf{u}$ is crossing the zero level and is directed into the resonator cavity. If it is assumed that the vortex shedding occurs at the moment $t = 0$, then the oscillating particle velocity $\mathbf{u}$ is given by

$$\mathbf{u} = -\mathbf{j}u_a\sin(\omega t), \qquad u_a > 0, \tag{2}$$

where $\omega$ is the oscillation frequency. As the vortex is convected downstream, it interacts with acoustic field and produces the acoustic energy which reinforces the oscillation. The instantaneous acoustic power generated by a vortex moving within the sound field can be calculated from Howe's formula [8]

$$P(t) = \mathbf{F}\cdot\mathbf{u}, \qquad \mathbf{F} = \int_V \mathbf{f}\,dV, \tag{3}$$

where $\mathbf{f} = -\rho(\mathbf{\Omega}\times\mathbf{v})$ is the force per unit volume which may be considered as the force acting on the acoustic field, $\mathbf{\Omega} = \nabla\times\mathbf{v}$ is the vorticity vector and $\mathbf{v}$ is the local fluid velocity. The integral is carried out over the volume $V$ where the vorticity $\mathbf{\Omega}$ is non-zero. A transfer of an energy from the vortical field to the acoustic field occurs if the average power $\mathscr{P}$ in the oscillation period is positive.

In order to describe a vorticity distribution, the model of Bruggeman [4] is used and the assumption model that all the vorticity, which is shed from the separation edge, is concentrated in a compact line vortex. The circulation of the vortex increases linearly with the time according to $\Gamma(t) = 0{\cdot}5U^2t$. The convective velocity $U_c$ of the vortex is constant and

TABLE 1

*Limit values of S for first five modes ($S_l < S < S_u$)*

| Mode number | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $S_l$ | 4·49 | 10·90 | 17·22 | 23·52 | 29·81 |
| $S_u$ | 7·72 | 14·06 | 20·37 | 26·67 | 32·96 |

close to a half of flow velocity. Now, consider the situation when the force **f** is produced by one vortex in the oscillation period $T$ only. This corresponds to the case $\tau \leqslant T$, where $\tau = l/U_c$ is the time at which the vortex travels from the separation edge to the downstream edge. The force **f** is then given by

$$\mathbf{f} = \begin{cases} \mathbf{j}\rho U_c \Gamma(t)\delta(x - U_c t)\delta(y), & 0 \leqslant t \leqslant \tau, \\ 0, & \tau < t \leqslant T, \end{cases} \tag{4}$$

where $0 \leqslant x \leqslant l$ and $0 \leqslant z \leqslant s$. When $\tau$ satisfies the condition $T < \tau \leqslant 2T$, the driving force is generated by two vortices in the oscillation period. In this case, the force **f** in the resonator mouth may be written as

$$\mathbf{f} = \begin{cases} \mathbf{j}\rho U_c\{\Gamma(t+T)\delta[x - U_c(t+T)] + \Gamma(t)\delta(x - U_c t)\}\delta(y), & 0 \leqslant t \leqslant \tau - T, \\ \mathbf{j}\rho U_c \Gamma(t)\delta(x - U_c t)\delta(y), & \tau - T < t \leqslant T. \end{cases} \tag{5}$$

In the same manner, the force **f** may be found when it is produced by a larger amount of vortices during the oscillation period. Calculations of the average power $\mathscr{P}$ for the force **f** predicted for one vortex, two vortices as well as for more vortices have shown that $\mathscr{P}$ is non-zero only for the fundamental Fourier component of the force **F**, so it may be treated as "external" force acting on the resonator. This component has the form

$$\mathbf{F}_1 = -\mathbf{j}\frac{\rho A_0 U^2}{2\pi S}[S\sin(\omega t - S) - \cos(\omega t - S) + \cos(\omega t)], \tag{6}$$

where $A_0$ is the mouth area and $S = \omega\tau$ is a Strouhal number determined for the convection speed of the vortex. Thus, the expression for the power $\mathscr{P}$ is given by

$$\mathscr{P} = \langle \mathbf{F}_1 \cdot \mathbf{u} \rangle = \frac{\rho A_0 u_a U^2}{4\pi S}[S\cos(S) - \sin(S)], \tag{7}$$

where the angle brackets denote an average over the period. From equation (7) it is clear that the power $\mathscr{P}$ is positive when the following condition is satisfied:

$$S\cos(S) - \sin(S) > 0. \tag{8}$$

If it is assumed that the left-hand side of this expression is equal to zero then after simple transformations one obtains the equations for the limit values of $S$:

$$S + \tan^{-1}(1/S) = 2n\pi \pm \pi/2, \quad n = 1, 2, 3 \ldots, \tag{9}$$

where $n$ is a hydrodynamic mode number. Calculation results of the lower ($S_l$) and the upper ($S_u$) limits of $S$ for first five modes are collected in Table 1. A solution of equations (9) for higher mode number yields the values of $S_l$ and $S_u$ close to $2\pi(n - \frac{1}{4})$ and $2\pi(n + \frac{1}{4})$, respectively.

Now, to simplify the analysis it is assumed that the velocity $u = u_a \sin(\omega t)$ represents the acoustic motion in the resonator neck. In this case from equation (6) it follows that the driving pressure is given by

$$p = \frac{\rho U^2}{2\pi S}\{[S\cos(S) - \sin(S)]\sin(\omega t) - [S\sin(S) + \cos(S) - 1]\cos(\omega t)\}. \tag{10}$$

Since $u = \mathrm{d}\xi/\mathrm{d}t$, substituting equation (10) into (1) yields

$$u_a = \frac{\rho U^2}{2\pi A_0 R S}[S\cos(S) - \sin(S)], \tag{11}$$

thus, the amplitude of acoustic oscillation is proportional to the square of the flow velocity $U$. It is important to note that this result is in accordance with experimental observations of Bruggeman [4] and Kriesels *et al.* [9]. To obtain a formula for the Strouhal number $S$ it is convenient to use the alternative form of equation (10)

$$p = \frac{\rho\beta U^2}{2\pi S}\sin(\omega t - S - \phi), \tag{12}$$

where

$$\beta = \sqrt{S^2 - 2[\cos(S) + \sin(S) - 1]}, \quad \phi = \tan^{-1}\left[\frac{1 - \cos(S)}{S - \sin(S)}\right], \tag{13}$$

which results from the fact that in equation (10) the expressions in square brackets are real and imaginary parts of $\exp(\mathrm{j}S)\{S - \sin(S) + \mathrm{j}[1 - \cos(S)]\}$. Taking into account the limit values of $S$ it can be immediately seen that an argument of the inverse tangent function always assumes non-negative values. Moreover, from condition (8) we have that for each $S$ the following relation must be satisfied:

$$\frac{1 - \cos(S)}{S - \sin(S)} < 1/S. \tag{14}$$

Thus, a maximum of $\phi$ does not exceed the value $1/S_l$. This means that $\phi$ is much smaller than $S$ and may be neglected in further considerations. In this case from equations (1) and (12) one obtains the following phase relationship:

$$S = \tan^{-1}\left[Q\left(\frac{1}{\Omega} - \Omega\right)\right] + 2n\pi, \quad n = 1, 2, 3\ldots, \tag{15}$$

where $\Omega = \omega/\omega_0$ is a non-dimensional frequency parameter, $\omega_0 = 1/\sqrt{CM}$ is a resonance frequency, $Q = \sqrt{M}/R\sqrt{C}$ is a quality factor and an inverse tangent function has the range $-\pi/2 < \tan^{-1}(\cdot) < \pi/2$. If acoustic parameters of the resonator and a relation between the vortex convection speed $U_c$ and the flow velocity $U$ are specified, a problem of finding the oscillation frequency $\omega$ resolves itself into a numerical solution of equation (15) for a given mode number $n$. Substitution of $\omega$ into equation (11) enables one to determine the oscillation amplitude.

It should be emphasized that in the proposed model a simple form of flow-acoustic coupling was assumed because force fluctuations, induced by vortices travelling across a resonator opening, excite an acoustic velocity which in turn triggers a periodic formation of new vortices. In previous theoretical methods a more complicated feedback mechanism was considered. For example, in the model of Mast and Pierce [3] it was assumed that a volume flow source associated with an interaction of discrete vortices with a downstream edge acts on a mass within the resonator opening and produces a driving pressure. An acoustic response of the resonator to this pressure is assumed to be similar to that of a lumped acoustic system. A total volume flow is a result of the excitation and it is considered as a sum of a source volume flow and an acoustic resonator flow. The resonator flow triggers a formation of new vortices at the upstream edge of the opening, closing a feedback loop. A criterion of sound excitation is that the loop gain is equal to unity. Since the presented model is similar to that of Mast and Pierce in some respects (the same phase relationship between an acoustic velocity and a vortex shedding, a driving pressure proportional to a square of flow velocity, a series lumped-element model of resonator), it will be interesting to compare equation (15) with the phase relationship

$$S = \tan^{-1}\left[\frac{Q^2 - \Omega^2(Q^2 - 1)}{Q\Omega^3}\right] + 2n\pi, \quad n = 1, 2, 3, \ldots, \tag{16}$$

obtained in reference [3] by means of a criterion of feedback loop gain. It is easy to see that for high-quality resonators the solutions of equations (15) and (16) are almost identical because the inverse tangent function in equation (16) can be expressed as

$$\tan^{-1}\left[\frac{Q^2 - \Omega^2(Q^2 - 1)}{Q\Omega^3}\right] = \tan^{-1}\left(\frac{\Omega}{Q}\right) + \tan^{-1}\left[Q\left(\frac{1}{\Omega} - \Omega\right)\right]. \tag{17}$$

## 3. COMPARISON BETWEEN THEORY AND EXPERIMENT

The theory presented here was compared to the experimental data of Nelson *et al.* [6] obtained for a Helmholtz resonator with the mouth dimension $l = 10$ mm. In the experiment, the oscillation frequency $f$ and the amplitude $p$ of a cavity pressure were measured as a function of $U_\infty$, the velocity far from the resonator mouth. The maximum cavity pressure was detected at a value of $U_\infty$ of 22 m/s. A velocity just above the mouth, which corresponds to the velocity $U$ in the theoretical model, was measured at the peak excitation only and it was found that $U = 12$ m/s. At this flow speed the convection velocity of the vortex inside the resonator mouth was 6 m/s. In comparison, we use the velocity $U$ instead of $U_\infty$ assuming that $U$ is proportional to $U_\infty$ by the factor 12/22. In the investigation of Nelson *et al.* parameters of the resonator needed by the theory were measured and these are

$$C = 3{\cdot}1 \times 10^{-9}\ \mathrm{m^4 s^2/kg}, \quad M = 22\ \mathrm{kg/m^2}, \quad Q_0 = 10, \tag{18}$$

where $Q_0$ is a quality factor at the resonance frequency. Since losses in the resonator were dominated by radiation, a value of $R$ was computed from the formula $R = \rho c k^2/2\pi$, where $k$ is the wave number.

By use of the theoretical method outlined above, calculations of frequency $f$ and amplitude $u_a$ of oscillation were carried out for the first mode number and the convection speed $U_c$ of the vortex equal to $U/2$. With a view to compare experimental data with the results of amplitude calculation, from the equation

$$p = u_a A_0/\omega C \tag{19}$$

the amplitude of a cavity pressure was computed using theoretical values of the velocity $u_a$. As shown in Figure 2(a), the theory predicts accurately the influence of the flow speed on the oscillation frequency. A larger difference between experiment and theory is noted in the case of oscillation amplitude, because the calculated pressure amplitude is about a factor three higher than the measured amplitude and the model predicts a peak excitation for the flow

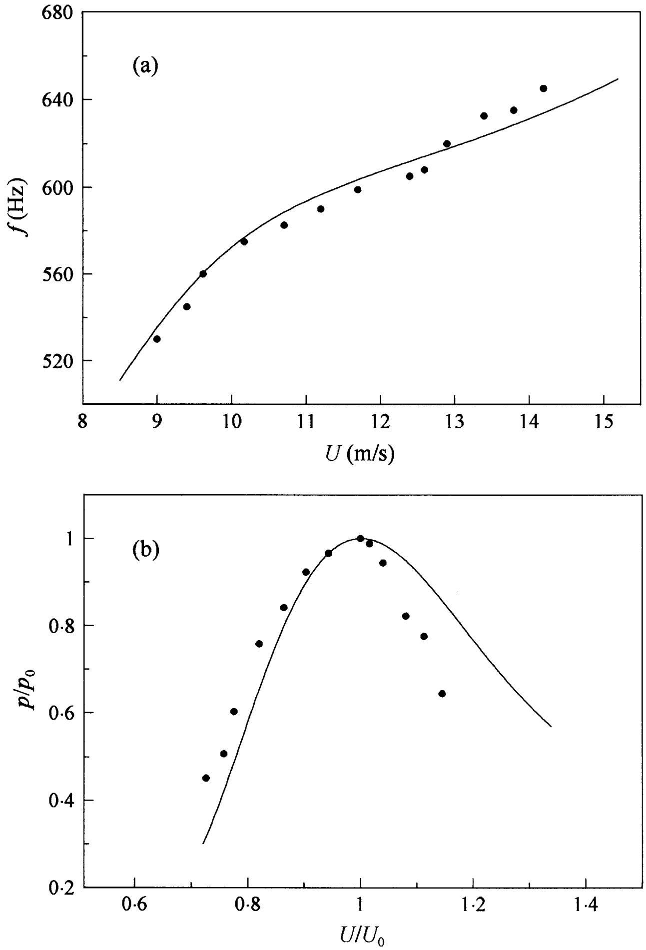


Figure 2. (a) Influence of flow speed on oscillation frequency, (b) normalized cavity pressure amplitude versus normalized flow speed. $(p_0)_{exp}/(p_0)_{theory} = 0{\cdot}306$, $(U_0)_{exp}/(U_0)_{theory} = 0{\cdot}889$. Lines: theory; dots: experimental data of Nelson *et al.* [6].

velocity $U = 13{\cdot}5$ m/s. However, if the pressure amplitude and flow velocity are normalized to $p_0$ and $U_0$, the values of $p$ and $U$ at the peak excitation, the agreement between the theory and the experiment seems to be reasonably good (Figure 2(b)).

## 4. SUMMARY AND CONCLUSIONS

A simple model of a flow-excited Helmholtz resonator has been presented. In the flow description, it was assumed that vorticity generation begins immediately downstream of the edge of flow separation and the shear layer disturbance is described as a concentrated vortex. The last assumption is supported by the fact that disturbances in the shear layer are large and cannot be modelled by the growth of small, wave-like disturbance in the shear layer as in the classical approach to this problem.

A phase between a sound field and the unsteady vortical field was determined by a relation between a shedding of the vortex and the acoustic particle velocity at the separation edge only. Thus, by contrast to models with a feedback loop explicitly taken into account, the phase relationship between the acoustic flow and the vortex–downstream edge interaction is not needed.

As was shown by the theory, a fundamental component of a force generated by moving vortices is effective in an acoustic power generation only; therefore, it may be treated as an external force driving a resonance system. From the final equations for oscillation, the frequency and the amplitude of cavity pressure can be calculated as a function of a flow speed. A support for the proposed theory is provided by favourable comparison between theoretical predictions and experimental data.

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