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DYNAMIC RESPONSE OF A BEAM WITH A CRACK SUBJECT TO A MOVING MASS

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An iterative modal analysis approach is developed to determine the effect of transverse cracks on the dynamic behavior of simply supported undamped Bernoulli–Euler beams subject to a moving mass. The presence of crack results in higher deflections and alters the beam response patterns. In particular, the largest deflection in the beam for a given speed takes longer to build up, and a discontinuity appears in the slope of the beam deflected shape at the crack location. Crack effects become more noticeable as crack depth increases. The effect of the inertia force due to the moving mass is, in general, qualitatively similar and additive to the effect of the crack. The exact effect of crack and mass depends on the speed, time, crack size, crack location, and the moving mass level. Other approximate methods, namely a stationary mass model and a single iteration technique, are also evaluated. The stationary mass approach is useful for light moving masses ($< 20\%$ of beam mass) and cracks at mid-span. For other cases, the errors can be unacceptably large. The results of the single-iteration approximation are quite close to the iterative modal analysis approach, which indicates that this approximate solution is an excellent tool for the analysis of the moving mass problem.

1. INTRODUCTION

The effect of moving loads and masses on structures and machines is an important problem both in the field of transportation and in the design of machining processes. A moving load (or moving mass) produces larger deflections and higher stresses than does an equivalent load applied statically. These deflections and stresses are functions of both time and speed of the moving loads. This problem has received considerable attention in the literature. Recent investigations include the work of Chen [1], who showed how a general finite element code may be used to efficiently model bridge superstructures (such as I-shaped girders) under variable moving load. Wu and Thompson [2] studied the non-linear properties of railroad track foundations under a single moving wheel load. Marchesiello *et al*. [3] studied the response of a multi-span plate subjected to a seven-degrees-of-freedom moving vehicle. The study included torsional modes, plate surface irregularities, and moving load speed. Todd and Vohra [4] presented a theoretical approach to reconstruct the beam shape under static or moving load from strain measurements at a number of locations along beam length and taking shear deformation into account. The method was successfully applied to a two-span beam under a static load, and a simply supported beam under a moving load.

On the other hand, there has been growing interest in studying the vibration of cracked components and structures. The presence of cracks in a structure introduces local flexibility

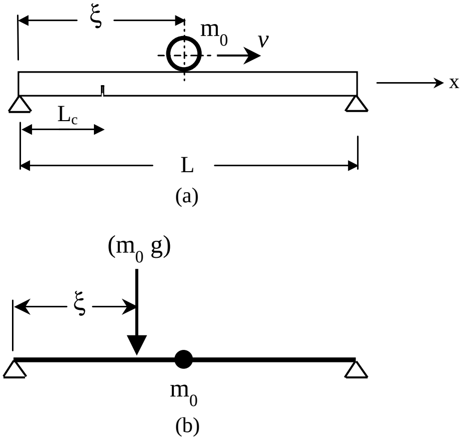

Figure 1. (a) Simple beam with a transverse crack subjected to a moving mass. (b) Stationary mass (*SM*) model.

and therefore alters the dynamic response of the structure. In one estimate, over 500 papers on the subject were published in two decades [5]. Recent investigations in this area include beams with multiple cracks [6], beams with breathing cracks [7], effect of cracks in rotating bladed disks [8], and non-linear response of a beam with a number of breathing cracks [9].

In most studies on transverse vibration of beams, the effect of a crack in the beam is determined based on modelling the cracked section as a rotational spring connecting two undamaged beam segments. Chondros and Dimarogonas [10] used finite element techniques to develop the terms of a (6×6) matrix for an arbitrary loading of a cracked beam section. In addition to our experimental evidence, there is a substantial body of evidence in the literature that confirms the usefulness of this spring model (see reference [5] for listing). Recently, Chondros and Dimarogonas [11] developed a continuous cracked beam theory to rigorously include singularities at the crack tip into the equations of motion. Their preliminary results confirm that the rotational spring model is a good approximation since its results agree well with the more rigorous theory. In this light, the rotational spring model will be used in this study to account for the crack compliance.

Very few studies were reported in the literature on the effect of cracks on the moving load or moving mass problems. Parhi and Behera [12] used the Runge–Kutta method to determine the deflection of a cracked circular shaft subjected to a moving mass. Recently, Mahmoud [13] used an equivalent static load approach to determine the stress intensity factors for a single- or a double-edge crack in a beam subjected to a moving load.

In this study, crack effects on the dynamic response of a beam subject to a moving mass are investigated. Attention is focused on the fundamental problem of simply supported, Bernoulli–Euler undamped beams, Figure 1(a). The solution of the undamaged beam is presented first. Then, an iterative modal analysis solution is developed to include crack effects on the beam response. In addition, an approximate stationary mass (*SM*) approach, Figure 1(b), recommended by Fryba [14] in his well-known monograph is also evaluated relative to the rigorous modal analysis developed herein. The *SM* method does not require iterations to account for the inertia force due to the moving mass. Numerical results are presented to show how the crack presence affects the deflection characteristics of the beam. The rotational spring model employed to model the crack is valid for open cracks only. The analysis does not include longitudinal motion or coupling between bending and torsional motions.

2. UNDAMAGED BEAM

The differential equation of a Bernoulli– Euler undamped beam subject to a moving mass is

$$EI\,\partial^4 w(x,t)/\partial x^4 + m\,\partial^2 w(x,t)/\partial t^2 = m_0(g - \ddot{w}(x,t))\,\delta(x-\xi), \quad (1)$$

where E is Young's modulus and I is the second moment of area of the beam cross-section about the Y-axis, w is the transverse deflection (in Z direction), $\ddot{w}(x,t)$ is the acceleration, t is the time ($t=0$ as the moving mass enters the beam from left to right, Figure 1(a)), m is the mass per unit length, m_0 is the moving mass, g is the gravitational acceleration, δ is the Dirac delta distribution, $\xi = vt$, and v is the constant mass speed. The boundary conditions for a simply supported beam are: $w(0,t) = w(L,t) = \partial^2 w(0,t)/\partial x^2 = \partial^2 w(L,t)/\partial x^2 = 0$, and the initial conditions are: $w(x,0) = \partial w(x,0)/\partial t = 0$.

The difficulty with equation (1) is that the unknown deflection $w(x,t)$ appears on both sides of the equation. Therefore, an iterative approach is used by rewriting the equation as

$$EI\partial^4 w_k(x,t)/\partial x^4 + m\,\partial^2 w_k(x,t)/\partial t^2 = P_{k-1}\delta(x-\xi), \quad (2)$$

in which

$$P_{k-1} = m_0(g - \ddot{w}_{k-1}(x,t)), \quad (3)$$

i.e., P represents both the weight of, and the inertia force due to, the moving mass.

In the first iteration ($k=1$), $\ddot{w}_0(x,t)$ is set to zero in the right-hand side of equation (3) and the resulting P is substituted into equation (2). Equation (2) is then solved to obtain $w_1(x,t)$, which, in fact, is the deflection due to a moving load of negligible mass. Then, $w_1(x,t)$ is numerically differentiated to obtain $\ddot{w}_1$, and then equations (3) and (2) are used to obtain $w_2(x,t)$, which is a better estimate of the deflection. This process is repeated until convergence occurs, i.e., stable values of w are obtained. A somewhat similar iterative approach was used for uncracked beams by Michaltos *et al.* [15].

The solution of the kth iteration of equation (2) may be written as

$$w_k(x,t) = \sum_{i=1}^{\infty} \Phi_i(x) q_i(t), \quad (4)$$

where $\Phi_i(x)$ is the eigenfunction, and $q_i(t)$ is the generalized displacement (the subscript i denotes the vibration mode number). Substituting equation (4) into equation (2), we get

$$m_{ii}\ddot{q}_i(t) + k_{ii}q_i(t) = Q_i(t), \quad (5)$$

where m_{ii} is the generalized mass, Q_i is the generalized force, and k_{ii} is the generalized stiffness;

$$Q_i(t) = \int_0^L P\delta(x-\xi)\Phi_i(x)\,\mathrm{d}x, \quad (6)$$

$$m_{ii} = m\int_0^L \Phi_i^2(x)\,\mathrm{d}x. \quad (7)$$

The solution of equation (5) is

$$q_i(t) = \int_0^t \zeta_i(t-\tau)Q_i(\tau)\,\mathrm{d}\tau, \quad (8)$$

where $\zeta_i(t)$ is the impulse response of the system which is (for undamped beams)

$$\zeta_i(t) = (1/m_{ii}\omega_i)\sin(\omega_i t), \tag{9}$$

$$Q_i(t) = P\Phi_i(vt) \quad \text{for } 0 \leqslant t \leqslant T \quad \text{and} \quad Q_i(t) = 0 \quad \text{for } t > T, \tag{10}$$

in which $T = L/v$ is the total time needed to traverse the beam. Substituting equations (9) and (10) into equation (8) one gets for the kth iteration

$$q_i(t) = (1/m_{ii}\omega_i)\int_0^t P_{k-1}\Phi_i(v\tau)\sin\omega_i(t-\tau)\,\mathrm{d}\tau, \tag{11}$$

where

$$P_{k-1} = m_0(g - \ddot{w}_{k-1}(v\tau, t)), \tag{12}$$

which shows that P_{k-1} is not constant, but changes as the moving mass traverses the beam from $\tau = 0$ to t.

For the undamaged beam, the eigenvalues ω_i, and the eigen-functions Φ_i are

$$\omega_i^2 = EI(i\pi)^4/mL^4, \quad \Phi_i(x) = \sin(i\pi x/L). \tag{13}$$

3. BEAM WITH A TRANSVERSE CRACK

A transverse crack in the beam increases the flexibility and therefore, reduces the eigenvalues and alters the eigenfunctions. The eigenvalues ω_i and the eigenfunctions Φ_i for the cracked beam must be used in the analysis instead of equation (13).

The eigenvalues and eigenfunctions for the cracked beam were calculated using a numerical approach that will be referred to as the $\mathscr{m}$-matrix approach. The beam with a transverse crack (Figure 1(a)) is discretized into n elements (or segments), as shown in Figure 2(a). The mass of each element (m_i) is lumped at the center of the element. These masses are then treated as being connected by massless rods having flexural rigidity EI. A typical segment is shown in Figure 2(b). Following Myklestad [16] and Tse *et al.* [17], the state at section i (immediately to the right of m_i) and the state at section ($i - 1$) (immediately to the right of m_{i-1}), with the inertia force on m_i added as shown in Figure 2(b), are related as follows (for undamped Euler–Bernoulli beams):

$$\begin{bmatrix} -w_i \\ \theta_i \\ M_i \\ V_i \end{bmatrix} = \begin{bmatrix} 1 & L & l^2/2EI & l^3/6EI \\ 0 & 1 & l/EI & l^2/2EI \\ 0 & 0 & 1 & l \\ \varepsilon & \Omega^2 m_i l & \beta & \gamma \end{bmatrix} \begin{bmatrix} -w_{i-1} \\ \theta_{i-1} \\ M_{i-1} \\ V_{i-1} \end{bmatrix}$$

or

$$\{\mathbf{S}_i\} = [\mathbf{Q}_i]\{\mathbf{S}_{i-1}\}, \tag{14}$$

where l is the distance between the masses m_i and m_{i-1}, w is the deflection, θ is the rotation, Ω is the frequency of vibration, $M = M_y$ is the bending moment, $V = V_z$ is the shearing force, $\varepsilon = (m_i\,\Omega^2)$, $\beta = (m_i\,\Omega^2\,l^2)/(2EI)$, $\gamma = 1 + (m_i\,\Omega^2\,l^3)/6EI$. The boundary conditions for a simply supported beam are: $w_0 = M_0 = M_{n+1} = w_{n+1} = 0$.

If a transverse crack exists in the beam, $[\mathbf{Q}]$ has to be modified to include the additional flexibility due to the crack presence. The steps to do this are as follows. First, the

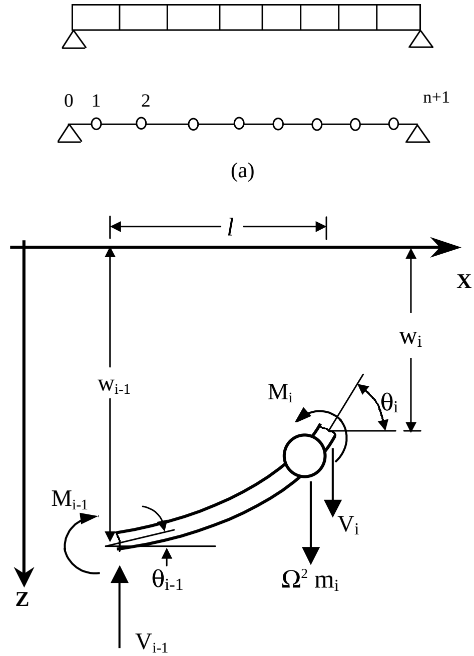

Figure 2. (a) Beam discretization. (b) Internal forces acting on a beam segment.

discretization of the beam (Figure 2(a)) should be made such that the crack is at the center of element j, and therefore, the crack is located at mass m_j. Due to the crack presence $\theta_j = \theta_{j0} + C\,M_j$, where θ_j is the rotation to the right of m_j in the presence of a crack, θ_{j0} is the rotation to the right of m_j if the crack were absent, C is the crack compliance, M_j is the bending moment at mass m_j. Now the modified recurrence equations for the section to the right of mass m_j are

$$\{\mathbf{S}_j\} = [\mathscr{m}_j]\,\{\mathbf{S}_{j-1}\}, \tag{15}$$

where

$$[\mathscr{m}_j] = \begin{bmatrix} 1 & L & l^2/2EI & l^3/6EI \\ 0 & 1 & (l/EI) + C & (l^2/2EI) + Cl \\ 0 & 0 & 1 & l \\ \varepsilon & \Omega^2 m_j l & \beta & \gamma \end{bmatrix}.$$

Equations (14) and (15) are used to determine the overall beam analysis in the form:

$$\{\mathbf{S}_{n+1}\} = [\mathbf{Q}_{n+1}]\,[\mathbf{Q}_n]\,\ldots\,[\mathscr{m}_j]\,[\mathbf{Q}_{j-1}]\cdots[\mathbf{Q}_2]\,[\mathbf{Q}_1]\,\{\mathbf{S}_0\}, \text{ i.e.,}$$

$$\{\mathbf{S}_{n+1}\} = [\mathbf{B}]\,\{\mathbf{S}_0\} \tag{16}$$

subject to the boundary conditions for a simply supported beam, i.e.,

$$\mathbf{B}_{12}\,\mathbf{B}_{34} - \mathbf{B}_{14}\,\mathbf{B}_{32} = 0. \tag{17}$$

Equation (17) is the basis for determining the eigenvalues (ω) using a root searching technique that was developed based on the well-known Newton–Raphson approach.

Convergence to the desired roots was achieved with no difficulty when the eigenvalues of the uncracked beam were used as initial values for the cracked beam cases. The eigenfunctions may then be obtained by using equation (14).

The crack compliance C may be determined by modelling the cracked section as a rotational spring connecting two undamaged beam segments. The stiffness of the rotational spring is determined using fracture mechanics and is incorporated in the $\mathscr{m}$-matrix approach. In this study, an expression for C was derived by using the results published by Tada [18] who gave the angle of rotation of a uniform strip with an edge crack under pure bending moment. In this case, for a rectangular section of height 'h' with a crack of depth 'a', C can be shown to be

$$C = \frac{2h}{EI}\left(\frac{a/h}{1 - a/h}\right)^2 \{5{\cdot}93 - 19{\cdot}69(a/h) + 37{\cdot}14(a/h)^2 - 35{\cdot}84(a/h)^3 + 13{\cdot}12(a/h)^4\}. \quad (18)$$

4. STATIONARY MASS (SM) MODEL

Fryba [14] recommended an approximate model (denoted herein SM) to account for the inertia force due to the moving mass. The model is based on adding a stationary concentrated mass (point mass), equivalent to the moving mass (m_0), at the beam center and then calculate the dynamic response due to a moving load equivalent to ($m_0 g$), Figure 1(b). In terms of the present formulation, the generalized deflection for the SM model is

$$q_i(t) = (1/m_{ii}\omega_i)\int_0^t (m_0\, g)\ \Phi_i(v\tau)\sin\omega_i(t-\tau)\,\mathrm{d}\tau. \quad (19)$$

The eigenvalues ω_i and the eigenfunctions Φ_i in this case are determined using the $\mathscr{m}$-matrix approach (equations (17) and (14)) but adding a concentrated mass at the beam center by setting $\varepsilon = \{(m_c + m_0)\Omega^2\}$ only at the center segment of the beam, where m_c is the mass of this center segment. Also, the generalized mass (m_{ii}) for the SM model is larger than m_{ii} for the uniform beam by the amount $[m_0\Phi_i^2(L/2)]$.

5. CRACK EFFECT ON THE DEFLECTION RESPONSE DUE TO A MOVING MASS

A transverse crack in the beam increases the flexibility and therefore, changes the free vibration response such that it reduces the eigenvalues and alters the eigenfunctions [19]. Crack presence also changes beam response to moving loads or masses. Equations (11) and (4) are used to obtain the dynamic response of a simple beam subjected to a moving mass. To illustrate this response, a beam of length 50 m, height $h = 1{\cdot}0$ m and width $b = 0{\cdot}5$ m was considered ($E = 2{\cdot}1 \times 10^{11}$ Pa, density $= 7860$ kg/m^3). The numerical results presented in the following are normalized in order to be valid for linear elastic, homogenous, isotropic materials. These results were obtained using six modes of vibration and the beam discretized into 100 segments. Results obtained in this way were found identical to the results obtained using nine modes of vibration with the beam discretized into 400 segments.

Figure 3 shows the deflection–time response at mid-span for the undamaged beam and for a cracked beam (crack mid-span with $a/h = 0{\cdot}5$) and for different moving mass speeds (the moving mass is 20% of the total beam mass). The deflection–time response (also known as the influence line for the deflection) is normalized relative to the value $(m_0 g)L^3/(48EI)$, which is the static deflection due to m_0 at mid-span. The figure also includes the response due to a moving load to show the effect of the inertia force due to the moving mass on the

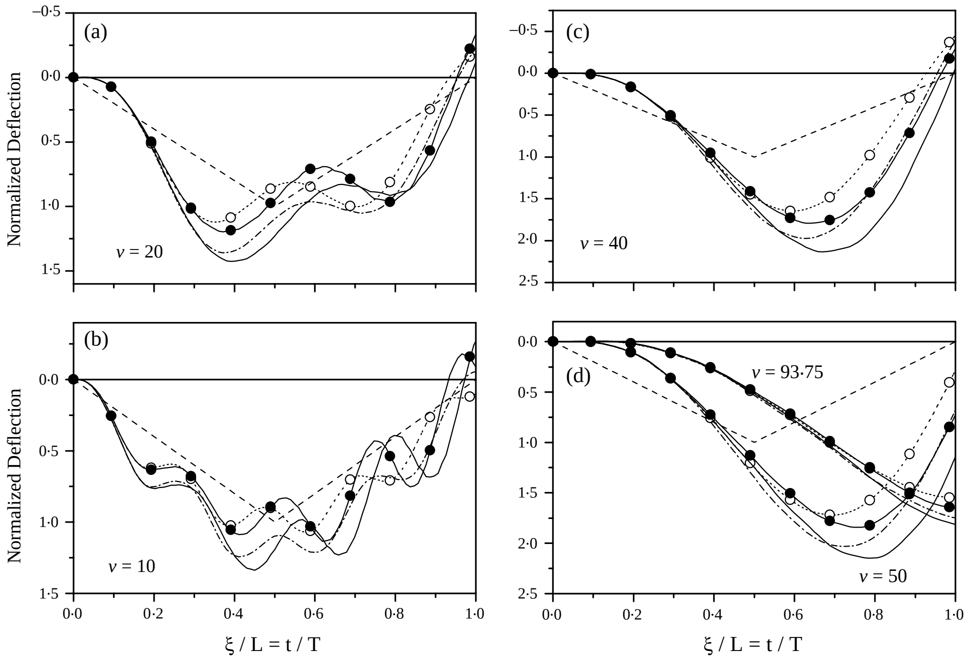

Figure 3. Effect of speed on the time response at mid-span (uncracked and $a/h = 0{\cdot}5$ mid-span); the moving mass is 20% of the beam mass; v in m/s. ---○--- moving load ($a/h = 0$), ---●--- moving mass ($a/h = 0$), -·-·-·-·- moving load, $a/h = 0{\cdot}5$, —— moving mass, $a/h = 0{\cdot}5$, ----- crawl ($a/h = 0$).

deflection. The figure shows that crack presence increases beam deflection and somewhat alters the response pattern. In particular, at a given moving mass speed it takes longer to build up the largest deflection in cracked beams than in the undamaged ones.

Figure 4 shows the effect of a crack at mid-span on the deflected beam shape for $v = 40$ m/s. When the moving mass reaches the mid-span region ($t/T = 0{\cdot}5$–0.8), the crack produces a noticeable discontinuity in the slope of the deflected beam. As the moving mass approaches the end of the beam ($t/T = 0{\cdot}96$), the cracked beam deflects downwards rather than upwards as in the undamaged beam.

Figures 3 and 4 show that the effect of crack presence is qualitatively similar for moving loads or moving masses, and that the inertia force due to the moving mass has an effect that is, in general, both similar and additive to the crack effect. Figure 5 illustrates the effect of the level of inertia force due to the moving mass on the normalized deflection (ND) at mid-span for a mass speed $v = 40$ m/s. The figure shows that for the undamaged beam, the heavier the moving mass the larger the ND, and the longer it takes to build up the maximum deflection. The crack presence compounds these effects.

Crack effect on the beam free vibration generally increases with the increase in crack depth [19]. The same trend is true for the dynamic response due to a moving mass; Figure 6(a) shows that the ND time response at mid-span for a given mass speed increases as the crack depth increases. In addition, the peak downward deflection takes longer to build up for larger cracks. Moreover, the beam changes its pattern of deflection as the mass leaves the beam from upward deflection for the undamaged beam to increasingly downwards as the crack size increases. Figure 6(b) shows a comparison of the moving load and the moving mass responses for crack sizes $a/h = 0{\cdot}2$, $0{\cdot}6$. In both cases, the inertia force due to the moving mass increases the ND relative to that of the moving load deflection.

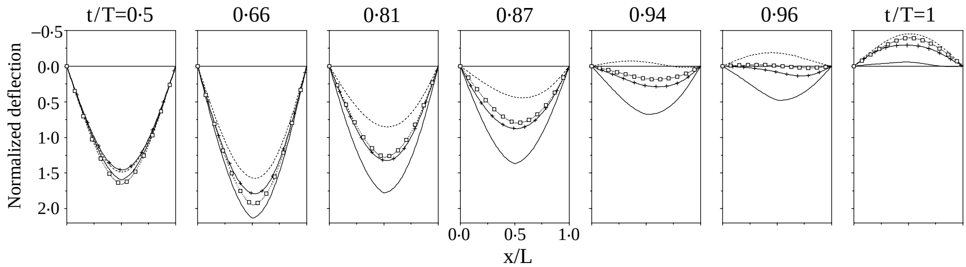

Figure 4. Beam shape for $v = 40$ m/s, $a/h = 0.5$ mid-span, mass 20%. ⋯⋯ moving load, —|— moving mass, ⋯□⋯ load, $a/h = 0{\cdot}5$ —— mass, $a/h = 0{\cdot}5$.

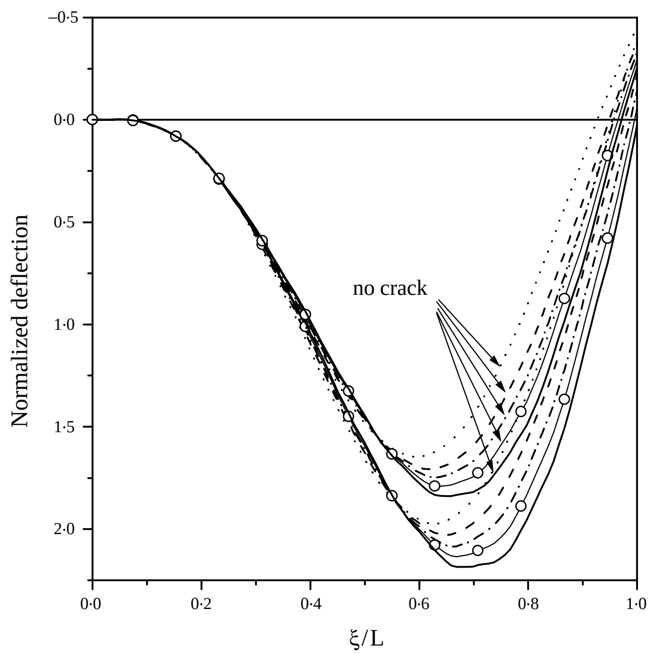

Figure 5. Effect of mass inertia-force on the time response at mid-span; $a/h = 0{\cdot}5$, $L_c/L = 0{\cdot}5$, $v = 40$ m/s. The percentiles in the legend indicate the ratio of the moving mass to the beam mass. ⋯⋯ moving load; – – – – – 10%; –·–·–·–·– 15%; —○— 20%; —— 25%.

As pointed out before, it was decided to evaluate the stationary mass (SM) approach recommended by Fryba [14], since it does not require any iterations to account for the inertia force due to the moving mass. However, the SM method requires the calculation of the eigenvalues and eigenfunctions for the beam with a concentrated mass (point mass) lumped at mid-span. This was achieved using the $\mathscr{m}$-matrix approach, and some results are presented in Figure 7. Figure 7(a) shows the variation of the first three odd eigenvalues (normalized relative to the undamaged beam without a concentrated mass) with the level of the point stationary mass (as a percentage of the total beam mass). The figure shows that the eigenvalues decrease with the increase of the magnitude of the point mass. It also shows the same trend for the beam when a crack ($a/h = 0{\cdot}5$) exists mid-span. Figure 7(b) depicts the effect of a point mass (20% of the total beam mass) on the mode shapes for both the undamaged and cracked beams. The results show that the crack produces a discontinuity in

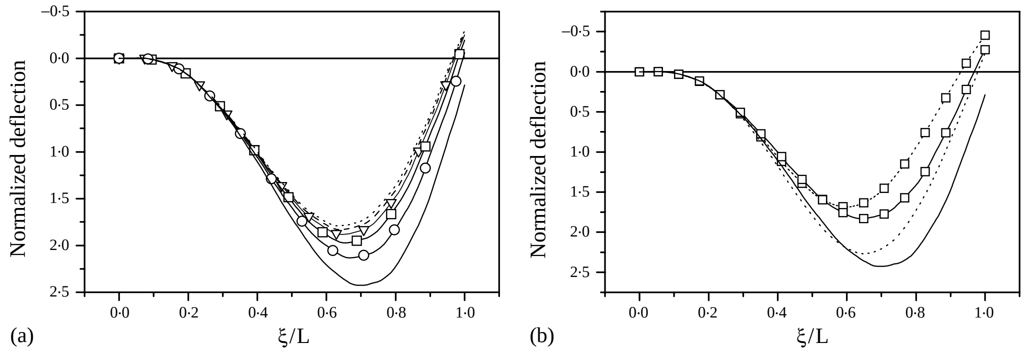

Figure 6. Effect of crack size on the time response at mid-span for $L_c/L = 0{\cdot}5$, $v = 40$ m/s, moving mass = 20% of beam mass: (a) for a moving mass ········ $a/h = 0$, – – – – – – $a/h = 0{\cdot}2$, —▽— $a/h = 0{\cdot}3$, —□— $a/h = 0{\cdot}4$, —○— $a/h = 0{\cdot}5$, —— $a/h = 0{\cdot}6$; (b) for a moving mass and a moving load. – – –□– – – load, $a/h = 0{\cdot}2$, – – – – load, $a/h = 0{\cdot}6$, —□— mass, $a/h = 0{\cdot}2$, —— mass, $a/h = 0{\cdot}6$.

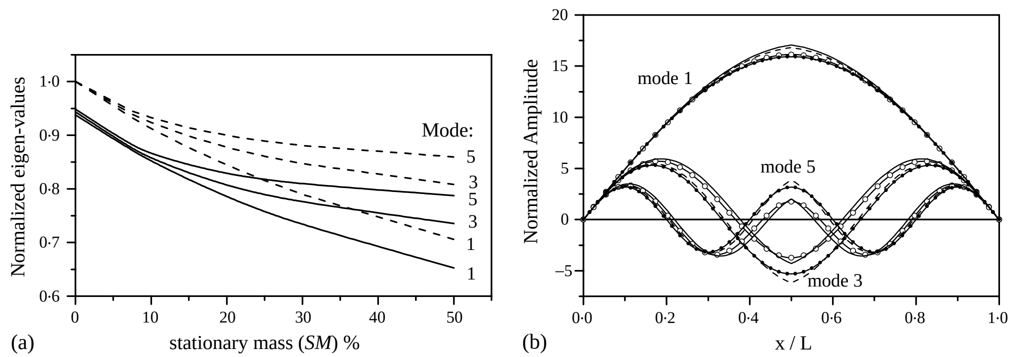

Figure 7. (a) Effect of the stationary mass (SM) level on the eigenvalues. —— undamaged, – – – – $a/h = 0{\cdot}5$, $SM = 20\%$ (b) Effect of crack and (SM) on the first three odd modes, $a/h = 0{\cdot}5$, $L_c/L = 0{\cdot}5$. —— $a/h = 0$, – – – – $a/h = 0$, SM = 20%, —○— $a/h = 0{\cdot}5$, ——●—— $a/h = 0{\cdot}5$, $SM = 20\%$.

the slope of the beam shape, and that the point mass distorts the mode shape in the middle. The modal results for the point mass were used with equations (19) and (4) to determine the beam response. Some results are shown in Figures 8–10.

Figure 8 shows a comparison of the time response at mid-span obtained by the SM method (equation (19)) and the iterative approach (equation (11)) for $v = 40$ m/s. The results of the SM are in fair agreement with the iterative approach for both cracked and uncracked beams. The figure also includes the results of equation (11) but with one iteration only. This was done to determine whether the accuracy of the modal analysis is acceptable if a first approximation replaced the iterative approach. The results presented in Figure 8, and other results for a large number of cases but not reported here for the sake of brevity, show that this approximation is quite acceptable since its results differed very little from results obtained after numerous iterations until convergence was achieved. This one-iteration approximation is useful since it permits us to generate graphs like Figures 9 and 10 that show the effect of speed on the beam dynamic response.

The effect of speed and crack presence on mid-span deflection is shown in Figure 9 for $t/T = 0{\cdot}6$ and in Figure 10 for $t/T = 1$. The speed is normalized as $\alpha = v/v_c$ to allow comparison of the responses of different undamaged beams, where $v_c = \omega_{10}L/\pi$ is the characteristic speed and ω_{10} is the fundamental natural frequency of the undamaged

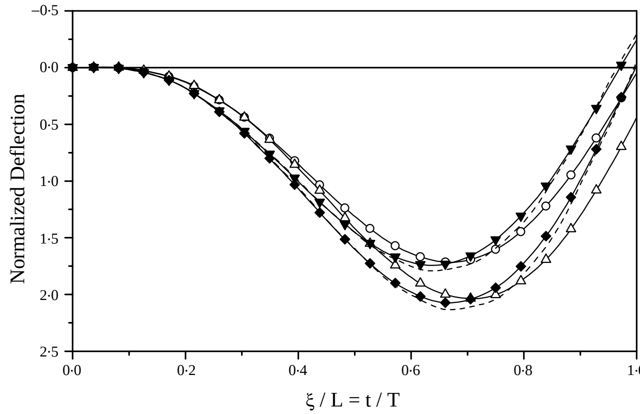

Figure 8. Comparison of the iterative approach and (*SM*) time response (crack mid-span), mass 20%, $v = 40$ m/s. —○— *SM* model ($a/h = 0$), ——△—— *SM* model ($a/h = 0{\cdot}5$), ——▼—— modal analysis ($a/h = 0$); —◆— modal analysis ($a/h = 0{\cdot}5$), - - - - - one iteration.

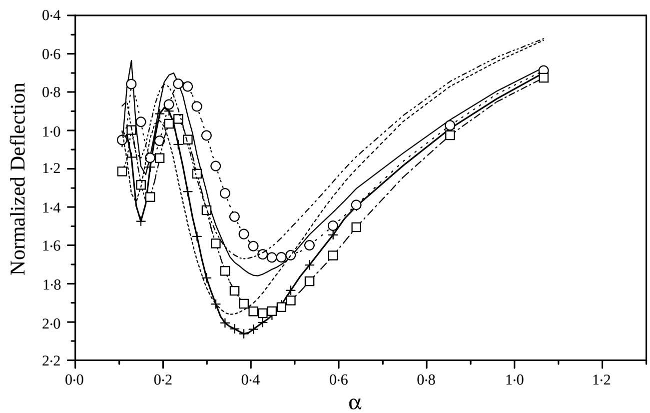

Figure 9. Comparison of modal analysis and (*SM*) deflection at mid-span for uncracked and $a/h = 0{\cdot}5$ mid-span, mass 20%; $t/T = 0{\cdot}6$ (α is the normalized moving mass speed). —— moving mass (no crack), —+— moving mass ($a/h = 0{\cdot}5$), - - -○- - - moving load (no crack), - - -□- - - moving load ($a/h = 0{\cdot}5$), - - - - - - *SM* (no crack), - - - - - *SM* ($a/h = 0{\cdot}5$).

uniform beam. When the moving load reaches 0.6 of the span length (i.e., $t/T = 0{\cdot}6$; Figure 9) and for slow speeds (small α), the *ND* oscillates around 1·0. This is because the time response oscillates about the quasi-static (crawling) time response curve as shown in Figure 3. For moderate speeds ($0{\cdot}2 < \alpha < 0{\cdot}6$), the largest deflection occurs when the moving mass is close to mid-span. The general effect of the inertia force due the moving mass is to increase the beam deflection, but the exact effect depends on the speed and time. For fast speeds ($\alpha > 0{\cdot}6$), the largest *ND* is achieved as t/T approaches 1. This is depicted in Figure 10 that shows also the effect of speed and crack presence on mid- span deflection for $t/T = 1$.

The results presented in Figures 8–10, and other results not reported here for the sake of brevity, show that the *SM* model is a useful approximation to a difficult engineering problem in the range $m_0 < 20\%$ of beam mass. For heavier moving masses, the error in the results of the *SM* approximation could become substantial.

The results presented in the foregoing sections show that crack presence, in general, leads to larger deflections in the beam, but the effect depends on the speed (as depicted in

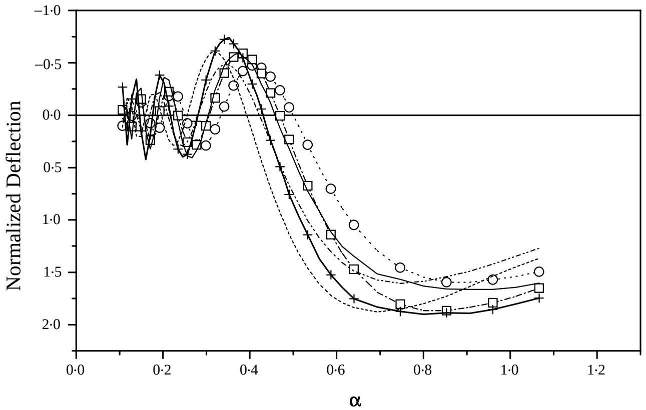

Figure 10. Comparison of modal analysis and (*SM*) deflection at mid-span for uncracked and $a/h = 0{\cdot}5$ mid-span, mass 20%; $t/T = 1$ (α is the normalized moving mass speed). ---○--- moving load (no crack), ---□--- moving load ($a/h = 0{\cdot}5$), —— moving mass (no crack), —+— moving mass ($a/h = 0{\cdot}5$), ------ *SM* (no crack), ----- *SM* ($a/h = 0{\cdot}5$).

Figures 3, 9 and 10), the time (Figures 3 and 4), the magnitude of the moving mass (Figure 5), and crack size (Figure 6). The deflection is largest at or close to beam mid-span for the simply supported beam investigated. The inertia force due to the moving mass has a similar effect.

The effect of crack location on the dynamic response was also investigated in the range $0{\cdot}5 < L_c/L < 0{\cdot}65$. The effects of crack size and the inertia force of the moving mass are qualitatively similar to the results presented above for $L_c/L = 0{\cdot}5$ (i.e., mid-span). The accuracy of the *SM* method, however, was less than that for the mid-span case. The single-iteration solution was in excellent agreement with the iterative modal analysis (equation (11)).

6. CONCLUSION

The effect of cracks on the dynamic behavior of simply supported undamped Bernoulli–Euler beams subject to a moving mass was determined. Crack presence results in higher deflections and alters the beam response patterns. In particular, the largest deflection in the beam for a given speed takes longer to build up, and a discontinuity appears in the slope of the beam-deflected shape at the crack location. Crack effects become more noticeable as crack depth increases. The effect of the inertia force due to the moving mass is, in general, qualitatively similar and additive to the effect of the crack. The exact effect of crack and mass depends on the speed, time, crack size, crack location, and the moving mass level.

The stationary mass (*SM*) approach is useful for light moving masses ($<20\%$ of the beam mass) and cracks at mid-span. For other cases, the errors can be unacceptably large. The results of the single-iteration approximation are quite close to the iterative modal analysis approach, which indicates that this approximate solution is an excellent tool for the analysis of the moving mass problem.

It should be borne in mind that the effects of the moving mass on the displacement are accompanied also by effects on the beam strength and crack stress intensity factors, which is the subject of an ongoing investigation.

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APPENDIX: NOMENCLATURE

a crack size
b beam width
h beam height
k_{ii} generalized stiffness
l distance between masses m_i and m_{i-1}
m mass per unit length
m_0 the moving mass
m_i mass of beam element
m_{ii} generalized mass

P	equivalent moving load
q_i	generalized displacement
t	time
v	speed of the moving load
v_c	characteristic speed or critical speed
w	transverse deflection
x	co-ordinate along the beam
C	crack compliance
E	Young's modulus
I	the second moment of area of the beam cross-section
L	beam span
L_c	crack location
M	bending moment
Q_i	generalized force
T	the total time needed to traverse the beam
V	shearing force
α	normalized speed
δ	Dirac delta function
θ	Rotation of beam cross-section
ω_i	eigenvalues
ξ	distance of the load from left support
ζ_i	the impulse response of the system
Φ_i	eigenfunction
Ω	frequency of vibration