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LETTERS TO THE EDITOR

POTHOLE-INDUCED CONTACT FORCES IN A SIMPLE VEHICLE MODEL

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1. INTRODUCTION

Road surface irregularities play a crucial role in problems related to the interaction of moving vehicles with structures carrying them. When a vehicle traverses an irregularity, sizable dynamic contact (tire) forces can occur, which affect pavement wear and, if the irregularity is located on the bridge or its approach, result in an increase in bridge vibration. Note that uneven road profile is the main cause of high-magnitude bridge vibration. Indeed, the analysis of numerical results reported in the literature, as well as our own numerical experiments, show that, for vehicle velocities of interest, the dynamic forces acting on the bridge are small compared to the vehicle weight in the case of a smooth bridge surface and zero bridge and vehicle initial conditions. As a result, the dynamic increment [1]

$$DI = \frac{(\delta_{dyn} - \delta_{static})}{\delta_{static}} \times 100\%,$$

where δ_{dyn} is the peak dynamic deflection and δ_{static} is the peak static deflection, is also small in this case. It can be shown, e.g., that if the bridge is modeled as a simply supported beam, the DI is not greater than 10–15% for vehicle velocities encountered in practice. On the other hand, some researchers report very high values of the DI (more than 100% in reference [1]) measured in field experiments, which can only be explained by large dynamic tire forces due to road surface irregularities on the bridge or its approaches. As stated in reference [2], "the surface profile of a bridge and its approaches are fundamental to the response of the truck suspension and in turn the dynamic response of the bridge".

The aim of this work is to derive an analytical expression for the magnitude of the contact force arising after the traversal of an isolated road surface irregularity by a vehicle modeled as a single-degree-of-freedom (s.d.o.f.) oscillator. The purpose of this analytical result is to provide a useful design formula, which explicitly shows dependence of the maximum contact force on the oscillator and irregularity parameters. Although a real vehicle can

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adequately be modelled only by a system with many degrees of freedom, this formula is still very important. Two basic reasons to consider the s.d.o.f. oscillator model are as follows. First, the problem of finding tire forces due to road surface irregularities for a m.d.o.f. vehicle model can be reduced to that for independent s.d.o.f. oscillators. An appropriate technique has already been developed by the authors of this work, and a paper on this subject is being prepared for publication. Second, the use of a s.d.o.f. vehicle model is often justified in problems related to bridge vibration. This is in view of the fact that only vehicle vibration with a frequency close to the bridge fundamental frequency considerably affects the bridge response; i.e., vehicle vibrations at other frequencies need not be taken into account. For example, for long-span bridges with low first eigenfrequency, the high-frequency (10–15 Hz) axle-hop vibration has negligible effect on the bridge vibration, such that the modeling of a vehicle as a sprung mass with its eigenfrequency corresponding to the body-bounce mode may be sufficient. On the other hand, for a short-span bridge with a high fundamental frequency (10 Hz or higher), the low-frequency (1·5–4 Hz) body-bounce or pitch modes have small effect on the maximum bridge response [1], and one can consider a s.d.o.f. oscillator corresponding to the axle-hop mode.

2. PROBLEM STATEMENT AND ANALYSIS

We consider an isolated irregularity, further referred to as *pothole*, of the form

$$r(x) = \begin{cases} -\dfrac{a}{2}\left[1 - \cos\dfrac{2\pi x}{b}\right], & 0 \leqslant x \leqslant b, \\ 0, & x < 0,\ x > b, \end{cases} \tag{1}$$

where a and $b > 0$ are the pothole "depth" and "width", respectively (negative values of a correspond to bumps). As discussed in reference [3], this function "is capable of expressing diverse types of irregularities".

In this paper, we restrict our consideration to undamped oscillators. The resulting equations in the case of a damped oscillator are much more complicated; and the extension of the results obtained to this case will be discussed in another paper. The equation governing vertical vibration of the oscillator is well known to be

$$m\ddot{z} = -\,k(z(t) - r(vt)). \tag{2}$$

The oscillator initial conditions are assumed to be zero, $z(0) = \dot{z}(0) = 0$. Our goal is to find the magnitude of the elastic force $kz(t)$ acting on the road from the oscillator after passing the pothole.

For $t > T = b/v$, $r(vt) = 0$, the oscillator freely vibrates, and the magnitude of the elastic force is $F = kZ$, where Z is the amplitude of the oscillator free vibration,

$$Z = \sqrt{z^2(T) + (\dot{z}(T)/\omega_0)^2}. \tag{3}$$

The solution to equation (2) is well known to be

$$z(t) = \int_0^t \frac{\sin\omega_0(t-\tau)}{\omega_0}\,\omega_0^2 r(v\tau)\,\mathrm{d}\tau = -\frac{a\omega_0}{2}\int_0^t \sin\omega_0(t-\tau)[1 - \cos\omega_p\tau]\,\mathrm{d}\tau, \tag{4}$$

where $\omega_0 = \sqrt{k/m}$ is the oscillator eigenfrequency and $\omega_p = 2\pi v/b$. Integrating the right-hand side of equation (4) gives

$$z(t) = -\frac{a}{2}\cos\omega_0(t-\tau)\Big|_{\tau=0}^{t} + \frac{a\omega_0}{2}\left\{\frac{1}{2(\omega_p+\omega_0)}\cos[\omega_0 t-(\omega_p+\omega_0)\tau] - \frac{1}{2(\omega_p-\omega_0)}\cos[\omega_0 t+(\omega_p-\omega_0)\tau]\right\}\Big|_{\tau=0}^{t}. \quad (5)$$

Substituting $t = T$, noting that $\omega_p T = 2\pi$, and simplifying give

$$z(T) = -\frac{1}{2}\frac{a\omega_p^2}{\omega_p^2-\omega_0^2}[1-\cos\omega_0 T]. \quad (6)$$

Similarly,

$$\dot{z}(T) = -\frac{1}{2}\frac{a\omega_0\omega_p^2}{\omega_p^2-\omega_0^2}\sin\omega_0 T. \quad (7)$$

Substituting equations (6) and (7) into equation (3), we find the amplitude of the free vibration

$$Z = |a|\frac{\omega_p^2}{|\omega_p^2-\omega_0^2|}\left|\sin\frac{\omega_0 T}{2}\right|. \quad (8)$$

Further, by expressing T in terms of ω_p, $T = 2\pi/\omega_p$, and introducing the parameter

$$\gamma = \frac{\omega_0}{\omega_p} \equiv \frac{b\omega_0}{2\pi v} \equiv \frac{bf_0}{v}, \quad (9)$$

where $f_0 = \omega_0/2\pi$ is the oscillator eigenfrequency in Hertz, the number of parameters reduces to two, and equation (8) takes the form

$$Z = |a|\frac{|\sin\pi\gamma|}{|1-\gamma^2|}. \quad (10)$$

As can be seen, the dependence of Z on the pothole depth a is linear, and the remaining three parameters (ω_0, v, and b) are combined into one such that the amplitude of free vibration is governed by the unique function of one variable,

$$\Phi(\gamma) = \frac{|\sin\pi\gamma|}{|1-\gamma^2|}. \quad (11)$$

The function $\Phi(\gamma)$, depicted in Figure 1 by solid line, may be called the *dynamic amplification factor of the pothole*; it shows how many times the amplitude of the free vibration ($t > T$) is greater (less) than the pothole depth.

Multiplying both sides of equation (10) by k, we get the magnitude of the elastic force

$$F = F_{st}\Phi(\gamma), \quad (12)$$

where $F_{st} = ka$ is the static force required to extend the spring by a. Thus, $\Phi(\gamma)$ is the value of the magnitude of the dynamic force in terms of the static force ka.

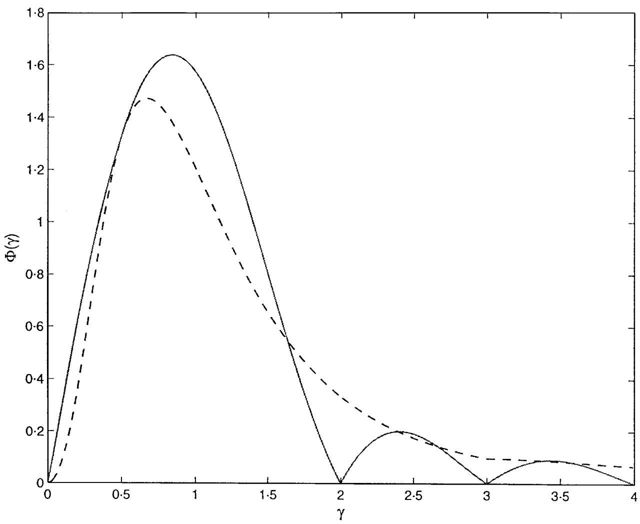

Figure 1. Dynamic amplification factor for the "cosine" pothole (1) (——) and the dimensionless contact force in the pothole (– – – –).

By using the plot for the dynamic amplification factor, one can immediately estimate the magnitude of the contact force arising after passing a pothole (bump) for any given set of parameters. Given the oscillator parameters, one can easily find the interval of "dangerous" pothole widths for a specified velocity, or, for a given pothole, the range of "dangerous" velocities.

In certain applications, the maximum of the contact force while the oscillator is in the pothole, $f_c(t) = k(z(t) - r(vt))$, $t < T$, may also be of interest. Substituting $t < T$ into equation (5) and simplifying the resulting equation, we find the contact force in the pothole,

$$f_c(t) = \frac{ka\omega_p^2(\cos\omega_0 t - \cos\omega_p t)}{2(\omega_p^2 - \omega_0^2)} \equiv \frac{ka(\cos\gamma\omega_p t - \cos\omega_p t)}{2(1-\gamma^2)}, \quad t < T.$$

Note that, since the total contact force is the sum of the oscillator weight and the dynamic contact force $f_c(t)$, only negative values of the latter force are of interest. Introducing the notation $\varphi = \omega_p t$ and noting that $0 \leqslant \varphi \leqslant 2\pi$, we find that the maximum of the dimensionless downward contact force is given by

$$\tilde{\Phi}(\gamma) = \frac{|\min_t f_c(t)|}{ka} = \left| \min_{0 \leqslant \varphi \leqslant 2\pi} \frac{\cos\gamma\varphi - \cos\varphi}{2(1-\gamma^2)} \right|.$$

The minimum on the right-hand side of the last equation has been found numerically, and the function $\tilde{\Phi}(\gamma)$ is depicted in Figure 1 by the dashed line. As can be seen, the dynamic amplification factor provides a sound estimate for the maximum contact force in the pothole (the upper bound in the range of γ, where the dynamic effect of a pothole is the most pronounced).

3. NUMERICAL ILLUSTRATION

The goal of this section is to demonstrate the application of the results obtained to the problem of bridge vibration due to a moving vehicle. Clearly, the presence of a pothole on the bridge or its approach results in excitation of vehicle vibration and, thus, in an additional dynamic force acting on the bridge, the effect of which is not known *a priori*. However, this effect can be readily estimated in the special case where the frequency of the dynamic force (oscillator eigenfrequency f_0) matches the fundamental frequency of the bridge. It is evident from physical considerations that the maximum bridge displacement grows with increasing amplitude of the harmonic force. Thus, in this case—the worst from the bridge standpoint—, we can estimate the relative effect of different potholes on the bridge vibration and determine "dangerous" pothole widths. Substituting the vehicle velocity v, eigenfrequency f_0 and $\gamma = 0{\cdot}8$ into equation (9) and solving for b, we find the pothole width for which the amplitude of the dynamic force and, thus, the peak bridge response are maximized for the given simple vehicle model.

The numerical experiments described below illustrate this. For the bridge model, we considered a proportionally damped simply supported beam with a smooth surface. The beam parameters were taken from reference [4] and are as follows: length $L = 40$ m, bending stiffness $EI = 1{\cdot}275 \times 10^{11}$ Nm^2, and mass per unit length $\rho = 1{\cdot}2 \times 10^4$ kg/m. The damping was set to 1·5% of critical. The fundamental frequency of the beam is 3·20 Hz. The initial conditions of the beam in all experiments were zero. The vehicle moving with velocity $v = 20$ m/s was modelled by an oscillator of mass $m = 4 \times 10^4$ kg. A spring stiffness $k = 1{\cdot}6 \times 10^7$ N/m was employed to make the oscillator eigenfrequency $f_0 \approx 3{\cdot}18$ Hz match the fundamental frequency of the beam. To numerically solve the moving oscillator problem, the method described in references [5, 6] was used, which is based on the expansion of the solution in a series in terms of the beam eigenfunctions.

We considered three potholes of the same depth $a = 1$ cm but different widths, $b_1 = 0{\cdot}5$ m, $b_2 = 5$ m, and $b_3 = 12$ m, located on the approach to the beam immediately before its left end. The amplitude of the dynamic force arising after passing the pothole in each case can easily be determined from equations (9), (11), and (12). Indeed, substituting f_0, v, and b_i, $i = 1, 2, 3$, into equation (9), we find the corresponding values of γ: $\gamma_1 \approx 0{\cdot}08$, $\gamma_2 \approx 0{\cdot}8$, and $\gamma_3 \approx 1{\cdot}9$. As can be seen from Figure 1, the dynamic force in the case of the second pothole is most considerable ($\Phi(\gamma_2) \approx 1{\cdot}6$), whereas the first ($\Phi(\gamma_1) \approx 0{\cdot}25$) and third ($\Phi(\gamma_3) \approx 0{\cdot}1$) potholes do not result in very large dynamic forces. By means of equation (12), we easily find that the amplitudes of the dynamic forces in the three cases are about 10%, 67%, and 4%, respectively, of the oscillator weight and, thus, get an idea of the increase in the maximum beam displacement in each case.

The results of numerical modelling shown in Figure 2 substantiate the *a priori* conclusions. The solid thin curve shows the displacement of the beam mid-point in the case of the pothole of width $b_2 = 5$ m. The dashed (1) and dotted (2) curves show the displacements corresponding to the short ($b_1 = 0{\cdot}5$ m) and long ($b_3 = 12$ m) wavelength potholes. For comparison, the bold line in Figure 2 shows the time history of the displacement of the beam mid-point in the case where the oscillator enters the left end of the beam with zero initial conditions (no potholes).

This figure clearly demonstrates that the dynamic effect of a pothole strongly depends on its width. Note also that, although not shown in the figure, the bold line corresponding to the absence of a pothole is very close to the moving force solution (the solution obtained by assuming the moving force acting on the beam equal to the weight of the vehicle) (this is illustrated, e.g., in reference [4]), which, in turn, is close to the static solution. This substantiates the statement in the Introduction about the importance of considering road surface irregularities and illustrates the observation that matching the oscillator and bridge eigenfrequencies alone does not necessarily result in an increase in bridge vibration. Rather, the increase takes place when the road surface on the bridge or its approaches contains irregularities of appropriate wavelengths.

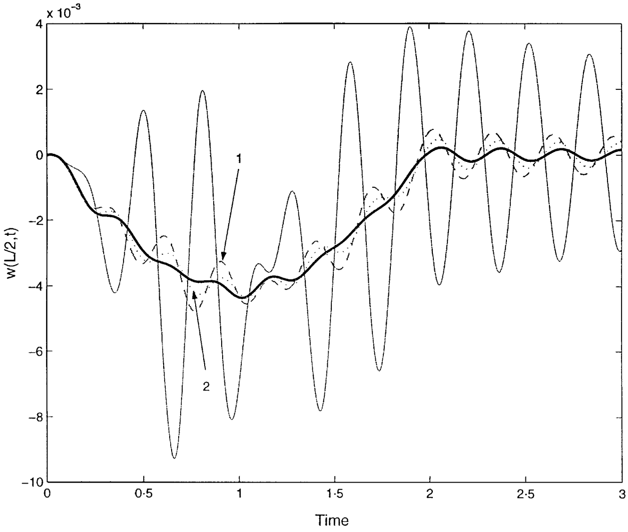

Figure 2. Displacements of the beam midpoint due to the moving oscillator: no potholes (——), pothole of width $b = 5$ m (——), $b = 0{\cdot}5$ m (– – – –, 1) and $b = 12$ m (· · · · · ·, 2).

It should be noted that, in practice, a small irregularity ($a = 1$ cm) cannot result in such an increase in bridge vibration, since, due to damping inherent in the vehicle suspension, the oscillator eigenvibrations reduce rapidly. However, a high-magnitude bridge response, similar to that depicted in Figure 2 by the thin solid line, may still take place if there are many road surface irregularities of appropriate wavelength located not only on the bridge approach but also on the bridge itself. The results of field experiments presented in reference [1, Figure 6] illustrate this point.

4. RELATIONSHIP TO SHOCK SPECTRUM

The reader may notice the resemblance of the plot in Figure 1 to a shock spectrum. For example, a plot similar to that in Figure 1 can be found in reference [7, Section 4.5, Figure 4.18]. It shows the dependence of the maximum response of a s.d.o.f. oscillator subjected to a half-sine force pulse on the oscillator eigenfrequency. This is not surprising, since traversing of a road surface irregularity by a moving oscillator can be interpreted as a force pulse acting on the stationary oscillator. It can be shown that, given that the pothole and the force pulse have the same shape, the dynamic amplification factor for the pothole and the shock spectrum are essentially the same functions. This implies, in particular, that one can take advantage of some results related to shock spectra available in the literature to get the dynamic amplification factor for a pothole.

To illustrate this, we will find the dynamic amplification factor for the pothole described by the half-sine function,

$$r(x) = \begin{cases} -a \sin \dfrac{\pi x}{b}, & 0 \leqslant x \leqslant b, \\ 0, & x < 0, \ x > b. \end{cases} \tag{13}$$

Potholes (1) and (13) have the same width and depth but differ in their first derivatives: function (1) is smooth, whereas the first derivative of function (13) has jumps at $x = 0$ and

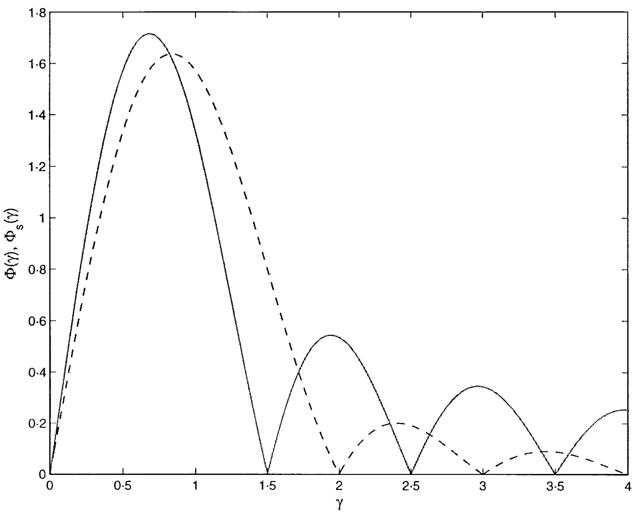

Figure 3. Dynamic amplification factor for the "half-sine" pothole (13) (——) and that for the "cosine" pothole (1) (- - - -).

$x = b$. Using the shock spectrum derived in reference [7, equation (4.53)], we immediately get the dynamic amplification factor for the "half-sine" pothole in terms of γ defined by equation (9),

$$\Phi_s(\gamma) = \frac{4\gamma}{|1 - 4\gamma^2|}|\cos \pi\gamma|. \tag{14}$$

The function $\Phi_s(\gamma)$ is shown in Figure 3. For comparison, the dynamic amplification factor for the "cosine" pothole (1) is depicted by the dashed line.

By means of the technique used in this paper (or that employed in reference [7]), one can find the dynamic amplification factors for potholes described by different functions. However, since any analytical description of an actual road surface irregularity is inevitably an approximation, the two types of potholes discussed above seem to be sufficient from a practical standpoint. Thus, pothole (1) can be used for modelling "smooth" isolated road surface irregularities, whereas equation (13) is appropriate for modelling an irregularity with "non-smooth" edges. For example, it can be checked directly that potholes described by polynomials of second and fourth degrees [8] are well approximated by the "half-sine" (13) and "cosine" (1) potholes. It is evident then from physical considerations that the dynamic amplification factors for those "polynomial" potholes are reasonably represented by the functions depicted in Figure 3.

5. CONCLUDING REMARKS

Analytical expressions for the magnitude of the dynamic contact force arising after traversing of a pothole by a moving s.d.o.f. oscillator have been derived for two types of potholes (bumps). It has been shown that the dependence of the magnitude of the dynamic force on pothole width, oscillator eigenfrequency and velocity is described by a unique, easily employed function of one variable, the dynamic amplification factor for the pothole. The dependence of the force on the pothole depth and the spring stiffness is shown to be linear.

The dynamic amplification factor can be efficiently used to evaluate "dangerous" pothole parameters for a given bridge and vehicle modelled as a s.d.o.f. oscillator or, for a given pothole, to find "dangerous" velocities of the vehicle. The analytical relations derived in this work can be used to predict dynamic forces in a more general case, where the vehicle is modeled as a m.d.o.f. system. This work is presently in progress.

The importance of considering uneven road profile in bridge-related problems has been noted and illustrated. Numerical results confirm that matching the vehile and bridge eigenfrequencies does not necessarily result in large bridge vibration, which will occur when the corresponding vehicle vibration is excited by road surface irregularities of appropriate wavelength. The relationship of the dynamic amplification factor to the shock spectrum has been noted.

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