



IS A SIMPLE SUPPORT REALLY THAT SIMPLE?

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A very frequently applied boundary condition in problems involving modelling the dynamic behaviour of beams and panels is a simple support. This type of boundary condition is particularly used in the research area of active control of vibrations on panels. However, the practical implementation of this boundary condition in the laboratory is not at all trivial, especially when high accuracy is required.

In this paper, various possible practical implementations of a simply supported panel are considered. A suitable method is selected and investigated experimentally. The results from a forced vibration test are compared with those obtained from theory in order to verify the relative accuracy of the approach.

The effect of the relative distance between the panel surface and a solid boundary on the fundamental resonant response frequency is experimentally investigated, as well as changes produced by fluctuations in the laboratory temperature.

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1. INTRODUCTION

Beams and panels are the most common structural elements, and when it comes to modelling their dynamics, a very frequently applied boundary condition is a simple support. For the reader not familiar with the topic, this condition consists of an out-of-plane constraint of the displacement of the edge (for example of a panel) which is simply supported [1]. At the same time, the rotations of the panel around an axis running along the simply supported edge are unrestrained (as if the edge was actually a long hinge), hence the bending moment at this edge is zero. Several examples of this supporting condition can be found in the research area of active control of vibrations on panels, where typically the models describing the dynamics of the structural element assume a simply supported panel [2–4]. One of the reasons for this choice of boundary condition is the relative ease with which the deformed shape of a simply supported uniform element can be approximated as a superposition of sine functions. The problems arise when these theories require experimental verification, and the theoretical simple support has to be reproduced in the laboratory. In particular, when a high level of accuracy is required in the experiment, to create a structural support that reproduces the theoretical condition is a relatively challenging task. This high accuracy is required, for example, when the effect of small changes in the response of the structure has to be investigated. A typical example is the effect of piezoelectric patches bonded onto structural members such as beams or panels [2, 5]. The piezoelectric patch can be used as a sensor, and therefore its impact on the dynamics of the elements onto which it is bonded should be minimal. The patches can also be used as actuators; however, in either case it is very important to quantify the change in the dynamics of the structure introduced by the patch.

In this paper, various possible practical implementations of a simply supported panel are considered. A suitable method is selected and investigated experimentally. The results from a forced vibration test are compared with those obtained from theory in order to assess the relative accuracy of the approach.

2. EXPERIMENTAL IMPLEMENTATION OF A SIMPLE SUPPORT FOR A THIN PANEL

There are several methods which can be used to build a physical support for a beam or a panel that approximates the effect of a theoretical simple support. Four possible methods are described in the following sections.

In this work, attention is focused on a rectangular panel 304.3 mm \times 203.2 mm, 1.453 mm thick, which has to be simply supported along all four edges. The panel is made of aluminium alloy with a Young's modulus of 71 GPa and a density of 2705 kg/m³. The density and the Young's modulus of the material were experimentally verified. The density was verified using a precision scale, after which the Young's modulus was verified by comparing the measured resonant response frequency of the freely supported panel with the calculated natural frequencies of vibration obtained using the finite element method (FEM).

The natural frequencies of vibration for a simply supported panel can be obtained using the well-known formula [6]

$$f = \frac{\pi}{2} \sqrt{\frac{D}{\rho h} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)}, \quad (1)$$

where

$$D = \frac{Eh^3}{12(1 - \nu^2)}. \quad (2)$$

E is the Young's modulus, ρ is the density, h is the thickness, a and b are the lengths of the two sides of the panel, and m and n are the number of half-sine waves along the two respective directions (x and y) of the panel, which identify a particular mode of vibration. For a panel with the specifications given in Table 1, the frequencies associated with the first four modes of vibration calculated using equation (1) are reported in the second column of Table 2.

The natural frequencies of vibration can also be calculated using finite element analysis (FEA). The FEM of the panel, with simply supported boundary conditions applied to the nodes along all four edges, is shown in Figure 1. The four lowest natural frequencies of vibration obtained are listed in the third column of Table 2. The code used for the FEA was Ansys [7], and the type of element used for the plate was Shell63, which is a four-node

TABLE 1

Simply supported panel specifications

Length, a	304.3 mm
Width, b	203.2 mm
Thickness, h	1.453 mm
Young's modulus, E	71 GPa
Density, ρ	2705 kg/m ³

TABLE 2

Natural frequencies of vibration for the “simply supported” aluminium panel

Mode	Theory	FEA			
		Ideal SS	Grooves	Knives	Shims
1,1	125.2	125.1 (1)	137.2 (0.999862101)	152 (0.998589318)	125.3 (0.999999983)
2,1	241.0	240.7 (1)	251.0 (0.999931018)	263.7 (0.998806012)	240.8 (0.999999991)
1,2	385.0	384.5 (1)	397.7 (0.999980766)	413.3 (0.999465058)	384.5 (0.999999929)
3,1	434.1	433.3 (1)	442.4 (0.999887152)	452.3 (0.998532263)	433.2 (0.999999856)

MAC in parenthesis.

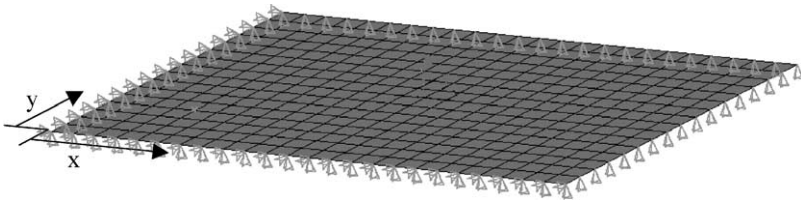


Figure 1. FEM of the panel.

elastic shell (details about the element formulation can be found in reference [8]). Within the FE code, the algorithm used for the extraction of the eigenvalues and eigenvectors (i.e., natural frequencies and mode shapes) was the Block Lanczos method, which is the software default method. Other methods (e.g., Subspace method) have also been tried in order to confirm the values obtained with the Block Lanczos method with identical results. The mesh of the model was refined until convergence was achieved, and only the converged results are shown in the table. As can be seen, there is a very good agreement between the FEM-calculated results (FE ideal SS) and those obtained using equation (1). These frequencies of vibration will be compared with the frequencies obtained by implementing the simple support using the various methodologies which are discussed in detail in the following sections.

2.1. PANEL EDGE GROOVES

A relatively simple method to produce a physical approximation of a simple support along the edge of a panel is to create a region with very low torsional stiffness running along the edge. In practice, this can be achieved by having two relatively deep grooves on opposite faces and running along the side of the panel, as shown in Figure 2(a). The portion of the panel surrounding the grooves is then clamped. With this configuration, out-of-plane displacements of the edge are avoided, whereas edge rotations are still possible due to the weakness introduced by the grooves.

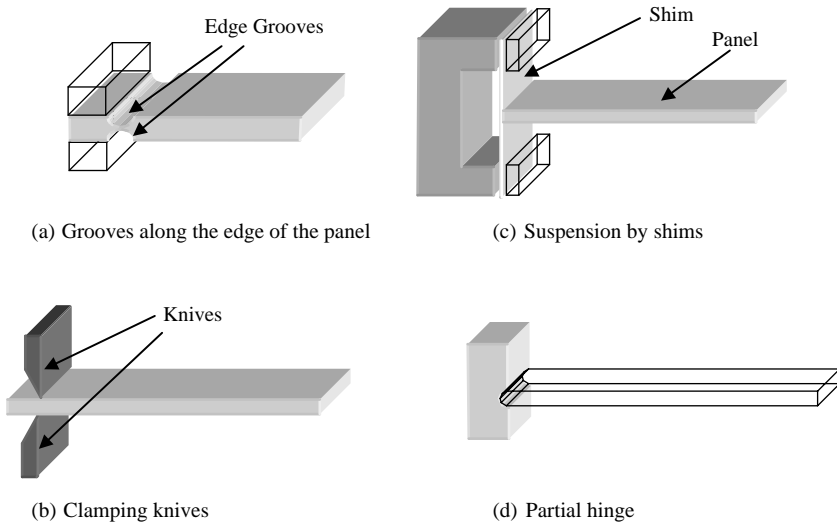


Figure 2. Schematic views of experimental methods to reproduce a simple support.

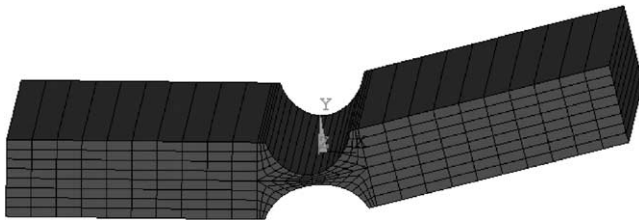


Figure 3. Detailed FEM of the grooves area used to calculate the rotational stiffness.

The amount of rotational (bending) stiffness along the edge is proportional to the thickness of the material left between the apexes of the grooves. For this type of support to reproduce the effect of a simple support (with zero rotational stiffness), it is crucial that the material left between the grooves is as thin as possible. Since the stiffness is proportional to the third power of the thickness, a panel that was 2 mm thick with 0.2 mm thickness between the grooves would result in a 1000-fold reduction in the rotational stiffness along the edge. However, this rotational stiffness, even if relatively small compared to the panel stiffness, can still produce a relatively large effect on the dynamics of the panel.

The rotational stiffness of the area of the structure incorporating the grooves was calculated using a detailed FEM of the region along the edge of the panel. In this model, shown in Figure 3, a small portion of unit length of the edge of the panel, which includes the groove, is modelled using Plane42 (2-D Structural Solid elements [8]). The side of the model on the left of the groove was then fully constrained (to represent physical clamping) and a load (bending moment) was applied to the right-hand side of the model. The ratio between the applied moment and the slope of the deformed section of the structure yields the rotational stiffness produced by the groove. This rotational stiffness was then included in the rotational degree of freedom (d.o.f.) constraint applied to the nodes along the outer edge of the FEM of the panel shown in Figure 1. The resultant stiffening of the panel produces an increase in the fundamental natural frequency of vibration (fourth column of Table 2) of approximately 10%. However, the mode shapes obtained with this implementation of

a simple support are still very similar to the mode shapes obtained for an ideal simple support. This is confirmed by means of the modal assurance criterion (MAC) reported in Table 2 which is very close to unity, for all the modes considered. In conclusion, although this methodology is relatively easy to implement and correctly reproduces the mode shapes, it does not accurately reproduce the theoretical natural frequencies of vibration. Furthermore, this implementation of a simple support is, in practice, similar to a pin joint which runs along the edge of the panel, constraining in-plane displacements of the edge. It is very likely that the supporting frame will be made of a material which is different from the panel; therefore, changes in temperature during the tests will produce an undesired pre-stressing of the panel.

2.2. CLAMPING KNIVES

A relatively popular method to reproduce a simple support along the edge of a panel is to clamp the edge between two knives, as shown in Figure 2(b). This method is probably the simplest to implement in terms of manufacturing and assembly, but it is quite difficult to calculate its accuracy in reproducing the ideal simple support. The problem with this support is in the contact area (ideally a line) between the edge (blade) of the knife and the panel. If the knives are clamping the panel too tightly, this would introduce certain constraints on the rotational movements of the panel between the knives. In addition, changes in temperature could introduce pre-stresses in the panel, whose in-plane expansion/contraction is constrained by the blades. However, if the knives do not hold the panel tightly enough, the surfaces of the panel could slide under the edges of the knives. Furthermore, there is also the undesired damping effect introduced by the friction between the knives and the panel, which would need to be included in the response. For these reasons, it is very difficult to produce an FEM which is able to reproduce all these effects accurately. Assuming that the force between the clamping knives is sufficient to avoid sliding between the panel and the blades, it is possible to model a unit length section of the region along the boundary of the panel. Figure 4 shows the FEM of this region. The model

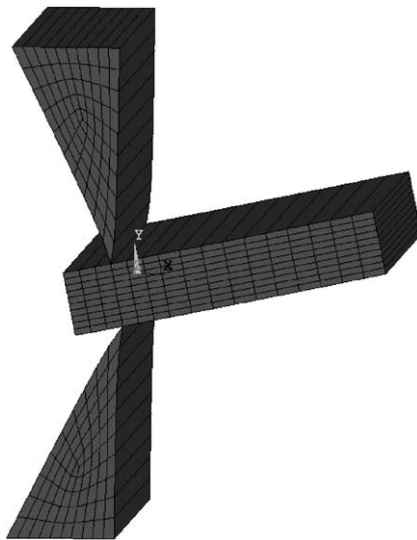


Figure 4. Detailed FEM of the panel edge supported by knives, used to calculate the rotational stiffness.

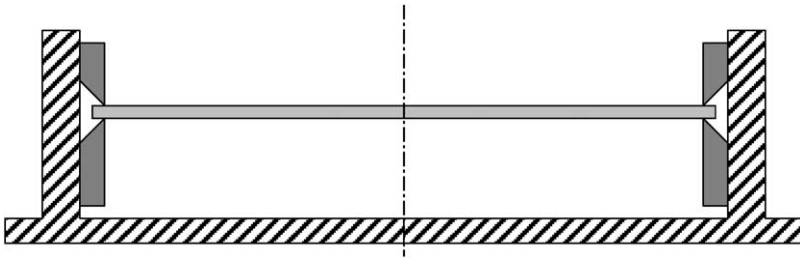


Figure 5. Cross-section of a typical simply supporting frame using "clamping knives".

was built using Plane42 (2-D Structural Solid elements [7, 8]), and the rotational stiffness calculated using this model was then applied along the supported edge of the model shown in Figure 1. The results are reported in Table 2, and it can be seen that the rotational stiffness introduced by the clamping knives produces a large increase in the natural frequencies of vibration. Reducing the clamping force and allowing the blades to slide will reduce this stiffness, but it is relatively difficult to quantify these factors in the FEM.

In experiments that use this type of support and using a frame such as that shown in Figure 5, the increase in the lowest natural frequency (in comparison to the ideal case) is not as large as suggested in Table 2. This is because the air cavity underneath the panel has the effect of reducing the frequency of the first mode of vibration, as shown in section 4.1.

2.3. SHIM SUPPORTS

Another possibility for the practical realization of a simply supported boundary condition along the edge of a panel is to exploit the high in-plane stiffness together with the low bending stiffness of a very thin sheet of steel (i.e., a shim) bonded perpendicularly along the edge of the panel.

The concept of using a thin strip of metal to reproduce a simple support was first implemented by White [9, 10]. However, this was applied to beams only, where thin strips of steel were bolted (perpendicularly) to the ends of the beam to reproduce simple supports. Here, this concept is applied to the edge of a thin panel (Figure 2(c)) rather than the end of a beam. This set-up produces a negligible rotational stiffness along the edge, due to the high bending flexibility of the shim. At the same time, the high in-plane stiffness of the shim, which is clamped along its upper and lower edges, restrains the out-of-plane movements of the edge of the panel.

An FEM of the panel supported with this technique is shown in Figure 6. The panel has the same characteristics as that discussed in the previous section, and the shims have a thickness of 0.052 mm. The distance between the upper and lower clamped edges of the shim is 40 mm, and the panel is bonded along the middle of the shim. The corner pieces (L-cross-section segments of steel beam) onto which the shims are constrained are then bolted to a rigid frame. The natural frequencies of vibration calculated using the FEM shown in Figure 6 (whose details are reported in section 3) are given in the sixth column of Table 2. As can be seen, there is excellent agreement between the theoretical results (equation (1)) and FEM results.

Another advantage of this type of support is that the flexibility of the shims allows expansion or contraction of the panel (due to temperature changes) without inducing large in-plane pre-stresses in the panel. Furthermore, the frame can also be positioned vertically

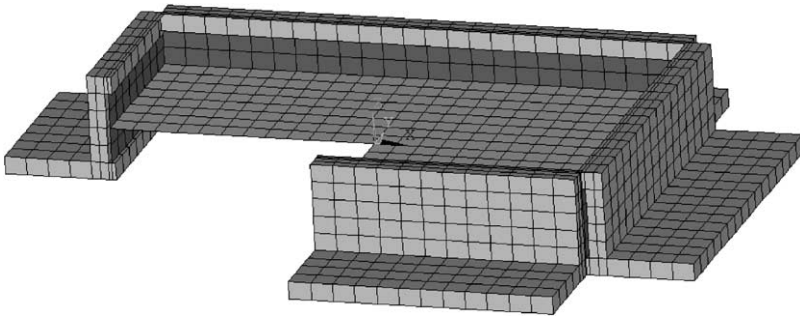


Figure 6. FEM of the panel suspended by the shims.

in order to assess the effect of pre-stresses produced by the mass of the panel and to avoid sagging of the panel. In the specific case treated in this paper, this effect was negligible.

2.4. PARTIAL HINGE

Another method which could be used to support the edge of the panel is that of a partial hinge, as shown in Figure 2(d). The support has a horizontal groove (with semi-circular cross-section) which mates with the edge of the panel, the latter being rounded to fit the groove. For this type of support to work properly, it is necessary that the groove and the edge of the panel mate precisely. A possible drawback of this method is the friction which is produced between the edge of the panel and the surface of the horizontal supporting groove. In addition, changes in temperature could produce high compressive pre-stresses in the panel.

The methods described above are just some of the techniques that can be put into practice to build a simple support. Each method has its advantages and disadvantages, but with regard to the accuracy with which these methods reproduce the theoretical simple support, the panel suspension by shims seems to be the most promising. For this reason, this type of support was adopted in this investigation, and its implementation and verification against theoretical predictions of the vibration response are presented.

3. FEM AND ANALYSIS OF THE ASSEMBLY

The FEM of the whole assembly, which was comprised of the panel and supporting structure, is shown in Figure 7. The panel, shims and frame have been modelled using shell elements (Shell63 [7, 8]), whilst the L-section beams, and the metal strips clamping the shims have been modelled using solid elements (both with 6 d.o.f. per node—Solid73 [7, 8]). The L-sections were constrained to the frame by merging the nodes in the areas where the connecting bolts between these elements were located.

The non-rigid-body modes and associated natural frequencies of vibration calculated in the FE analysis of the whole assembly, with the supporting structure freely supported, are reported in the second column of Table 3. As expected, these frequencies are slightly higher than those obtained by “grounding” the L-sections (without the base frame included in the analysis). The addition of a lumped mass (2.2 g) to the model, representing the accelerometer

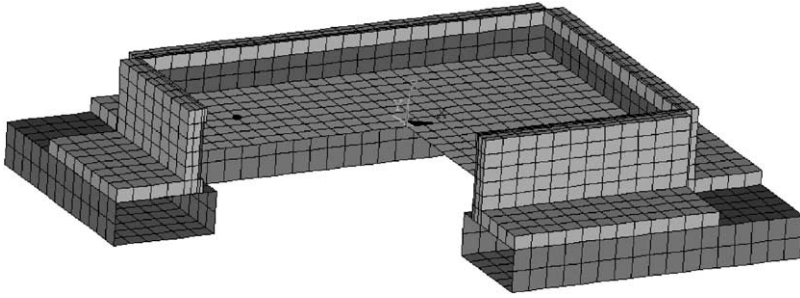


Figure 7. FEM of the panel suspended by the shims assembled on the frame.

TABLE 3
Natural frequencies of vibration for the assembly

Mode	FEA			Difference FF-acc/ experimental (%)
	Frame free-free	Frame F-F incl. accelerometer	Experimental	
1,1	125.63	125.37	125.3	0.05
2,1	240.97	239.46	240.5	0.43
1,2	384.68	382.91	383.9	0.26
3,1	433.26	429.51	431.5	0.46

positioned on the panel at $x = 50.8$ mm and $y = 50.8$ mm, produces a small decrease in the natural frequencies of vibration (third column of Table 3). The accelerometer was positioned in an area of high acceleration response for all the modes considered, away from their nodal lines.

The FEM was also used to calculate the harmonic response to a point force acting perpendicularly to the panel at $x = 50.8$ mm and $y = 152.4$ mm. The power spectral density of the applied force used as an input to the model is shown in Figure 8.

The FEM described above was validated by comparing the natural frequencies and acceleration curves with those obtained with the experimental arrangement described in the next section, with excellent results.

In addition, other FEMs were built to investigate the suitability of this method to simply support more flexible panels (e.g., plastic panels) or panels with edge length ratios different from the one considered in this work (specifically a ratio of 2:3 was investigated).

Concerning panels with different edge length ratios, Table 4 reports the results obtained using FEM of this type of assembly for panels with edge length ratios of 2 and 2.5 respectively. It can be seen that in these cases, the support gives a very good approximation of the ideal simple support.

Regarding the use of this type of support for more flexible panels, a 1.5 mm thick Polyetheretherketon (PEEK) panel (PEEK type: 450G, density 1300 kg/m^3 , Young's modulus 3.5 GPa) was analyzed. In this case, to compensate for the higher bending flexibility of the panel, the flexibility of the shims was increased to maintain the negligible rotational stiffness produced by the support. This can be done by simply using thinner shims. The results obtained with 0.026 mm thin shims are reported in Table 5. The shims

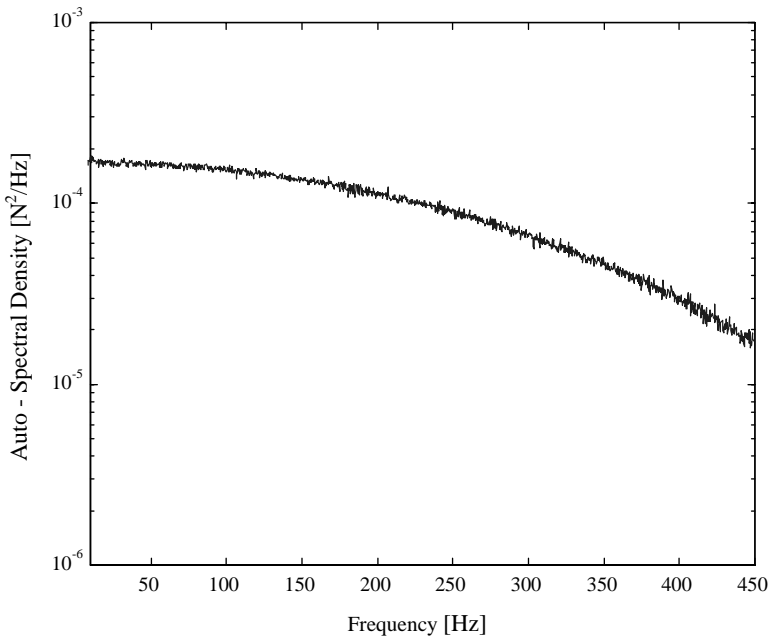


Figure 8. Spectrum of the force applied by the hammer during the tap tests.

TABLE 4

Natural frequencies of vibration for “simply supported” panels with different edge length ratio

Panel 203.2 mm × 406.4 mm ratio 1:2			Panel 203.2 mm × 508.0 mm ratio 1:2.5		
	FEA			FEA	
Theory	Ideal SS	Shims	Theory	Ideal SS	Shims
108.3	108.4	108.2	100.5	100.4	100.5
173.2	173.3	173.2	142.0	141.9	142.0
281.5	281.4	281.4	211.3	211.0	211.1
368.1	367.4	367.3	308.3	307.9	307.9

TABLE 5

Natural frequencies of vibration for the “simply supported” PEEK panel

Mode	Theory	FEA	
		Ideal SS	Shims
1,1	41.4	41.4	41.5
2,1	79.7	79.6	79.7
1,2	127.3	127.1	127.3
3,1	143.5	143.2	143.3

also produce an increase in stiffness in the direction along the edges themselves. However, this effect is negligible unless large-amplitude, non-linear vibrations are considered.

4. EXPERIMENTAL ARRANGEMENT

The experimental arrangement of the simply supported panel is shown in Figures 9 and 10. A 13-mm-thick steel plate, with a rectangular cut-out machined in the centre of the plate to the dimensions of the aluminium alloy panel, was used as the base. Four steel U-section beams were welded to the bottom of this plate to increase the stiffness of the supporting structure, and the shims were clamped to four L-section steel beams that were machined with a channel to produce a gap behind the shims, as shown in Figures 7 and 9. The aluminium panel was bonded to the shims using epoxy adhesive. The panel was supported while the two long side shims were bonded first. Packing pieces were located behind the shims to prevent movement of the shims during the bonding process. Once the epoxy had cured, the short side shims were bonded to form the complete arrangement.

The complete test rig, which is shown in Figure 10, was suspended in a frame using four tension springs to provide freely supported boundary conditions to the supporting structure. A series of forced vibration tests were carried out using an Endevco impact hammer (30927-1671). An aluminium tip was used for the impact hammer, in order to deliver energy in the entire frequency range under investigation, as can be seen from the spectrum shown in Figure 8. A B&K accelerometer (Type 4344) which had a mass of 2.2 g was used, the signal from which was conditioned and amplified using a B&K Conditioning Amplifier (Type 2626).

Data were recorded using a Signal Processing Ltd four-channel data acquisition suite [11] connected to a personal computer which operated using the Matlab software [12]. A total of three taps were recorded per measurement with a sample rate of 3000 Hz over

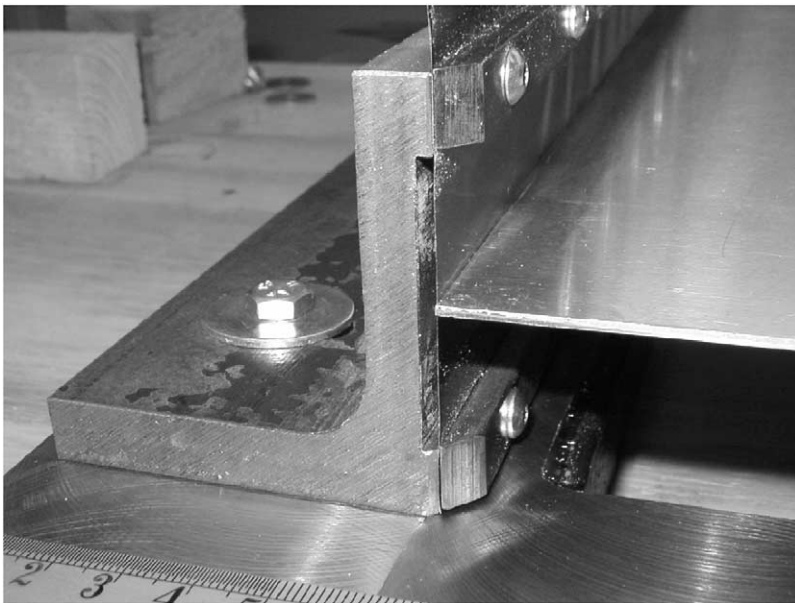


Figure 9. Experimental implementation of the simple support based on shims.

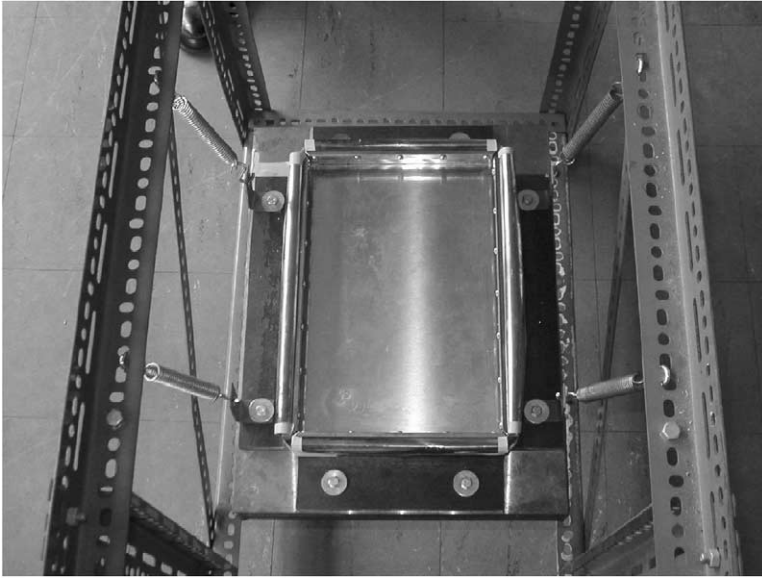


Figure 10. Test rig supporting frame.

a period of 3 s. The force and acceleration data were post-processed using the Matlab “spectrum” function. Three measurements were taken by successively tapping the panel. Each force and acceleration time history vector was transformed into the frequency domain using a radix-2 FFT. A rectangular window was used prior to carrying out this transformation, since the data were observed to decay significantly within the 3 s data window. Finally, the results from the FFT of the three successive taps were averaged.

4.1. EXPERIMENTAL RESULTS

Initially, the first four resonant response frequencies obtained from the experiment were different from those obtained from the FEA (which was presented in the previous section) with a difference of approximately 5%. This was particularly true for the first mode of vibration, which one would have expected to give the closest agreement when compared with the FE results for the natural frequency of vibration. In addition to this, it was noted that the measured frequencies tended to vary over time. It was concluded that the welded steel supporting structure possibly contained pre-stresses, which were being relieved over time. During the manufacture of the steel frame (after that the U-channel sections were welded under the steel plate) a small curvature of the upper surface of the steel plate was detected. In order to obtain a flat area (which was necessary to be able to bolt the L-section segments), the surface of the steel plate was machined thus introducing pre-stresses in addition to those previously created during the welding process. The whole structure was then put together and stored in an area affected by large fluctuation in temperature (between 15 and 60°C). In the days following the assembly, the pre-stresses that had accumulated in the steel frame were progressively released producing a slight warping of the frame, which was transferred to the aluminium panel by the shims. This ultimately resulted in a shift in the measured resonant response frequencies, particularly that associated with

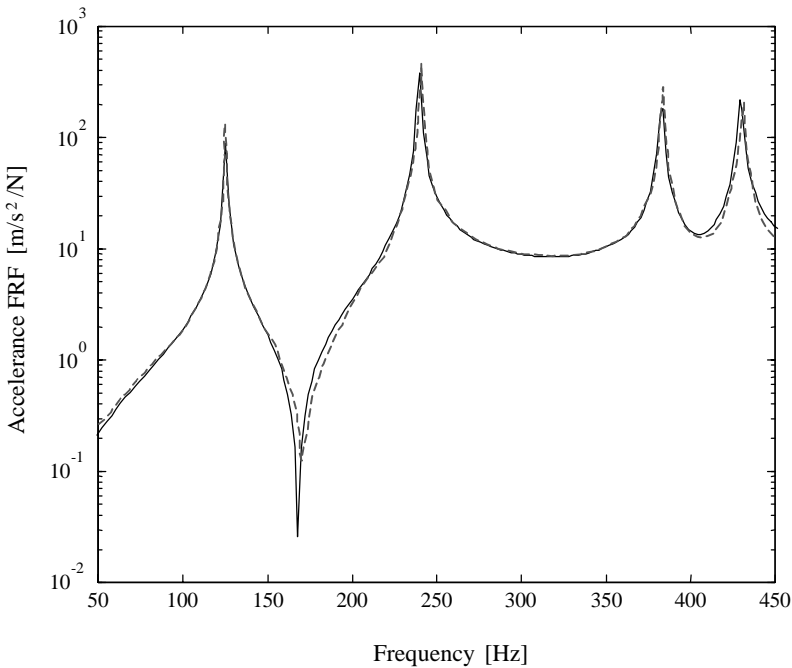


Figure 11. Comparison of measured (dashed line) and calculated (continuous line) accelerance FRF for the simply supported panel.

the first mode of vibration, which would certainly be the most sensitive to any slight twist in the panel.

By releasing the tension in the shims (by releasing and then re-tightening their clamping devices), it was possible to relieve the pre-stresses in the aluminium panel. The new results obtained for the first four resonant response frequencies are given in Table 3. The accelerance frequency response function is shown in Figure 11, and a comparison is made with that predicted by the FE harmonic analysis, which was described in the previous section. As can be seen, excellent agreement is obtained between the experimental and theoretical results, which clearly demonstrates the suitability of this type of arrangement for producing a simple support experimentally.

It was also verified experimentally that, as long as the amplitude of the force pulse delivered by the impact hammer did not excite large (possibly non-linear) vibrations, the amplitude of the force pulse did not influence the natural frequencies of vibration and mode shapes, which is to be expected. Large values of force have not been used to avoid damaging the bonded joint; however, this type of arrangement was not designed for experiments involving large-amplitude dynamic response.

During the course of the experiments, several interesting effects were noted which relate to the sensitivity of the lower resonant response frequencies to changes in distance from a parallel surface and temperature. These effects (often neglected in this type of experiment) are described and qualified in the next two sections in order to show the degree to which they affect the percentage accuracy of the results.

4.1.1. Variation in the resonant response frequencies with distance from a parallel surface

Experiments were conducted to investigate the effect of varying the distance between the simply supported panel and a parallel surface, such as the floor, on the first resonant

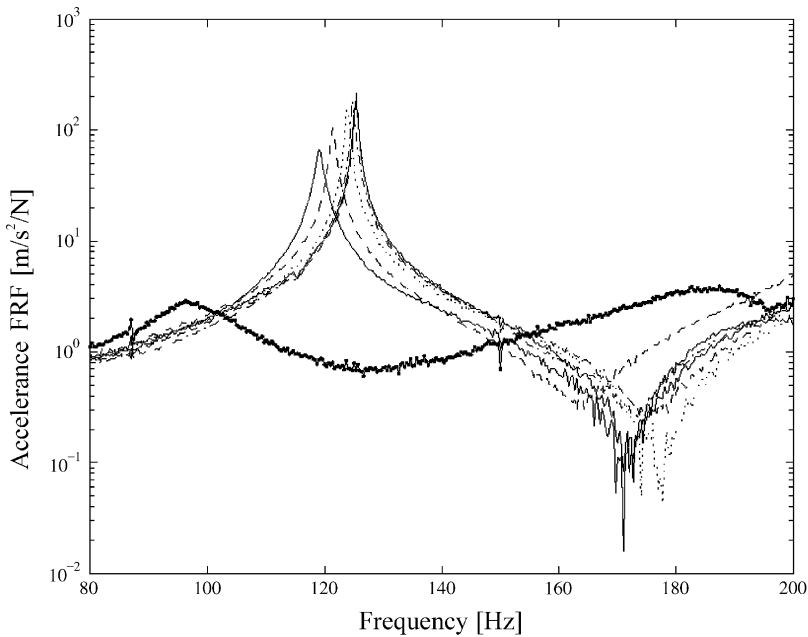


Figure 12. Effect of relative distance between the simply supported panel and a parallel surface on the measured resonant response frequency. —●— 0 mm; — 8 mm; - - - 16 mm; ····· 32 mm; - · - · - 64 mm; ——— 128 mm; - - - 256 mm.

response frequency. The panel supporting structure was first placed directly on the floor (in this condition the distance between the panel and the floor is 77 mm), and then the assembly was raised a further 8, 16, 32, 64, 128, and 256 mm from the floor. The response was measured again using an impact hammer and accelerometer in the same locations as in the previous tests. The results are shown in Figure 12, and clearly demonstrate that the resonant response frequency is very sensitive to distance from a parallel surface when the distance of that surface is less than the minimum length of the side of the panel. This effect is due to the fluid–structure interaction of the cavity of air beneath the panel. When the panel is at some distance from the parallel surface (i.e., 256 mm) the volume of air in the cavity is unbounded and therefore mass-loads the structure, and provides acoustic radiation damping [13], albeit a negligible effect in this instance. As the panel is moved closer to the parallel surface, this effect becomes more pronounced resulting in a decrease in the resonant response frequencies of the panel. When the supporting structure is placed on the floor, the volume of air beneath the panel becomes bounded, which results in a strongly coupled fluid–structure system. In this instance, both the structure and fluid can sustain standing waves and natural frequencies and hence there is an interaction between these two systems which is evident in the results in Figure 12.

4.1.2. Variation in the resonant response frequencies with changes in temperature

At a very early stage in the experiments, the sensitivity of the resonant response frequencies to small changes in temperature was highlighted. To this end, experiments were conducted at various laboratory temperatures to ascertain the degree of sensitivity. Thermocouples were used to monitor the laboratory, panel, and supporting structure temperature, and when all of these were within 0.5°C of each other, measurements of the

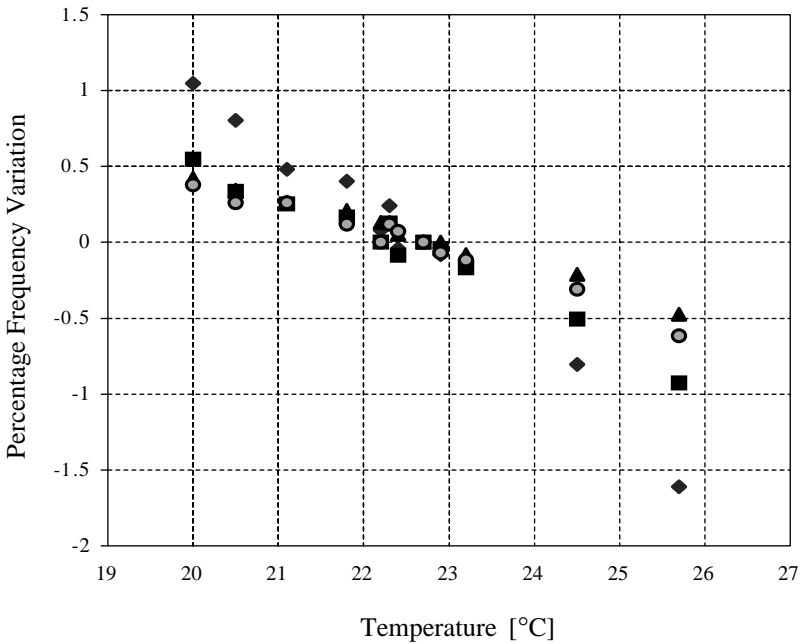


Figure 13. Percentage variation of the first four natural frequencies of vibration of the panel as a function of the laboratory temperature. \blacklozenge Mode 1,1; \blacksquare mode 2,1; \blacktriangle mode 1,2; \circ mode 3,1.

resonant response were taken, again using the impact hammer and accelerometer. The results are shown in Figure 13 for the first four resonant response frequencies. The results clearly indicate that the simply supported panel is reasonably sensitive to small temperature changes. The first mode (1,1) appears to be the most sensitive, with a 2.5% decrease in frequency, for a 6°C increase in temperature. This is particularly significant since it shows that even with this method, which is the least sensitive to temperature changes, the laboratory temperature has to be controlled carefully if high accuracy is required. The next few modes considered appeared to be less sensitive to changes in temperature, as shown in Figure 13.

5. CONCLUSIONS

In this paper, various methods commonly used to produce a simple support along the edge of a panel have been briefly reviewed. With some of these methods it is relatively difficult to predict the level of accuracy with which the physical support will approximate the theoretical condition. Suspension of the panel by thin sheets of steel (shims) was identified as the most promising method to reproduce the simple support. Furthermore, it is relatively easy to produce an FEM of the support to investigate the level of accuracy which can be expected. A test rig used to implement this method was built and various tests were performed. The experimental results were compared with theoretical calculation of the forced vibration response, proving that this method is able to reproduce the theoretical condition very accurately.

The effect of a boundary parallel to the panel was investigated, and the large influence that the distance of the boundary has on the lowest natural frequency of vibration was

highlighted. Finally, the effect of laboratory conditions, such as the temperature, on the natural frequencies of vibration was investigated. In particular, it was shown that a small change in the laboratory temperature has a relatively large effect on the frequency of the fundamental mode of vibration. This is particularly important since it shows that even with this method, which is the least sensitive to temperature changes, the laboratory temperature has to be controlled carefully if high accuracy is required.

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