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VIBRATION STUDIES OF COMPOSITE THIN-WALLED CURVED BOX-BEAM USING STRUCTURAL TAILORING

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1. INTRODUCTION

The requirements of weight saving and structural efficiency in many applications of aeronautical, naval and mechanical engineering have stimulated an extensive and wide use of structures with the configuration of thin-walled beams, specially when they are made of composite materials. Among these kind of structures, composite thin-walled curved box-beams play an important role in the analysis of basic cores of turbomachinery blades, robotic manipulator arms, reinforcement of composite curved panels and many mechanical engineering applications, as well. There are several studies [1–3] devoted to study the static behavior of horizontally curved thin-walled beams, where the shear flexibility is considered [2, 3] or neglected [1]. However, studies focusing the dynamic aspect of horizontally composite thin-walled curved beams appear to be scarce [4] and constrained to a limited type of motion, specifically out of plane. In order to enhance the dynamic response properties of these structures, some possibilities arise. One of the possible options is the use of tailored anisotropic laminates [5]. The subsequent elastic couplings have an advantageous passive effect on the dynamic behavior of the curved beams, which can be used to avoid some circumstances of resonance.

In this letter, parametric studies of the natural frequencies of tailored composite thin-walled curved box-beams are carried out by means of a finite element procedure. The structural model applied in this analysis takes into account the shear flexibility due to warping as well as due to bending, and it was recently developed by the authors [9]. These shear effects play an important role in the behavior of thin-walled composites straight or curved beams [4]. Thus, in the structural tailoring of thin-walled composite straight box-beams, two configurations appear as the most relevant. These are the circumferentially asymmetric stiffness (CAS) and circumferentially uniform stiffness (CUS) configurations. The CAS configuration produces a bending–twisting coupling [6] and the CUS configuration produces extension– twisting coupling. The CAS and CUS configurations are also known as symmetric and antisymmetric configurations, respectively [7]. However, in the context of curved beams, the mentioned selective coupling induce other kind of couplings between in-plane and out-of-plane motions. In order to avoid misunderstandings in this note, the concept of in-plane motion implies movements constrained to the plane π in which the curve of reference centers is defined. Conversely, the out-of-plane motion implies perpendicular movements to that plane, as it is shown in Figure 1.

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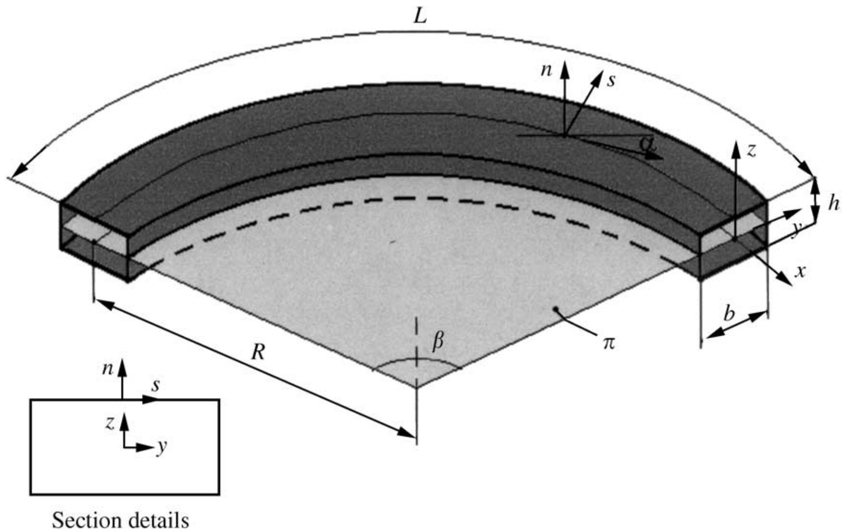

Figure 1. Structural element.

2. STRUCTURAL MODEL

The structural model used in this analysis was recently developed [9]. Its main assumptions are: (1) the cross-section is rigid in its own plane, (2) only the stress components σ_x, σ_{xs} and σ_{xn} are taken into account, and (3) the curved beam is supposed to be with low curvature. In these circumstances the governing equations are:

$$-Q'_X - \frac{M'_Z}{R} + \mathscr{M}_1 = 0, \quad -Q'_Y + \frac{Q_X}{R} + \mathscr{M}_2 = 0, \tag{1a, b}$$

$$M'_Z - Q_Y + \mathscr{M}_3 = 0, \tag{1c}$$

$$-Q'_Z + \mathscr{M}_4 = 0, \quad M'_Y - Q_Z - \frac{B' + T_{SV}}{R} + \mathscr{M}_5 = 0, \tag{1d, e}$$

$$-T'_W - T'_{SV} - \frac{M'_Y}{R} + \mathscr{M}_6 = 0, \quad B' - T_W + \mathscr{M}_7 = 0. \tag{1f, g}$$

In these equations $\mathscr{M}_j$ are inertia terms, Q_X is the axial force; Q_Y and Q_Z are the transverse shears forces; M_Y and M_Z are bending moments; T_W is the flexural-torsional moment; T_{SV} is the Saint Venant torsional moment, and B is the bimoment. The variables u_{xc}, u_{yc}, u_{zc} mean the displacements in directions x, y, z of the geometrical center of reference (Figure 1), θ_y and θ_z are out-of-plane and in-plane bending rotations, ϕ_x is the torsional rotation and θ_x is a warping variable.

In the case of a generic cross-section and stacking sequence, equations (1) are fully coupled. However, equations (1) can be split into out-of-plane and in-plane movements, if cross-section and elastic symmetries are applied. The in-plane motion is appropriately described by equations (1a–c) whereas the out-of-plane motion is governed by equations (1d–g). The selective coupling and decoupling conditions will be explained in the next paragraphs.

In the general case of lamination for a box-section, the stress resultants are defined as follows:

$$\begin{Bmatrix} Q_X \\ M_Y \\ M_Z \\ B \\ Q_Y \\ Q_Z \\ T_W \\ T_{SV} \end{Bmatrix} = \begin{bmatrix} J^{11}_{11} & J^{11}_{12} & J^{11}_{13} & J^{11}_{14} & J^{16}_{15} & J^{16}_{16} & J^{16}_{17} & J^{16}_{18} \\ & J^{11}_{22} & J^{11}_{23} & J^{11}_{24} & J^{16}_{25} & J^{16}_{26} & J^{16}_{27} & J^{16}_{28} \\ & & J^{11}_{33} & J^{11}_{34} & J^{16}_{35} & J^{16}_{36} & J^{16}_{37} & J^{16}_{38} \\ & & & J^{11}_{44} & J^{16}_{45} & J^{16}_{46} & J^{16}_{47} & J^{16}_{48} \\ & & & & J^{66}_{55} & J^{66}_{56} & J^{66}_{57} & J^{66}_{58} \\ & & sym & & & J^{66}_{66} & J^{66}_{67} & J^{66}_{68} \\ & & & & & & J^{66}_{77} & J^{66}_{78} \\ & & & & & & & J^{66}_{88} \end{bmatrix} = \begin{Bmatrix} u'_{xc} + u_{yc}/R \\ -(\theta'_y + \phi_x/R) \\ -(\theta'_z - u'_{xc}/R) \\ -(\theta'_x - \theta'_y/R) \\ u'_{yc} - \theta_z \\ u'_{zc} - \theta_y \\ \phi'_x - \theta_x \\ \phi'_x - \theta_y/R \end{Bmatrix} \tag{2}$$

and the inertia terms are expressed as

$$\begin{Bmatrix} \mathscr{M}_1 \\ \mathscr{M}_2 \\ \mathscr{M}_3 \\ \mathscr{M}_4 \\ \mathscr{M}_5 \\ \mathscr{M}_6 \\ \mathscr{M}_7 \end{Bmatrix} = \begin{bmatrix} J^{\rho}_{11} + J^{\rho}_{33}/R^2 & 0 & -J^{\rho}_{33}/R & 0 & 0 & 0 & 0 \\ & J^{\rho}_{11} & 0 & 0 & 0 & 0 & 0 \\ & & J^{\rho}_{33} & 0 & 0 & 0 & 0 \\ & & & J^{\rho}_{11} & 0 & 0 & 0 \\ & & & & J^{\rho}_{22} & 0 & -J^{\rho}_{44}/R \\ & sym & & & & J^{\rho}_{22} + J^{\rho}_{33} & 0 \\ & & & & & & J^{\rho}_{44} \end{bmatrix} \begin{Bmatrix} \ddot{u}_{xc} \\ \ddot{u}_{yc} \\ \ddot{\theta}_z \\ \ddot{u}_{zc} \\ \ddot{\theta}_y \\ \ddot{\phi}_x \\ \ddot{\theta}_x \end{Bmatrix}. \tag{3}$$

The stiffness coefficients in equation (2) and the inertia coefficients in equation (3) are obtained as follows:

$$J^{kh}_{ij} = \int_S [\bar{A}_{kh}(\bar{g}^{(a)}_i \bar{g}^{(a)}_j) + \bar{B}_{kh}(\bar{g}^{(a)}_i \bar{g}^{(c)}_j + \bar{g}^{(c)}_i \bar{g}^{(a)}_j) + \bar{D}_{kh}(\bar{g}^{(c)}_i \bar{g}^{(c)}_j)]\,\mathrm{d}s, \tag{4a}$$

$$J^{\rho}_{ij} = \int_A \rho\, \bar{g}^{(d)}_i \bar{g}^{(d)}_j \mathrm{d}A, \tag{4b}$$

where the vectors $\bar{g}^{(j)}$ are defined as

$$\bar{g}^{(a)} = \left\{1, Z, Y, \omega_P, \frac{\mathrm{d}Y}{\mathrm{d}s}, \frac{\mathrm{d}Z}{\mathrm{d}s}, r(s) - \psi, \psi\right\}, \quad \bar{g}^{(b)} = \left\{0, 0, 0, 0, \frac{\mathrm{d}Z}{\mathrm{d}s}, -\frac{\mathrm{d}Y}{\mathrm{d}s}, -l(s), 0\right\}, \tag{5a, b}$$

$$\bar{g}^{(c)} = \left\{0, \frac{\mathrm{d}Y}{\mathrm{d}s}, -\frac{\mathrm{d}Z}{\mathrm{d}s}, l(s), 0, 0, 1, -2\right\}, \quad \bar{g}^{(d)} = \left\{1, Z + n\frac{\mathrm{d}Y}{\mathrm{d}s}, Y - n\frac{\mathrm{d}Z}{\mathrm{d}s}, \omega\right\}. \tag{5c, d}$$

In equation (4a) the laminate elastic coefficients $\bar{A}_{ij}$, $\bar{B}_{ij}$ and $\bar{D}_{ij}$ are defined like in reference [8], by making assumptions of plane stress state. In equation (4b), ρ denotes the density, which is assumed to be constant. In equation (5), $\{Y, Z\}$ are the coordinates of a point lying on the middle line of the wall (where co-ordinate $n = 0$). ω is the whole warping function, which is composed of two terms: primary or contour warping (ω_p) and secondary or thickness warping (ω_s) defined by

$$\omega = \omega_p(s) + \omega_s(s, n), \tag{6a}$$

$$\omega_p(s) = \int_{S_\circ}^{S} [r(s) - \psi(s)]\,\mathrm{d}s, \quad \omega_s(s, n) = n\,l(s); \tag{6b, c}$$

$r(s)$ and $l(s)$ are defined as

$$r(s) = -Z(s)\frac{\mathrm{d}Y}{\mathrm{d}s} + Y(s)\frac{\mathrm{d}Z}{\mathrm{d}s}, \quad l(s) = Y(s)\frac{\mathrm{d}Y}{\mathrm{d}s} + Z(s)\frac{\mathrm{d}Z}{\mathrm{d}s}. \tag{7}$$

The function $\psi(s)$ is the torsional shear flow with reference to a closed contour section. It accounts for variable laminates along the contour, and it is defined analogously to one of the reference [7].

3. STRUCTURAL TAILORING

The CAS and CUS configurations are represented in Figure 2(a, b), respectively, where the angle α is arranged according to Figure 1. The CAS configuration involves the following conditions in the constitutive relations (2):

$$J^{11}_{12} = J^{11}_{13} = J^{11}_{14} = J^{11}_{23} = J^{11}_{24} = J^{11}_{34} = J^{16}_{17} = J^{16}_{18} = J^{16}_{25} = J^{16}_{26} = 0,$$

$$J^{16}_{35} = J^{16}_{36} = J^{16}_{45} = J^{16}_{46} = J^{16}_{47} = J^{16}_{48} = J^{66}_{56} = J^{66}_{57} = J^{66}_{58} = J^{66}_{67} = J^{66}_{68} = 0. \tag{8a}$$

These conditions imply, according to the constitutive behavior, that out-of-plane bending and torsion induce in-plane bending and extension. In fact, the CAS configuration shown in Figure 2(a) produces a coupling between Q_X and Q_Y because $J^{16}_{15} \neq 0$; T_W and T_{SV} are coupled with M_Y because of non-vanishing coefficients J^{16}_{27} and J^{16}_{28}. On the other hand, T_W and T_{SV} are coupled with the in-plane bending moment M_Z because of non-vanishing coefficients J^{16}_{37} and J^{16}_{38}. The first and second couplings have in-plane and out-of-plane character, respectively, but the third one allows for the coupling between both movements.

The CUS configuration imposes the following conditions in the constitutive relations (2):

$$J^{11}_{12} = J^{11}_{13} = J^{11}_{14} = J^{11}_{23} = J^{11}_{24} = J^{11}_{34} = J^{16}_{15} = J^{16}_{16} = J^{16}_{26} = J^{16}_{27} = J^{16}_{28} = 0,$$

$$J^{16}_{35} = J^{16}_{37} = J^{16}_{38} = J^{16}_{45} = J^{16}_{46} = J^{16}_{47} = J^{66}_{48} = J^{66}_{56} = J^{66}_{57} = J^{66}_{58} = J^{66}_{67} = J^{66}_{68} = 0. \tag{8b}$$

Expression (8b) indicates that in-plane extension and bending induce out-of-plane bending and torsion. In this case, the in-plane axial force Q_X is coupled with the torsional moments T_W and T_{SV} and the in-plane and out-of-plane bending moments M_Z and M_Y are coupled with the out-of-plane and in-plane shear forces Q_Z and Q_Y, respectively.

It has to be noted that, if $\alpha = 0$, i.e., in the case of a specially orthotropic staking sequence, it is possible to decouple equation (1) into in-plane and out-of-plane movements.

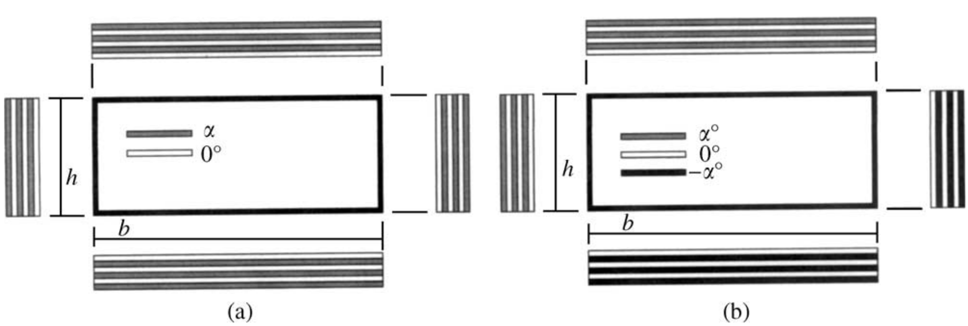

Figure 2. (a) CAS and (b) CUS configurations.

4. NUMERICAL ANALYSES

In order to perform the appropriate vibration analysis about the tailoring patterns of curved box-beams, a former finite element procedure developed by the authors [2] is employed. The considered curved beam is clamped at one end and free at the other. Consequently, the boundary conditions are

$$u_{xc} = u_{yc} = \theta_z = u_{zc} = \theta_z = \phi_x = \theta_x = 0, \text{ at } x = 0, \tag{9a}$$

$$Q_X = Q_Y = Q_Z = B = M_Z = M_Y = T_W + T_{SV} = 0, \text{ at } x = L. \tag{9b}$$

If the set of unknowns $\{u_{xc}, u_{yc}, \theta_z, u_{zc}, \theta_z, \phi_x, \theta_x\}$ is considered to vary harmonically, then the common eigenvalue equation (10) has to be solved in order to obtain the frequencies

$$|[K] - \Omega^2 [M]| = 0, \tag{10}$$

where $\Omega = 2\pi f$ is the circular frequency and f is the frequency measured in hertz. $[K]$ is the stiffness matrix and $[M]$ the mass matrix. The case of a rectangular cross-section is considered. The cross-sectional has the following parameters: $h = 0{\cdot}05$ m, $b = 0{\cdot}10$ m, $e = 0{\cdot}008$ m (see Figures 1 and 2), whereas the physical properties of the material (Graphite-Epoxy AS4/3501-6) are shown in Table 1. The circumferential length (see Figure 1) of the curved beam is $L = 1{\cdot}0$ m and calculations for three different length/radius parameters $L/R = \{1{\cdot}0, 1{\cdot}5, 2{\cdot}0\}$ are carried out. In order to obtain very accurate results, models with 40 elements are employed. The variations of the first frequency with respect to angle α, for different L/R ratios, are shown in Figures 3 and 4 for CAS and CUS configurations, respectively. It is possible to see in these figures, that the behavior of CAS and CUS configurations is similar with respect to the parameter L/R. In fact, lower frequencies are obtained for increasing L/R ratios. However, differences between CAS and

Table 1

Material properties

$E_1 = 144$ MPa $\quad E_2 = 9{\cdot}68$ MPa
$G_{12} = G_{13} = 4{\cdot}14$ MPa $\quad G_{23} = 3{\cdot}45$ MPa
$\nu_{12} = 0{\cdot}3 \quad \nu_{23} = 0{\cdot}5$
$\rho = 1389$ kg/m^3

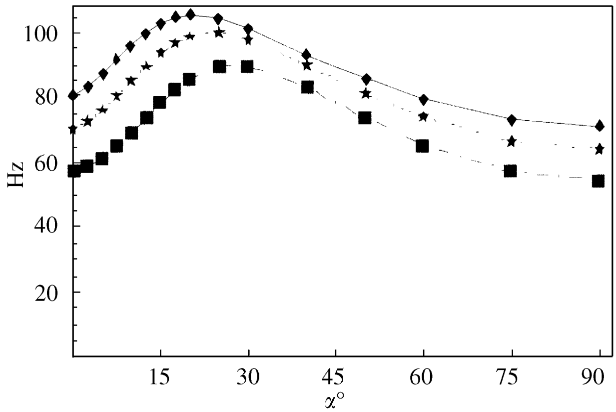

Figure 3. Variation of the first frequency with the winding angle of CAS laminates: ◆, $L/R = 1{\cdot}0$; $*$, $1{\cdot}5$; ■, $2{\cdot}0$.

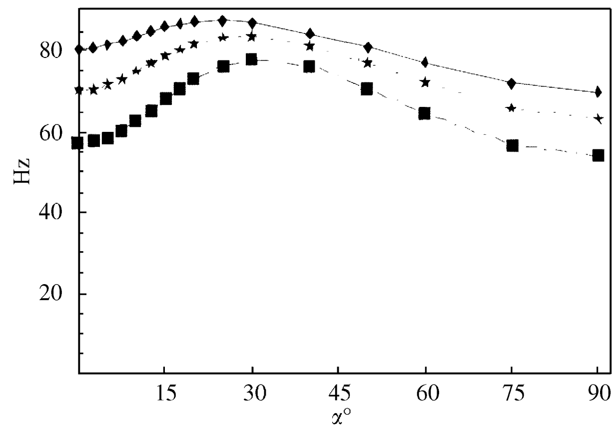

Figure 4. Variation of the first frequency with the winding angle of CUS laminates: ◆, $L/R = 1{\cdot}0$; $*$, 1·5; ■, 2·0.

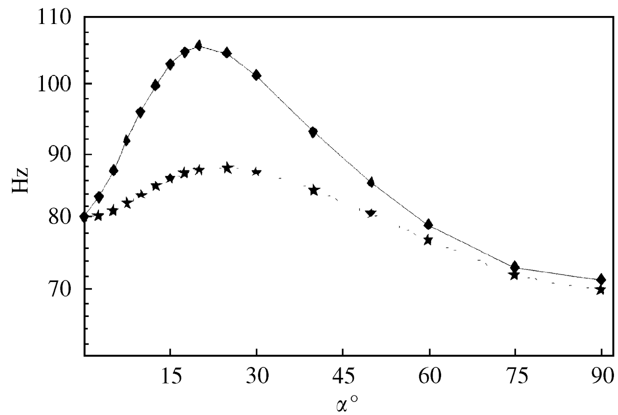

Figure 5. Comparison of CAS (◆) and CUS ($*$) responses for a curved beam with $L/R = 1{\cdot}0$.

CUS responses can be observed together in Figure 5 for the ratio $L/R = 1{\cdot}0$. As it can be seen in Figure 5, frequencies of the CAS configuration are greater than the frequencies of the CUS configuration in the range $\alpha \in [5^\circ, 45^\circ]$, offering a remarkable frequency spread. For example, at $\alpha = 20^\circ$ there is a maximum frequency spread of about 20 Hz.

The selective employment of tailoring techniques offers the possibility to perform analysis of different responses in the design stage without changing the beam shape. This is useful when some vibrating conditions have to be avoided, especially near resonant conditions. At this level, the analysis of mode shapes could be highly clarifying in order to understand the nature of different selective elastic couplings. However, this is the task of future work.

5. CONCLUSIONS

In this note, a vibration analysis of structurally tailored anisotropic thin-walled curved box-beams in performed. A new model, which accounts for shear flexibility in a full form, is employed. A finite element formulation is used in order to obtain the frequencies. CAS and CUS configurations are considered. It is possible to take advantage of the different performance of these tailoring configurations in order to obtain the desired response in the structural member. Although both configurations have a similar behavior with respect to the parameter L/R, the CAS configuration is shown to be stiffer than the CUS configuration.

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