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AUTHOR'S REPLY

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The author thanks N. G. Stephen for his comments. The writer correctly draws attention to his reference [8], the paper by G. R. Cowper, of which the author was unaware. This is essentially the same solution, although it only covers the lowest frequency mode ($\gamma < 1$) for which the arguments of the hyperbolic functions remain real. Other real solutions exist on substituting circular functions for these hyperbolic functions. However, using the expansions of equation (9), all the possible modes associated with the sinusoidal flexural wave form can be examined without difficulty. The author does not agree with the conclusion drawn by the writer that Figure 1 shows that the other root of Timoshenko's solution (TBT2) is meaningless. In fact, it shows that this solution, marked T2, converges on the exact second mode, marked 2, for wavelengths of the order of the lengths of normal beams. At a wavelength of five times the beam depth, Timoshenko's predicted second frequency is only 1·5% above the exact value. For shorter wavelengths, it approaches the values of the third flexural mode, marked 3. The fourth and fifth flexural modes could have been plotted in the figure, but they do not seem relevant to a discussion of Timoshenko's theory. Likewise, there seems no need to make comparisons with longitudinal vibration modes, as suggested by the writer, because a sinusoidal flexural mode is assumed in the solution yielding both Timoshenko's roots.

The assumption by the writer that a value of κ equal to $\frac{5}{6}$ was used in the author's analysis is correct. Also, using a more accurate computer program, the author agrees with the curve marked A in his Figure 1. However, it would be misleading to justify any particular value of κ on the basis of a best fit to this particular problem. The author is aware of at least 15 different values of κ, no doubt each of which is justifiable as the best, in particular circumstances. Many of these can be found in a review by Kaneko [1]. He lists the expression attributed by the writer to Hutchinson [3] and himself [5, 6] as implicit in the works of Timoshenko (1922) and Higuchi *et al.* (1957).

It is possible to derive all beam stiffnesses on the basis of energy methods applied to the exact characteristic responses to resultant loads, found by applying Saint-Venant's principle. For homogeneous isotropic beams, this yields the usual bending stiffness, EI, axial stiffness, EA, and torsional stiffness, GJ. It also gives a unique shear stiffness, $G\kappa A$. If it were accepted that this is the κ which should be used in general when applying Timoshenko's beam theory, then it takes the value of $\frac{5}{6}$ in the present case. (For beams of a finite thickness, κ is then a function both of the breadth/depth ratio and of Poisson's ratio.) It must be borne in mind that engineering beam theory is almost always an approximation, so that to adjust it to fit a particular case is unlikely to lead to a general improvement.

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REFERENCE

1. T. KANEKO 1975 *Journal of Physics D*: *Applied Physics* **8**, 1927–1936. On Timoshenko's correction for shear in vibrating beams.