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FORCED AND SELF-EXCITED VIBRATIONS OF PIPES CONTAINING MOBILE BOILING FLUID CLOTS

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Numerical modelling of the dynamic behaviour of a pipe containing inner non-homogeneous flows of a boiling fluid has been carried out. Inasmuch as the efforts to solve this problem analytically are confronted by considerable difficulties connected with varying system mass, geometry and discontinuity of equation coefficients, computational techniques for simulating pipe dynamics have been developed based on using of numerical time integration methods and transfer matrix methods together with orthogonalization procedures relating to the space variables. The system vibrations at different values of the parameters of the flow non-homogeneity and its velocity are observed. The possibility of forming stable and unstable flows depending on the character of the non-homogeneity and the velocity of fluid clots has been found.

1. INTRODUCTION

The tubes of a heat exchanger containing non-homogeneous mobile masses of a boiling fluid, vapour and their mixture is an important feature of modern heat and nuclear power plants. The heat exchanger sections possessing an initial curvature or taking a curvilinear shape due to dynamic bending, generate centrifugal inertia forces playing the role of active forces and acting in the osculating plane. They are proportional to the pipe curvature, the mass of the moving fluid element and the square of its velocity [1]. In the case of non-steady processes of boiling these forces change in time and lead to pipe-line vibration.

As experimental studies carried out in connection with the analysis of boiling fluid motions in glass tubes heated externally testify, so-called slug flows appear at certain thermodynamical states and values of the geometrical and mechanical system parameters. They reside in the fact that in heat exchanger tubes, fluid boiling regimes are possible when the vapour–water mixture is not homogeneous but consists of fluid and vapour segments alternating and moving at high velocities. As the mixture flows, the process of boiling continues, thus the lengths of the tube segments filled with a fluid (called fluid clots) decrease and the lengths of cavities filled with a vapour (gas slugs) increase. In this case, their velocities considerably increase.

The observations carried out on heated glass tubes show that the lengths of fluid clots change approximately from 10 internal diameters of the pipe on their formation to zero on complete evaporation, and the volume of a fluid, as it evaporates, increases tenfold. On boiling, the volume of gas cavities can change from zero to 50 diameters of the pipe and then, as a result of clot evaporation, mix.

The motion of a liquid clot inside a curvilinear channel is accompanied by the action of a centrifugal inertial force on its walls in the direction opposite to the orientation of a

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principal normal. In addition, as each element of the fluid is also influenced by the slewing motion together with tube vibrations, additional gyroscopic forces of interaction between the fluid and pipe walls are generated. If the stiffness of the curved pipe is relatively small, its interaction with the moving fluid clot can cause noticeable dynamical effects. There are some cases, for example, when the effects of these vibrations erode the walls of the tubes, where they are in contact with the supporting structures. As a result, the whole heat exchanger unit can fail and radioactive heat-transfer agents released.

As the relationship between the lengths of the fluid clots and vapour slugs has changes, the functioning of mechanical systems can be accompanied by complex dynamical effects attributed to the possibility of the system bodies participating in several forms of these motions together with gyroscopic interaction between them. For example, there is the possibility of static (divergent) loss of stationary motion stability, the appearance of unstable oscillatory motions (of a flutter-type) and parametric resonances [1, 2]. These stability losses depend on the relationships between the geometrical and inertial parameters of the system, the clot velocity as well as the presence or absence of an initial curvature, so, if a pipe is curved the fluid motion can result from forced resonances and ordinary resonances. If the initial curvature is absent, self-sustained vibrations associated with parametric resonances can be excited. These vibrations are attributed to the fact that as a non-homogeneous fluid flows inside the pipe, the internal characteristics of the system vary continuously and so might be an additional cause of vibration excitation.

The development of the solution to this problem was first stimulated by the need to eliminate considerable vibrations of the Trans-Arabic oil pipe line [3]. By considering a simplified circuit design, the authors [4, 5] developed equations for straight pipe-line dynamics and showed the possibility of loss of stability on attaining critical flow velocities.

In subsequent papers general questions of the stability of pipe systems conveying fluid were studied [6, 7], together with the dependancy of the peculiarities of pipe vibrations on boundary conditions [8,9]; the publishing of records of continuously changing fluid pressure and flow velocity [10, 11]; the development of more accurate theory of beams [12, 13]; the influence of tension, damping and attached mass at the lower end of a vertical pipe [14, 15] the effect of an elastic support in an intermediate cross-section [16, 17] and the evaluation of elastic foundations [18], torsional stiffness of clamping [19] and other factors [20–22].

This paper studies the influence of an initial curvature of a pipe, the size of fluid clots and vapour cavities and the velocity of their flow on the character of dynamic loss of a pipe system stability.

2. STATEMENT OF THE PROBLEM

Consider the problem concerning transverse vibrations of an elastic pipe having an initial curvature. A non-homogeneous fluid flows inside the pipe. Let us assume that its non-homogeneity might be caused, for example, by the change of its modular state associated with heating, boiling and conversion into vapour–water mixture. If typical dimensions of liquid clots and vapour cavities dividing them exceed typical dimensions of the pipe line, for example, the diameter of its channel (see Figure 1), one must consider discontinuities in the parameters of density and inner flow velocity. In this case as the pipe line vibrates, the fluid particles have an accelerated flow both along, and transversly to the pipe axis, thus forming a dynamical load on the pipe. To calculate inertial forces acting on the pipe elements one assigns the formulae governing fluid-clot flow and motion of the

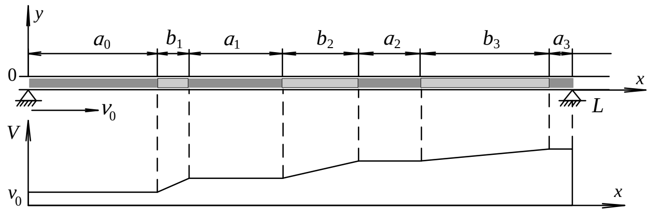

Figure 1. The diagram of fluid clot flows and change in internal flow velocity.

vapour-filled cavities in a tube by applying the condition of overall vapour–water mixture flow mass rate conservation from inlet to outlet. The model of changing the flow parameters of motion can be formed by assuming that the clots of length a_0 enter the channel at a velocity of V_0. At the inlet, the gap between two neighbouring clots is zero. The motion caused by boiling varies the clot length as $a_1 = a_0 e^{-kt}$ and decreases at the rate of $\dot{a} = da_1/dt = -k\, a_0 e^{-kt}$. As a result, the lengths of the spaces (cavities) between clots increase at the rate of $\dot{b} = db_1/dt = c\, k\, a_0 e^{-kt}$. The volume of vapour in a space is considered to be c times as much as that of a fluid from which it was formed, therefore the relation $\rho_f = c\rho_v$ is applied between the densities of the fluid and the vapour.

As the volume of a cavity increases, the velocity V_{i+1} of the $(i+1)$th clot increases relative to the previous one as $V_{i+1} = V_i(c-1)\,\dot{a}$. The velocity of vapour in the cavity between clots is assumed to be distributed linearly (see Figure 1). The influence of the initial pipe curvature on the character of excited vibrations and their stability is investigated.

In studying the dynamical interaction between an elastic pipe and an inner flow, Benjamin [6] showed that viscous friction forces occurring during flow appeared to be relatively small. As these forces are directed along the axis of a pipe, they may be neglected in the investigation of its transverse vibration. Thus, the fluid can be considered to be perfect and while investigating its influence on the dynamics of the tube, only its inertial properties will be considered. Therefore, solution of the problem of vibrations of a pipe with an inner non-homogeneous flow, can be undertaken by consideration of the motion of a fluid element along the vibrating and dynamically bending pipe line, calculation of its acceleration in the direction perpendicular to the pipe axis and determination of those inertial force acting on the fluid element transferring to the pipe walls.

Let a fluid element of mass m move along the vibrating pipe at a predetermined velocity $V(x)$ (see Figure 1). Considering its motion in the transverse direction, $m d^2 y_f / dt^2 - N = 0$. Here y_f is the displacement of the fluid element together with the pipe in the direction of the Oy-axis, N the force with which the pipe acts upon the element. In this equation, the function $y_f(t)$, determining the fluid element co-ordinate, must be transposed to the deflection function $y(x,t) + y_0(x)$ of the pipe with an initial curvature $y_0\,(x)$ at point x, the location of the element. To do so, one should consider that the fluid element takes a new position in the pipe at each instant of time, therefore, its velocity in a vertical direction is determined not only by the velocity of the pipe point in which the element is located, but also by the fact that the element moves to a neighbouring point in the pipe with another co-ordinate y and velocity $\dot{y}$:

$$dy_f/dt = \partial y/\partial t + (\partial y_0/\partial x + \partial y/\partial x)\partial x/\partial t = \dot{y} + (y_0' + y')\ V. \tag{1}$$

Differentiating once more both members of equation (1) with respect to t, one finds the vertical component of absolute acceleration of the fluid element in the vibrating pipe with an initial curvature

$$\mathrm{d}^2 y_f/\mathrm{d}t^2 = \ddot{y} + 2\dot{y}'V + Vy'V' + y''V^2 + y_0''V^2 + y'\dot{V} + y_0'\dot{V} + Vy_0'V'. \tag{2}$$

This formula can be correlated with the formula of the Coriolis theorem [23] for absolute acceleration of a particle, where $\ddot{y}$ is the bulk acceleration, $2\dot{y}'V$ is the Coriolis acceleration, $y''V^2$ the centripetal acceleration, $\dot{y}'$ the angular velocity of the pipe element and V the relative velocity of the fluid element.

When constructing the equation of transverse vibration of a tubular rod with an inner flow of a non-homogeneous fluid, one models it as an Euler–Bernoulli beam, neglecting the internal friction forces and the beam friction on interaction with the environment. The resulting equation of plane transverse vibrations of the pipe can be presented as

$$EJy^{\mathrm{IV}} + \rho_t a_t + \rho_f a_f = 0. \tag{3}$$

Here EJ is the pipe bending stiffness, ρ_t the tube linear density, ρ_f the linear density of the inner flow and a_t, a_f are the accelerations along the Oy-axis of the tube and fluid elements respectively.

Using formulas $a_t = \mathrm{d}^2 y/\mathrm{d}t^2$, $a_f = \mathrm{d}^2 y_f/\mathrm{d}t^2$ and taking into account equation (2) and (3), the equation of pipe vibrations in the sections containing the fluid can be written as

$$EJy^{\mathrm{IV}} + \rho_f V^2 y'' + (\rho_t + \rho_f)\,\ddot{y} + 2V\rho_f\dot{y}' + \rho_f\dot{V}y' = -\rho_f y_0''V^2 - \rho_f y_0'\dot{V}. \tag{4}$$

In the sections containing the vapour spaces, $\rho = \rho_v$ and equation (4) takes the form

$$\begin{aligned} &EJy^{\mathrm{IV}} + \rho_v V^2 y'' + (\rho_t + \rho_v)\ddot{y} + 2V\rho_v\dot{y}' + \rho_v\dot{V}y' + \rho_v VV'y' \\ &= -\rho_v y_0''V^2 - \rho_v y_0'\dot{V} - \rho_v y_0' VV'. \end{aligned} \tag{5}$$

The distinguishing feature of the assigned problem described by equations (4) and (5) resides in the fact that when the fluid clots flow, either equation (4) or (5) is alternately used for one and the same points of the pipe. Thus, the chosen mechanical system belongs to the systems with variable parameters (with approximately periodical coefficients and right member). Due to this fact on varying the velocity V, both ordinary and parametrical resonance vibrations, typical of such systems, can be excited as the result of the dynamical loss of stability. For the case considered, the problem of studying parametric vibrations is complicated by the presence in equation (4) of the component $2\rho_f\dot{y}'V$ describing the internal force resulting from the gyroscopic effect. Their presence considerably complicates the mode of the pipe motion because its elements start vibrating at different phases.

The second peculiarity of the process studied is that, because of the changes of inertial properties of the pipe as fluid clots travel in it, there is no frequency spectrum and modes of free vibrations do not exist, and natural frequencies in the vicinity where resonances could be realized are lost. Therefore, it is difficult to predict a dynamical loss of stability in such systems. Finally, the difficulty of studying such a dynamical system increases whilst the discrete character of the clot flow is lost resulting in the coefficients of the united set of equations (4) and (5) becoming discontinuous.

The above peculiarities illustrate the difficulties in using analytical methods for studying the dynamic instability of pipes with inner flows, based on the Liapunov and Floquet approaches [24]. Thus this investigation uses direct numerical modelling methods of the system motion at chosen initial disturbances and the assigned velocity V of the flow.

3. INVESTIGATION PROCEDURE

Let one consider two problems, namely that of motion of a non-homogeneous flow in a straight pipe and that of flow in a pipe with an initial curvature. By imparting the system with some small initial perturbation in the form of a preset deflection, it is possible to analyze the possibility of self-excitation of the straight pipe-line vibrations with non-homogeneous inner fluid flow, though performing numerical modelling of its dynamical behaviour at various clot lengths and differing velocities, V_0, at the inlet. If the vibrations of the perturbed pipe-line decay, then its initial state is considered to be stable. When the amplitude of vibrations and divergent deflections increase indefinitely, the system is considered to be dynamically unstable. The fluid velocity V_0, at which periodic motion is established in the system, is thought to be critical.

In order to investigate the dynamics of the tube with initial curvature, it is not necessary to introduce the additional perturbations into the system, in that the constitutive equations are non-homogeneous in advance.

When boundary conditions are being preset, one of the considerations is that the pipe line be represented as a multispan beam with equal span lengths and hinged supports. The system vibrations are modelled by the least power-intensive modes having skew symmetry relative to the support cross-sections. It is then assumed that the vibrations of neighbouring sections of the pipe have opposite phases and in studying them one arbitrarily separates one span of the pipe applying boundary zero conditions to deflections and bending moments at its support points:

$$y(0) = y(L) = 0, \quad y''(0) = y''(L) = 0. \tag{6}$$

If the tube is assumed to have preliminary curvature $y_0(x)$, the initial conditions are chosen in the form $y(x,0) = 0$, $\dot{y}(x,0) = 0$. If the tube is initially straight, the initial conditions are chosen as the initial static perturbation $y(x,0) = w_0 \sin \pi x/L$, $\dot{y}(x,0) = 0$. Here the coefficient w_0 is considered to be very small.

For numerical integration of the equations with preset boundary conditions (6) and initial perturbations, one uses the Houbolt implicit finite difference method characterized by the approximation of pinpoint accuracy and stability [25]. In this case for the time t, the time derivatives in equations (4) and (5) are substituted by finite differences in the form of

$$\dot{y}(x,t) = \dot{y}|_t = [11y_t(x) - 18y_{t-1}(x) + 9y_{t-2}(x) - 2y_{t-3}(x)]/6\Delta t.$$

$$\ddot{y}(x,t) = \ddot{y}|_t = [2y_t(x) - 5y_{t-1}(x) + 4y_{t-2}(x) - y_{t-3}(x)]/\Delta t^2. \tag{7}$$

Here $y_t(x) = y(x,t)$, $y_{t-1}(x) = y(x, t-\Delta t)$, $y_{t-2}(x) = y(x, t-2\Delta t)$, $y_{t-3}(x) = y(x, t-3\Delta t)$, Δt is the numerical time integration step.

By considering the above relationships, equations (4) and (5) can be written as

$$\begin{aligned} &EJ\frac{\mathrm{d}^4y}{\mathrm{d}x^4}\bigg|_t + \rho_f V_f^2 \frac{\mathrm{d}^2y}{\mathrm{d}x^2}\bigg|_t + \frac{2(\rho_t+\rho_f)}{\Delta t^2}y|_t + \frac{11\rho_f V_f}{3\Delta t}\frac{\mathrm{d}y}{\mathrm{d}x}\bigg|_t \\ &= \frac{5(\rho_t+\rho_f)}{\Delta t^2}y|_{t-\Delta t} - \frac{4(\rho_t+\rho_f)}{\Delta t^2}y|_{t-2\Delta t} + \frac{(\rho_t+\rho_f)}{\Delta t^2}y|_{t-3\Delta t} + \frac{6\rho_f V_f}{\Delta t}\frac{\mathrm{d}y}{\mathrm{d}x}\bigg|_{t-\Delta t} \\ &\quad - \frac{3\rho_f V_f}{\Delta t}\frac{\mathrm{d}y}{\mathrm{d}x}\bigg|_{t-2\Delta t} + \frac{2\rho_f V_f}{3\Delta t}\frac{\mathrm{d}y}{\mathrm{d}x}\bigg|_{t-3\Delta t} - \rho_f V_f^2 y_0''|_t - \rho_f y_0' \dot{V}_f|_t, \end{aligned}$$

$$
\begin{aligned}
&EJ\frac{d^4y}{dx^4}\bigg|_t+\rho_v V_v^2\frac{d^2y}{dx^2}\bigg|_t+\frac{2(\rho_t+\rho_v)}{\Delta t^2}y|_t+\frac{11\rho_v V_v}{3\Delta t}\frac{dy}{dx}\bigg|_t+\rho_v VV'\frac{dy}{dx}\bigg|_t\\
&=\frac{5(\rho_t+\rho_v)}{\Delta t^2}y|_{t-\Delta t}-\frac{4(\rho_t+\rho_v)}{\Delta t^2}y|_{t-2\Delta t}+\frac{(\rho_t+\rho_v)}{\Delta t^2}y|_{t-3\Delta t}+\frac{6\rho_v V_v}{\Delta t}\frac{dy}{dx}\bigg|_{t-\Delta t}\\
&\quad-\frac{3\rho_v V_v}{\Delta t}\frac{dy}{dx}\bigg|_{t-2\Delta t}+\frac{2\rho_v V_v}{3\Delta t}\frac{dy}{dx}\bigg|_{t-3\Delta t}-\rho_v V_v^2 y_0''|_t-\rho_v y_0'\dot{V}_v|_t-\rho_v y_0' V_v V_v'|_t. \qquad (8)
\end{aligned}
$$

By knowing the states $y_{t-1}(x)$, $y_{t-2}(x)$, $y_{t-3}(x)$ of the system at times $t-\Delta t$, $t-2\Delta t$, $t-3\Delta t$ one can find the state $y_t(x)$ of the system at time t using (8) with appropriate boundary conditions and then extend this to determination of the system states at times $t+\Delta t$, $t+2\Delta t$, etc. Inasmuch as equations (8) represent the four-layer difference scheme but one has only two initial conditions, the first step of the calculational processes is performed by using the three-layer Newmark difference scheme.

Equations (8) with boundary conditions (6) are solved using the transfer matrix method. To do this, the fourth order equations (8) were transformed to first order equations. For the first equation (8),

$$
dy_1/dx=y_2,\quad dy_2/dx=y_3,\quad dy_3/dx=y_4,
$$

$$
\begin{aligned}
EJ\frac{dy_4}{dx}\bigg|_t=&-\rho_f\dot{V}_f y_{2,t}-\rho_f V_f^2 y_{3,t}-\frac{2(\rho_t+\rho_f)}{\Delta t}y_{1,t}-\frac{11\rho_f V_f}{3\Delta t}y_{2,t}\\
&+\frac{5(\rho_t+\rho_f)}{\Delta t}y_{1,t-1}-\frac{4(\rho_t+\rho_f)}{\Delta t^2}y_{1,t-2}+\frac{(\rho_t+\rho_f)}{\Delta t^2}y_{1,t-3}+\frac{6\rho_f V_f}{\Delta t}y_{2,t-1}\\
&-\frac{3\rho_f V_f}{\Delta t}y_{2,t-2}+\frac{2\rho_f V_f}{3\Delta t}y_{2,t-3}-\rho_f V_f^2 y_0''|_t-\rho_f y_0'\dot{V}_f|_t. \qquad (9)
\end{aligned}
$$

This system can be written in a general form

$$
d\vec{y}/dx=A(x)\vec{y}+\vec{f}(x). \qquad (10)
$$

Here $\vec{y}=\vec{y}(s)$ is the four-dimensional vector of the unknown functions, x the independent variable changing within the limits of $0\leqslant x\leqslant L$; $A(x)$ the known discontinuous matrix function of the independent variable x and $\vec{f}(x)$ the preset vector of right members determined by the known solution functions at previous steps in time.

The solution to equation (9) must be subset to boundary conditions (6) in the interval bounds, which are predetermined at the beginning $x=0$ and at the end $x=L$ of the integration interval.

One represents them in the general form as

$$
\mathbf{B}\vec{y}(0)=0,\quad \mathbf{D}\vec{y}(L)=0, \qquad (11)
$$

where matrices $\mathbf{B}$ and $\mathbf{D}$ measure 2×4.

To construct the solution $\vec{y}(x)$, choose two components $y_j(x)$ among $y_i(x)(i=\overline{1,4})$ components, such that any values $y_j(0)$ of do not violate the first equation (11) at zero values of the other components. After renumbering the unknown values $y_i(x)(i=\overline{1,4})$ in such a way that the index j could take on the values $j=\overline{1,2}$, the solution to problems (10) and (11) can be given as $\vec{y}(x)=\mathbf{Y}(x)\vec{C}+\vec{y}_0$, where $\vec{y}_0$ is the solution to the Cauchy problem for system (10) at zero initial conditions, $\mathbf{Y}(x)$ is the 4×2 matrix of particular solutions $\vec{y}_{ij}(x)$ to the homogeneous matrix differential equation

$$
d\mathbf{Y}/dx=A(x)\mathbf{Y}, \qquad (12)
$$

with initial conditions $y_{ij}(0) = \delta_i^j\,(i = \overline{1,4}, j = \overline{1,2})$ for the independently modified variables, and with initial conditions chosen from the first equation of system (11) for the other variables $y_{ij}(0)(j = \overline{3,4})$. Here δ_i^j is the Kronecker symbol.

As $\mathbf{Y}(x)$ is the solution to the homogeneous equation (12), then on choosing initial conditions for the predetermined vectors, special attention is paid to their linear independence. This is achieved by assuming the matrix of initial conditions $\mathbf{Y}(0)$ to have the unit elements $y_{11}(0) = 1$, $y_{22}(0) = 1$. In doing so, any pair of vectors $y_{ij}(0)$ are mutually orthogonal so providing their linear independence.

The vector of the constants $\vec{C} = (C_1, C_2)^{\mathrm{T}}$ is so chosen that the equality $\mathbf{DY}(L)\vec{C} + \mathbf{D}\vec{y}_0(L) = 0$, following from the second conditions of system (11) is satisfied.

The construction of the matrix function $\mathbf{Y}(x)$ and the vector function $\vec{y}_0(x)$ is made by integrating equations (10) and (12) by the fourth order Runge–Kutta method. The peculiarity of using such an approach is that due to the presence of large factors in the coefficients of system (8), it is rigid and there are rapidly growing functions among its particular solutions. Therefore, in constructing the matrix of its fundamental solutions, the method of discrete orthogonalization by Godunov is additionally used making it possible to obtain a stable computational process by orthogonalizing the vector solutions to the Cauchy problems in the finite number of argument change interval points. Its essence is in the fact that the integration interval is divided into sections, and the numerical integration of the initial differential equation is carried out on each of these sections in as with the method of transfer matrix. The lengths of the sections are such that the particular solutions to a homogeneous equation within the limits of one section remain linearly independent. When passing from one section to another, the matrix of the solutions is subject to linear transformation so that the vectors of particular solutions of the homogeneous and non-homogeneous equations become orthogonal. Thus, it is possible to preserve the linear independence of the equation solutions in the whole interval of integration. To avoid excessive increase of the numerical values of the non-homogeneous equation solutions, the normalization factor is introduced at the section boundaries.

4. RESULTS AND DISCUSSIONS

The calculation algorithms and computer programs for carrying out numeric modelling pipe vibrations at various values of their geometrical parameters were developed on the basis of the described procedure.

To study the influence of the initial curvature on the character of vibrations of a pipe system, the cases, when in the initial state the pipe was straight $(y_0(x) \equiv 0)$ and when its centreline was curved according to the law $y_0(x) = (L/400)\sin(\pi x/L)$ were considered. For the first problem non-trivial solutions may appear as a result of either divergent or flutter bifurcations. The results of the calculations for the above cases are given in Table 1, where L is the length of the pipe, h the thickness of its wall, a_0 the clot length at the inlet and k the parameter determining the velocity of fluid evaporation. It was assumed or all the pipes that $E = 2 \times 10^{11}\,\mathrm{Pa}$, $\rho_t = ((R)^2 - (R-h)^2)\rho$, $\rho = 7800\,\mathrm{kg/m^3}$, $\rho_f = \pi(R-h)^2\rho_w$, $\rho_w = 1000\,\mathrm{kg/m^3}$, $R = 0{\cdot}015\,\mathrm{m}$, and $c = 10$.

Using suitable parameters, eight problems were solved (see Table 1) with differing lengths of the clots at the inlet a_0 and the value k determining the velocity of evaporation of the boiling fluid. Here the value of a_0 were $L/8$ and $L/4$, and the values of k were chosen so that during the flow in the pipe channel a fluid clot decreases in length by 15–40%.

TABLE 1

Velocities values and periods of forced vibrations of a straight-line pipe

No.	L (m)	h (m)	a_0	k (s^{-1})	Dynamical parameter values					
1	5	0·003	$L/8$	0·1	V_0 (m/s)	1·1	1·2 ($V_{0,cr}$)	20	40	87·5
					T_v (s)	0·244	0·245	0·262	0·289	1·301
					T_c (s)	0·568	0·52	0·031	0·016	0·007
2	5	0·003	$L/8$	0·5	V_0 (m/s)	1·8	1·9 ($V_{0,cr}$)	20	40	80
					T_v (s)	0·24	0·242	0·26	0·292	0·783
					T_c (s)	0·347	0·329	0·031	0·016	0·008
3	5	0·003	$L/4$	0·5	V_0 (m/s)	5	5·1 ($V_{0,cr}$)	10	20	40
					T_v (s)	0·246	0·247	0·25	0·262	0·298
					T_c (s)	0·25	0·245	0·125	0·063	0·031
4	5	0·003	$L/4$	1	V_0 (m/s)	6·9	7 ($V_{0,cr}$)	10	20	40
					T_v (s)	0·245	0·245	0·25	0·262	0·297
					T_c (s)	0·18	0·179	0·125	0·063	0·031
5	8	0·001	$L/8$	0·1	V_0 (m/s)	0·5	0·6 ($V_{0,cr}$)	4	10	25
					T_v (s)	0·623	0·63	0·7	0·783	1·518
					T_c (s)	0·2	1·66	0·25	0·1	0·04
6	8	0·001	$L/8$	0·5	V_0 (m/s)	1·2	1·3 ($V_{0,cr}$)	4	10	20
					T_v (s)	0·61	0·613	0·66	0·763	1·325
					T_c (s)	0·833	0·77	0·25	0·1	0·05
7	8	0·001	$L/4$	0·5	V_0 (m/s)	3	4·4	4·5 ($V_{0,cr}$)	10	20
					T_v (s)	0·66	0·675	0·69	0·768	1·253
					T_c (s)	0·667	0·454	0·444	0·2	0·1
8	8	0·001	$L/4$	1	V_0 (m/s)	3	4	6·1	6·2 ($V_{0,cr}$)	10
					T_v (s)	0·643	0·655	0·668	0·703	0·758
					T_c (s)	0·667	0·5	0·328	0·323	0·1

For each problem, at a fixed value of V_0, the dynamics of the pipe at a time interval equal the time of arrival of at least 300 was studied. It was assumed that the pipe was given some initial excitation in the form of a low initial velocity. If then the vibrations were decaying, the initial state was considered to be stable, but if the amplitude of vibrations increased, the initial state was unstable. To find resonance flows, the velocity V_0 was varied and modelling the flow was repeated at a new value of V_0. The least value of V_0 at which the amplitude of vibrations began to increase without limit was considered to be critical. The step ΔV_0 of variation V_0 was $\Delta V_0 = 0{\cdot}2$ m/s. In the vicinity of a critical state, the calculations were made specific with the step $\Delta V_0 = 0{\cdot}1$ m/s.

The predetermined velocity values (T_c) of the arrival of clots in the pipe (see Table 1) were calculated to correspond with the values of the time T_v between two neighbouring maximum values of the pipe middle point displacement along Oy-axis .

Note that for problems 3 ($V_0 = 5$ m/s, $V_0 = 5{\cdot}1$ m/s), 7 ($V_0 = 3$ m/s) and 8 ($V_0 = 3$ m/s) value T_v is equal to period T_c but for problems 3 ($V_0 = 10$ m/s, $V_0 = 20$ m/s), 4 ($V_0 = 10$ m/s), 8 ($V_0 = 6{\cdot}1$ m/s) value T_v is approximately a multiple of T_c.

The results of the investigation of the dynamics of pipes with an initial curvature (see Table 2) show that the interaction of forced and parametric vibrations does not lead to the

TABLE 2

Velocities values and periods of forced vibrations of a pipe with curvature

No.	L (m)	h (m)	a_0	k (s^{-1})	Dynamical parameter values					
1	5	0·003	$L/8$	0·1	V_0 (m/s)	1·1	1·2 ($V_{0,cr}$)	20	40	87·5
					T_v (s)	0·24	0·245	0·261	0·287	1·32
					T_c (s)	0·568	0·52	0·031	0·016	0·007
2	5	0·003	$L/8$	0·5	V_0 (m/s)	1·8	1·9 ($V_{0,cr}$)	20	40	80
					T_v (s)	0·245	0·247	0·26	0·293	0·785
					T_c (s)	0·347	0·329	0·031	0·016	0·008
3	5	0·003	$L/4$	0·5	V_0 (m/s)	5	5·1 ($V_{0,cr}$)	10	20	40
					T_v (s)	0·244	0·246	0·25	0·258	0·29
					T_c (s)	0·25	0·245	0·125	0·063	0·031
4	5	0·003	$L/4$	1	V_0 (m/s)	6·9	7 ($V_{0,cr}$)	10	20	40
					T_v (s)	0·246	0·247	0·25	0·258	0·297
					T_c (s)	0·18	0·179	0·125	0·063	0·031
5	8	0·001	$L/8$	0·1	V_0 (m/s)	0·5	0·6 ($V_{0,cr}$)	4	10	25
					T_v (s)	0·624	0·63	0·73	0·783	1·532
					T_c (s)	0·2	1·66	0·25	0·1	0·04
6	8	0·001	$L/8$	0·5	V_0 (m/s)	1·2	1·3 ($V_{0,cr}$)	4	10	20
					T_v (s)	0·615	0·614	0·66	0·768	1·326
					T_c (s)	0·833	0·77	0·25	0·1	0·05
7	8	0·001	$L/4$	0·5	V_0 (m/s)	3	4·4	4·5 ($V_{0,cr}$)	10	20
					T_v (s)	0·665	0·68	0·683	0·765	1·268
					T_c (s)	0·667	0·454	0·444	0·2	0·1
8	8	0·001	$L/4$	1	V_0 (m/s)	3	4	6·1	6·2 ($V_{0,cr}$)	10
					T_v (s)	0·63	0·63	0·665	0·69	0·75
					T_c (s)	0·667	0·5	0·328	0·323	0·1

displacement of critical velocities values. Values T_v of the curved pipe have not been changed practically. These peculiarities for values T_v of straight pipe vibrations are also characteristic for the pipe with an initial curvature. Figure 2 gives vibration graphs for the point $x = L/2$ of the pipe centreline along the Oy-axis for problem 8 (see Table 2). The associated states of a flow (the arrangement of clots and their velocities) for the instant of time, when a clot arriving at the channel at the velocity of V_0 reaches its full length a_0 and starts separating from its main flow at the point $x = 0$, are shown in Figure 3.

One notices that at $V_0 = 3{\cdot}3$ m/s (see Figure 2) the vibrations decay with additional beats. With further increase in the velocity to $V_0 = 6{\cdot}0$ m/s, the pipe vibrations are stable in nature and assume a beating mode. In the critical case $V_{0,cr} = 6{\cdot}2$ m/s, the pipe loses its stability by modal flutter, but not according to the linear law and with additional vibrations. In the postcritical state ($V_0 > V_{0,cr}$) the elastic system remains unstable and in doing so begins to vibrate with less frequency. Figure 4 illustrates the modes of the curved tube plane vibrations which occur for problem 8 during time T_v. The pipe was found to vibrate according to the combination of the first and the second modes of natural vibrations of a pipe without a fluid flow.

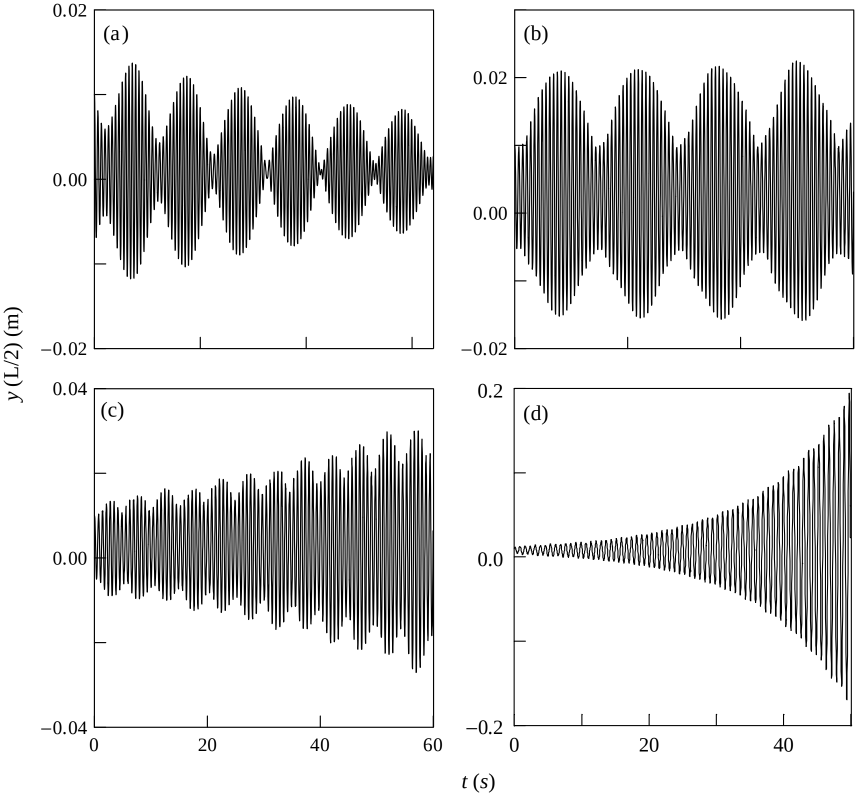

Figure 2. The forms of vibrations in time of a central cross-section of a pipe with mobile "boiling-away" clots ($L = 8{\cdot}0\,\mathrm{m}, a = L/4$): (a) $V_0 = 3{\cdot}3\,\mathrm{m/s}$; (b) $V_0 = 6{\cdot}0\,\mathrm{m/s}$; (c) $V_0 = 6{\cdot}2\,\mathrm{m/s}$; (d) $V_0 = 10{\cdot}0\,\mathrm{m/s}$.

In conclusion, one notes a peculiarity, characteristic of the dynamical process under discussion. The case is that when vibrational motions of a pipe are excited by inner mobile clots, a joint action of two affecting mechanisms is shown, each of them having its own nature. First, one observes here only the dynamical action of inertial centrifugal forces on an elastic pipe, which in this case play the role of active forces. The action of these forces determines the presence of the right member in the constitutive equations and their non-homogeneity. Second, the characteristic effects of a parametric vibration excitation mechanism appear here.

Indeed, as the dynamic system carries mobile masses, its inertial parameters periodically change, and this may cause an additional source of excitation of vibrations. The dynamical action of parametric effects is shown up by the appearance of time dependent coefficients in the constitutive equations.

It is known [2] that the resonance vibrations (i.e., ordinary and parametric resonances) excited by these two factors develop and proceed with time in different ways. If in the first case, the amplitudes of vibrations increase with time according to the linear law, then in the second case, they increase according to the square law. As seen from the calculations obtained (Figure 2(a); $V_0 = 6{\cdot}2\,\mathrm{m/s}$; (b) $V_0 = 10\,\mathrm{m/s}$) the amplitudes of vibrations with the resonance of the system under study are built up non-linearly. Thus, it can be concluded that the manifestation of the parametric vibration excitation is predominant here.

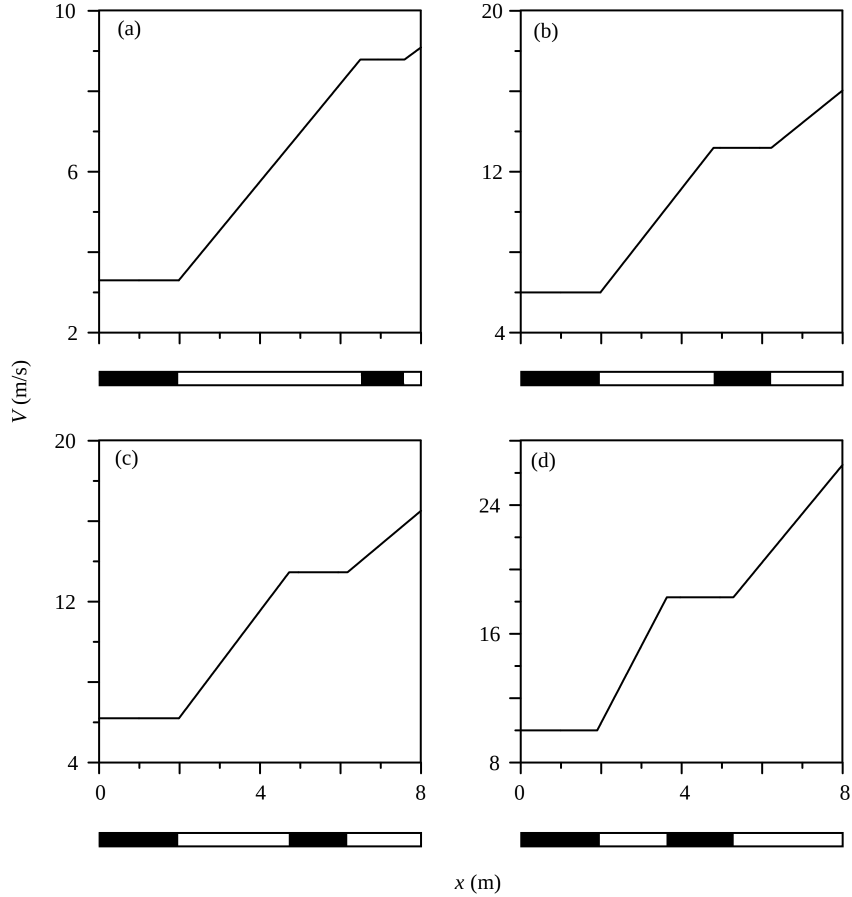

Figure 3. The fluid-clot velocities along the pipe for: (a) $V_0 = 3{\cdot}3$ m/s; (b) $V_0 = 6{\cdot}0$ m/s; (c) $V_0 = 6{\cdot}2$ m/s; (d) $V_0 = 10{\cdot}0$ m/s.

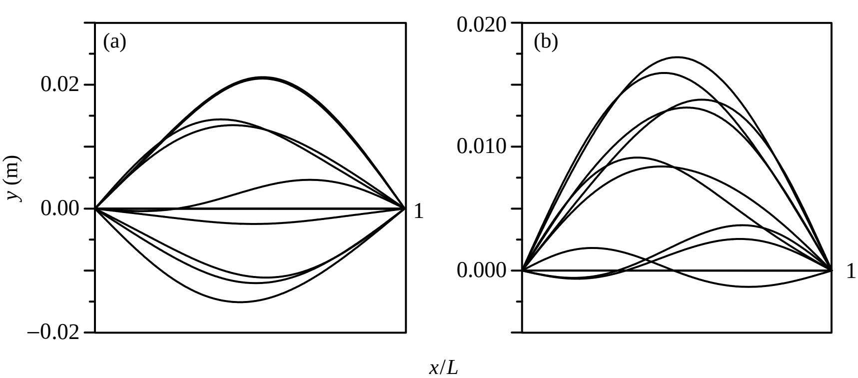

Figure 4. Modes of self-excited vibrations of a curved pipe ($L = 8{\cdot}0$ m, $a = L/4$) for: (a) $V_0 = 6{\cdot}0$ m/s and (b) $V_0 = 10{\cdot}0$ m/s.

5. CONCLUSIONS

The purpose of this paper is to carry out the numerical modelling of self-excited vibrations of tubes containing inner flows of non-homogeneous boiling fluid. Straight tubes and curved tubes have been considered. Dynamic flow model is suggested with allowance made for the discontinuous character of its density and the fluid-clot flow mode in the process of their heating and evaporations. The action of inertial forces of positional and gyroscopical effects is taken into account. The analysis of the results obtained makes it possible to conclude:

(1) Unstable equilibrium states accompanied by self-excitation of vibrations and flutter type loss of stability can arise in a pipe from the action of inertial forces of non-homogeneous non-stationary inner flows on the pipe walls. In a number of cases the divergent conditions of losing the straight-line stability were realized in supercritical states.
(2) The mechanism of distorting the normal shape of a pipe results from the action of centrifugal and Coriolis' inner flow inertial forces which can be classified as positional and gyroscopical ones.
(3) The non-homogeneity of an inner fluid flow manifests itself both in the non-homogeneity of centrifugal inertial forces acting on a pipe in the transverse direction and in the change with time of the general system mass geometry. In this connection purely dynamical and parametrical excitations of vibrations take place.
(4) Gyroscopic inertial forces caused by the interaction between the slewing movement of pipe elements and linear flows of fluid masses have a marked influence on the dynamic process character. They lead to the system loss of a general motion phase and to essential complication of the modes of the pipe transverse vibrations.
(5) The calculations testify that in the general case the transverse motions of a pipe constitute non-stationary vibrations in which one can distinguish a conventional period T_v. As a rule this period does not appear to be comparable to the period of fluid clots arriving in the pipe channel although in some cases these values were almost equal or multiple.

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