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CALCULATION OF NON-LINEAR FUNDAMENTAL FREQUENCY OF A CANTILEVER BEAM USING NON-LINEAR STIFFNESS

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1. INTRODUCTION

Large amplitude vibrations of cantilever beams are of interest to engineers in many fields of engineering. The linear theory of vibrations predicts the frequencies of natural vibration to be independent of the amplitude. In many instances, if the amplitude of the vibrations is large, then the above statement is not justified due to the non-linear effects involved. Wagner [1] has obtained approximate solutions for free non-linear oscillations of an initially straight, uniform elastic bar with clamped–free and free–free end conditions. The problem of large amplitude vibrations has been presented for clamped–free and free–free uniform beams [2] and a tapered cantilever beam [3]. Recently, a simple relationship has been presented [4] to determine the first mode linear natural frequency of linearly tapered cantilever beam as a function beam stiffness (small deformation theory), the beam mass, and a mass distribution parameter.

In this note, the fundamental frequency, when the amplitudes are large, has been evaluated for a uniform cantilever beam undergoing large amplitude utilizing the methodology of reference [4]. The present results compare well with those of Wagner [1] and Rao and Rao [2]. However, these are in better agreement with those obtained by Wagner [1].

2. METHODS OF ANALYSIS

2.1. LOADS AND NON-LINEAR DEFLECTION FUNCTION

The approximate formula for the large deflection of a cantilever beam by linearizing the elliptic integral solution are presented explicitly in reference [5]. For the case of a cantilever beam of length L and with a vertical tip load P (Figure 1), the non-linear solution is expressed in terms of α the tip slope of the beam. The expressions useful to the present study are reproduced below from reference [5].

$$\sqrt{B} = (2\sin\alpha)^{1/2}\left(1 + \frac{1}{2}\alpha + \frac{2}{3}\alpha^2\right)\left(1 - \frac{\alpha}{2} + \frac{17}{120}\alpha^2\right)$$

$$\cong (2\sin\alpha)^{1/2}\left(1 + \frac{4}{15}\alpha^2\right), \tag{1}$$

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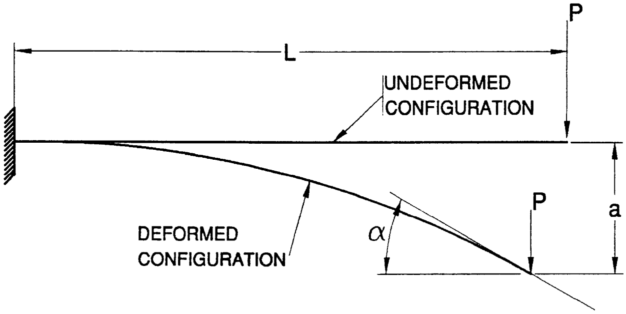

Figure 1. A uniform cantilever beam undergoing large amplitudes.

where $\alpha < B/2$ and $B = PL^2/EI$.

$$\frac{a}{L} = 1 - \frac{(2\sin\alpha)^{1/2}\left(1 - \frac{\alpha}{2} - \frac{\alpha^2}{8}\right)\left(1 - \frac{\alpha}{6} + \frac{37}{120}\alpha^2\right)}{(2\sin\alpha)^{1/2}\left(1 + \frac{4}{15}\alpha^2\right)}$$

$$\cong \frac{2}{3}\alpha, \tag{2}$$

where $2/3\alpha < B/3$,
where a is the tip lateral deflection due to tip load P, E is the Young's modulus of elasticity and I is the area moment of inertia.

2.2. LINEAR FREQUENCY EQUATION

The equation for the fundamental frequency of a cantilever beam can be represented as [4]

$$f_L = C\sqrt{\frac{S}{M}}, \tag{3}$$

where f_L is the linear frequency in Hz, S is the linear stiffness of the beam (P/a) in N/m, M is the mass of the beam in kg, C is the mass distribution parameter, and for a uniform cantilever beam, $C = 0{\cdot}323316$, which is taken from reference [4].

2.3. LARGE AMPLITUDE FREQUENCY RELATION

If the cantilever beam is undergoing large amplitude vibrations, the non-linear fundamental frequency can be approximately calculated by using the non-linear stiffness in place of the linear stiffness in equation (3), as

$$f_{NL} = C\sqrt{\frac{S_{NL}}{M}}, \tag{4}$$

where S_{NL} is the non-linear stiffness, the calculation of which is explained in the next section.

TABLE 1

Non-linear fundamental frequency of a cantilever beam

α (deg.)	Ω		% difference
	Present	Reference [2]	
0·01	3·5167	3·5160	0·02
10	3·5377	3·5393	0·04
20	3·5960	3·6110	0·41
30	3·6388	3·7373	2·71
40	3·8286	3·9306	2·79
50	3·9661	4·2136	5·87

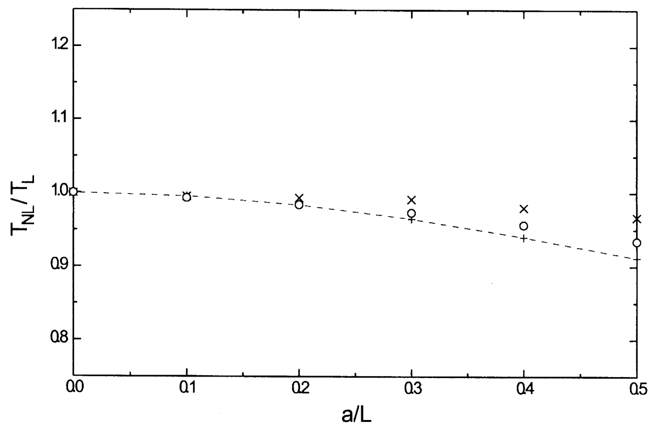

Figure 2. The ratio of the non-linear period to the linear period (T_{NL}/T_L) versus amplitude (a/L) for a uniform cantilever beam. ×××, Reference [2]; ○○○, reference [1]; --+---+--, present.

3. RESULTS AND DISCUSSIONS

A uniform cantilever beam (Figure 1), made of steel having a cross-section of 2·54 cm wide by 0·099 cm deep with a length (L) of 0·693166 m has been considered [6]. The flexural stiffness (EI) of the beam and mass per unit length ($\bar{m}$) are 0·4107599 N m^2 and 0·196138 kg/m respectively.

The large amplitude fundamental frequency of a cantilever beam has been obtained for a range of tip slope ($\alpha = 0{\cdot}01°$, which represents almost a linear case, to 50°) using mass distribution parameter (C) in the computation. First, using equations (1) and (2), the tip load (P) and tip amplitude (a/L) are calculated for a given end slope (α) of the cantilever beam. And the corresponding non-linear stiffness (P/a) is determined. Using equation (4) the first mode non-linear frequency (f_{NL}) is computed for various end slopes. The fundamental frequency parameter (Table 1) $\Omega = \sqrt{\bar{m}/EI}\,\omega L^2$ (where $\bar{m}$ is the mass per unit length, ω is the radian frequency given by $2\pi f_{NL}$) are compared with those of reference [2] for the case where only transverse inertia is considered. The present results are not compared with those of Wagner [1], as the frequency parameter versus tip slope data is not readily available in reference [1]. The difference in present results with that of reference [2] are shown in Table 1. The fundamental frequency parameters ($\alpha = 0{\cdot}01$–40°) are less than 2·8% difference and for $\alpha = 50^{\circ}$, the difference is around 5·9% when compared with the values of reference [2]. This difference in the present results are compared to reference [2], is due to the different methods of analysis of the problem considered. It is observed that the non-linear fundamental frequency parameter (Ω) increases with tip slope (α) or amplitude (a/L). This shows that the first mode of vibration of a cantilever uniform beam shows a hardening type of non-linearity.

The ratio of the non-linear (large amplitude) period to the linear period (T_{NL}/T_L) versus amplitude (a/L) for a cantilever beam is shown in Figure 2. The present results are compared with those of Wagner [1] and Rao and Rao [2]. The comparison shows that the present results are closer to those of Wagner [1] than Rao and Rao [2].

4. CONCLUDING REMARKS

In this note, the free large amplitude fundamental frequency has been studied for a uniform cantilever beam using a very simple approach. The present approach gives reasonably accurate results with much less computational efforts as compared to other methods [1, 2]. However, the present analysis may be restricted to lower range of tip slope (α) or amplitude (a/L).

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