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EFFECTIVE VIBRATION ANALYSIS OF IC ENGINES USING CYCLOSTATIONARITY. PART II—NEW RESULTS ON THE RECONSTRUCTION OF THE CYLINDER PRESSURES

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This part addresses the feasibility of reconstructing the pressure trace in IC engine from non-intrusive structural vibration measurements by means of an optimal inverse filter. The problem is formally phrased into the general framework of cyclostationary processes introduced in Part I of this paper. It is proved that the optimal inverse filter is periodically varying. A number of guidelines are given for efficiently applying the proposed deconvolution scheme.

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1. INTRODUCTION

The first part of this paper addressed the issue of using structural vibration measurements as a means of condition monitoring of internal combustion (IC) engines. This second part gives more emphasis to the use of structural vibration for assessing the combustion process which happens to be the most critical phenomenon taking place in IC engines. Indeed, for both diesel and spark ignition engines, the quality of the combustion in each of the engine cylinders is an essential factor for achieving optimal operation. Due to its extreme sensitivity to most malfunctions that are likely to affect the power-train system, the combustion process itself serves as an excellent indicator for condition monitoring the engine. Therefore, any effort to determine it is usually justified. This objective requires *ad hoc* methods that are more specialized than those discussed in Part I of this paper [1], but may be used simultaneously in the same condition monitoring procedure.

The paper is structured as follows. In the first section, the usefulness of the cylinder pressure as a diagnostic indicator of the combustion process is briefly recalled, along with the difficulties associated with its practical measurement. The details of its reconstruction from external structural vibration measurements on the engine block are then discussed in detail. In the second section, the problem is formally phrased in a unified theoretical framework that encompasses the previous work and gives new insights into the issue. An important result is deduced which states that the cylinder pressure can be recovered by applying an inverse filter of a specific structure to the vibration signal. Indeed, such a filter is proved to be periodically varying in the most general case. Finally, the proposed

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deconvolution scheme is illustrated on actual signals from a diesel engine. A number of guidelines are given in order to get statistically sound results that really demonstrate the effectiveness of the approach.

2. USING THE PRESSURE TRACE AS A DIAGNOSTIC INDICATOR

2.1. MOTIVATIONS

The cylinder pressure traces in IC engines are recognized to be indicators of value, most often to be used for controlling a set of parameters [2, 3], but for diagnostic purposes as well [4]. Actually, many faults can be detected by simply inspecting the shape of the pressure traces: this is typically the case when incomplete combustion, misfire, knock, abnormal advanced or delayed combustion, or compression losses occur, but also when some subsystems experience indirect faults such as injector cloggings, valve leakages, etc. The knowledge of the pressure traces can then be used to derive advanced indicators dedicated to the surveillance and the diagnosis of the engine. For instance, by solving the equation of energy conservation, the heat release rate may be deduced to provide more relevant information on the combustion process in each of the cylinders. Also, the well known P–V or Clapeyron diagram may be readily set up from the pressure trace to give valuable information on the indicated power furnished by the engine over the thermodynamic cycle.

Although attractive, the direct measurement of the pressure trace is hardly tract-able in practice as it usually requires dismantling or drilling some parts of the engine, especially in the case of diesel engines. A number of alternative strategies have been sought in the last decades for measuring the cylinder pressure by means of non-intrusive devices.

2.1.1. *Measuring the angular speed*

One option is based on the measurement of the instantaneous angular speed of the crankshaft and then converting it into torques and forces acting on the pistons. The effectiveness of this procedure has been demonstrated since the mid-1980s [5], yet it should be pointed out that it is only able to return an image of the pressure traces in a narrow frequency band, typically below one-fourth of the first natural frequency of the crankshaft [6]. Even if this frequency is sufficient in practice for tracking rough malfunctions such as misfires and knocks [7], it is too poor for detecting finer faults that are likely to affect the combustion process as well.

2.1.2. *Measuring the structural vibration of the engine block*

Another option is based on the measurement of structural vibrations on the engine block, which are supposed to result from the suddenly applied pressure forces inside the cylinders when combustion takes place. It has been shown in the literature that appropriate post-processing of the vibration signals can then help to recover the pressure traces partially or fully.

A pioneer approach based on cepstral deconvolution was proposed by Lyon in the early 1980s and seems to have prevailed over classical Wiener filtering for a time [8–10]. Reasons of this supremacy are due to the inherent robustness of cepstral deconvolution with respect to variations and errors in the transfer function estimate. Other approaches have been introduced more recently that achieve robust smoothing in the time domain [11] or concentrate on a better modelling of the input–output relationship by accounting either for non-linearities [12] or for time-variations of structural parameters [13].

The option of indirectly reconstructing the pressure traces from structural vibrations is far less advanced than that of using angular speed measurements. As a matter of fact, it is still an open field of research that deserves much attention before a versatile solution is found to be easily applicable to most industrial cases. This paper aims at introducing some results that may help toward this prospect.

2.2. PRESSURE RECONSTRUCTION THROUGH VIBRATION DECONVOLUTION: PROSPECTS AND PITFALLS

Part I of this paper evidenced the importance of associating the vibration signatures with their generating sources in the engine cycle. Here the vibration signatures of interest result from the rapid pressure rise due to combustion in each of the cylinders. Figure 1 displays an example of a pressure trace with its resulting vibration signature on the engine block. The challenge is then to transform this vibration signature back to its excitation force, i.e. to infer the upper curve from the lower one in Figure 1. Feasibility of this procedure naturally leads to a number of questions:

(1) Does the vibration signature contain enough information so that the full pressure trace that generates it can be fully recovered from its measurement? In other words, is the pressure trace observable from the vibration signature?
(2) Is the vibration signature well isolated in the vibration signal so that its contribution can be separated out from other vibration signatures? In other words, is the vibration signature corrupted by some additive noise and if so, is there any way to cope with that?

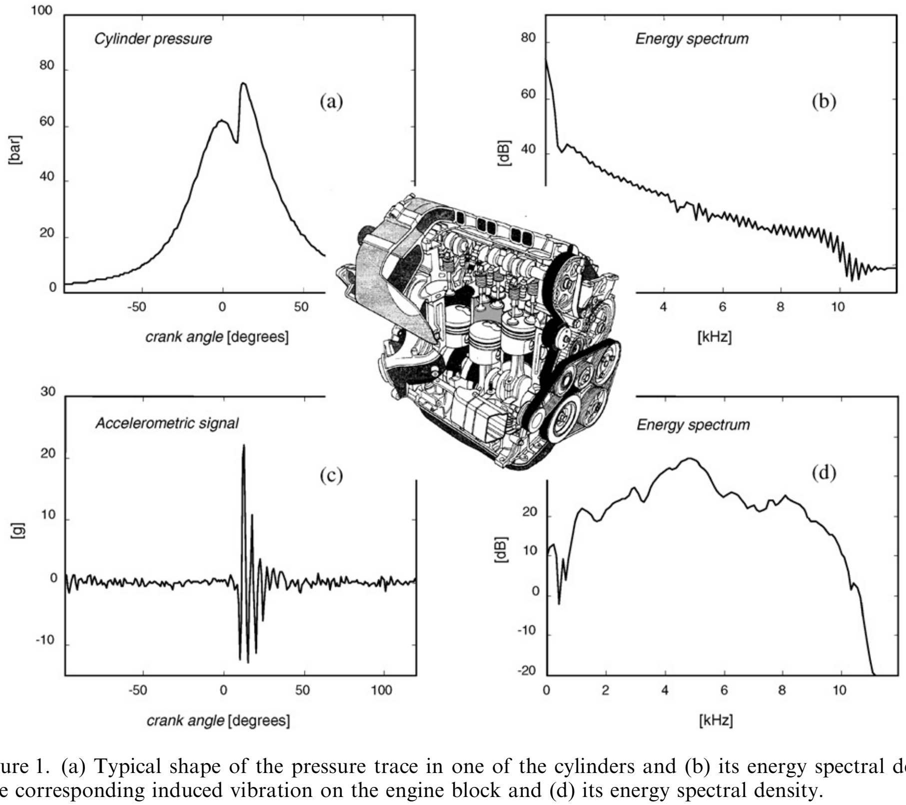

Figure 1. (a) Typical shape of the pressure trace in one of the cylinders and (b) its energy spectral density. (c) The corresponding induced vibration on the engine block and (d) its energy spectral density.

(3) Does one have any information on the direct input–output relationship between the pressure trace and its resulting vibration signature that may help to design the inverse relationship?
(4) Having devised an inverse input–output relationship, how to make it robust enough with respect to variability among engines (of the same design) so that it can be used on a large number of them?

2.2.1. *Observability of the pressure trace*

The issue of observability is first conditioned by the type of vibration measurements. Actually, most measurements are of the velocity or acceleration type, which means that the transduced information is that of an oscillation around an equilibrium point. Thus, in no case can the equilibrium point itself be recovered. As a matter of fact, when only velocity or acceleration are concerned, it is theoretically not possible to recover the actual magnitude of the pressure trace. For atmospheric engines, this problem is easily corrected because the minimum value of the pressure trace is known to equal the atmospheric pressure (except during the intake stroke where some depression is possible). Unfortunately, no such reference is available for turbocharged engines so that large uncertainties should be accounted for with the recovery of the absolute pressure magnitude.

A second problem in the issue of observability is that the pressure trace has most of its energy concentrated at low frequencies, say below a few hundred hertz, whereas its vibration signature is usually found to have little energy there due to the high rigidity of the engine block. This is well illustrated in Figure 1 where the spectrum of a pressure trace and that of its corresponding vibration signature are compared. It follows therefore that real difficulties are expected when trying to reconstruct the low-frequency pressure trace from the higher-frequency vibration measurement. From a mathematical viewpoint, reconstruction of the pressure may be viewed as an *ill-posed problem*, i.e., with no guarantee of *uniqueness*, *stability* or even *existence of a solution*. Another way to be aware of this difficulty is to note that the vibration signature shown in Figure 1 is actually shorter than its excitation, a fact that clearly evidences some loss of information.

2.2.2. *Additive noise*

It was argued in Part I of this paper that several vibration signatures stemming from different excitation sources are very likely to overlap in the vicinity of the top-dead-centres where combustion arise.

In particular, piston slap is recognized to be a major source of disturbance which covers the vibration signature of combustion both in the time and frequency domain. Furthermore, at high speed, piston slap can display more vibration energy than combustion does [14]. Most probably the best way to minimize the effect of piston slap is to carefully choose the location of the vibration transducer. It was found that the cylinder head bolts give satisfactory signal-to-noise ratio [10], but other locations underneath the crankshaft bearings may also prove successful.

Another important corrupting noise to deal with is that coming from inertial forces that act on the engine block. These forces are located at low frequencies—a few harmonics of the engine speed—so that they are very likely to interact with the reconstruction of the pressure trace. Here, the only solution to diminish their effects is to set the experiments at a moderate engine speed. Indeed, this will lower the piston slap effects at the same time. In brief, the slower the engine speed, the lesser the additive noise.

Whatever the strategy, the presence of additive noise will never be cancelled out totally, yet it will be shown in the following how to counteract its effect by making use of the cyclostationary property of the vibration signals to process.

2.2.3. *Input–output relationship and structural variability*

The method for reconstructing the pressure trace will tightly depend on the nature of the input–output relationship between the pressure trace and its resulting vibration signature on the engine block. For simplicity, a linear relationship will be assumed, even though this is nothing other than an approximation. Indeed, non-linearities are clearly evidenced by such phenomena as the non-linear stiffness of the oil film between the piston skirt and the cylinder liner or coupling between axial and transverse vibrations. For instance, readers are referred to reference [12] for an implicit non-linear deconvolution scheme.

Apart from linearity, the classical assumption of time invariance of the input–output relationship should be questioned as well. Reciprocating engines evidence modifications of their geometrical configuration within the cycle, so that time invariance is hardly acceptable in an interval larger than a few degrees around the top-dead-centre. Strictly speaking, due to the rotation of the crankshaft, one should refer to a *periodically angle varying* input–output relationship. In most of the previous studies, angle invariant relationships have been assumed as a first approximation. It will be shown in the following that there is no necessity to rely on this simplification as soon as cyclostationarity is explicitly taken into account.

Having assumed a linear periodically varying relationship, one should finally address the issue of statistical variability. Indeed, although most deconvolution schemes presume the input–output relationship to be non-random, it was verified that this ascertainment hardly holds true for IC engines. Actually, two kinds of variability should be recognized:

(1) *Inter-sample structural variability*: from one engine to another—pertaining to a given population with the same design—the input–output relationship will never be exactly the same [8],
(2) *Inter-cycle structural variability*: from one cycle to another the input–output relationship can vary slightly due to some non-predictable effects (changes in the temperature field, changes in the homogeneity of the oil film, etc.).

Structural variability should not be ignored when designing an inverse input–output relationship for recovering the pressure trace. In fact, an effective way to integrate it is to reduce the number of degrees of freedom of the inverse relationship by some kind of smoothing [8, 11].

Having recognized the actual difficulties associated with the reconstruction of the pressure trace, the problem will now be stated from a more mathematical standpoint.

3. A UNIFIED FRAMEWORK FOR THE DECONVOLUTION OF CYCLOSTATIONARY SIGNATURES

This section aims at fitting the issue of deconvolving the vibration signatures measured on IC engines into a unified framework. The framework is large enough so that it actually includes a formalization of former studies and enables new breakthroughs to be deduced. The material used in this section essentially relies on the definition and properties of cyclostationary processes that were introduced in Part I of this paper [1].

In the aforementioned reference, the basic idea was to recognize that the vibration signals have periodic statistical patterns, i.e., are *cyclostationary* in the engine cycle, and then to make use of this property for designing specific signal processing tools. Indeed, the paradigm of cyclostationarity was shown to be perfectly suited for IC engines since it enables the processing of the vibration signals by means of *cyclic statistics* that are locked onto the engine kinematics. This was conveniently achieved by use of constant *angular sampling* instead of constant *time sampling*. However, when dealing with deconvolution problems, the resort to angular sampling is to be investigated again.

3.1. TEMPORAL VERSUS ANGULAR SAMPLING

Angular sampling is a very convenient solution for obtaining a discretized signal with a constant number of points per cycles. This is true whatever the fluctuations of the engine speed. Thus, its use is recommended when one wants to track vibration signatures that are expected to occur at constant angular positions in the engine cycle. From this viewpoint, angular sampling can conveniently simplify the deconvolution process as well. However, angular sampling sustains some counter-effects which can mitigate against its use in deconvolution applications.

Precisely, if the input–output relationship between the pressure trace and its vibration signature has invariant characteristics (e.g., modal parameters), then angular sampling will make them vary with the engine speed fluctuations. That is, a time invariant input–output relationship will be turned into an angle varying input–output relationship. Therefore, if on the one hand one wants to design an invariant inverse relationship, then time sampling is better recommended than angular sampling. On the other hand, if one wants a periodically varying inverse relationship, then time or angular sampling may just do it equally well. This latter case can be shown to be theoretically true as long as the engine speed is itself a cyclostationary process. This includes constant, periodic and stationary speeds as particular cases.

Besides, if the identified inverse relationship acts in the angle domain, it will be valid only for some given nominal speed, so that extreme care must be taken to correctly tune the engine speed before applying it. The identified inverse relationship in the time domain may not be so stringent with respect to this requirement.

3.2. THEORETICAL FORMALIZATION

The theoretical formalization of the deconvolution issue is illustrated in Figure 2. For a given direct input–output relationship, one wants to design an *inverse filter* that enables recovery of an estimate of the pressure trace as close as possible to the true one, and robust enough with respect to additive perturbations on the measured vibration signatures. Once identified, this inverse filter could then be used on other engines. Note that the solution to this problem essentially relies on the correct definition of some inverse filter. Indeed, it is advantageous to directly identify this inverse filter instead of first identifying the direct input–output relationship and then to invert it, because this would lead to a different and weaker solution. Let us look at these matters in detail.

3.2.1. *Direct input–output relationship*

Assume the discretized measured vibration signal is generated by an underlying stochastic process $\{Y[m]\}$, $m \in \mathbb{Z}$. Here, the sample number m refers either to time sampling, i.e., $t = mT_e$ or to angular sampling, i.e., $\theta = m\Theta_e$. As depicted in Figure 2, it is believed that process $Y[m]$ is the filtered response to the pressure excitation process $P[m]$

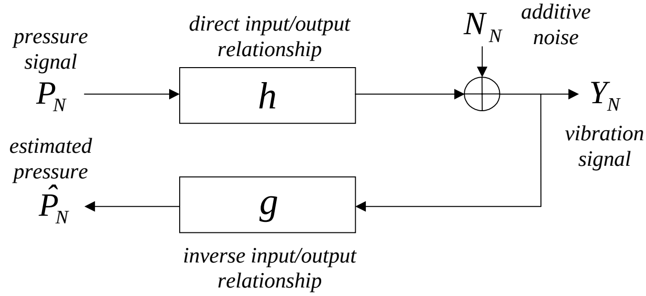

Figure 2. Theoretical formalization of the deconvolution scheme.

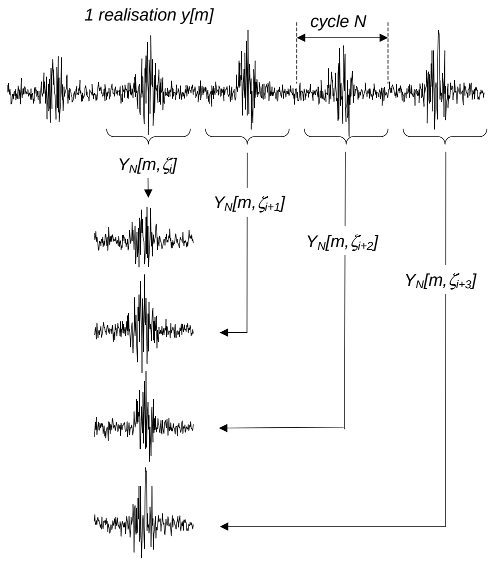

Figure 3. Schematic principle of Wold's isomorphism.

and is corrupted by some additive noise $N[m]$. Under the assumption of cyclostationarity with a cycle period of N samples, the statistics of the triplet $\{Y[m], P[m], N[m]\}$, $m \in \mathbb{Z}$ are identical to those of the triplet $\{Y_N[m], P_N[m], N_N[m]\}$, $m = 0, \ldots, N-1$ made up of new stochastic processes whose supports are restricted to the cycle period N of the engine. This is well formalized by Wold's isomorphism as discussed in Part I of the paper and summarized in Figure 3.

If the effective durations of processes $Y_N[m]$ and $P_N[m]$ are shorter than the cycle length N, then the direct input–output relationship may be written as

$$Y_N[m] = \sum_{n=0}^{m} h[m,n]P_N[n] + N_N[m], \quad m = 0, \ldots, N-1, \tag{1}$$

where the causal Green's function $h[m,n], m \leqslant n$, is the impulse response of the engine block to an impulse at sample n. From the cyclostationarity of the underlying process, $h[m,n]$ is a periodic function such that

$$h[m,n] = h[m+N, n+N]. \tag{2}$$

It embodies the more classical case of an invariant input–output relationship for which

$$h[m,n] = h[m-n]. \tag{3}$$

In order to take account of small structural variability, $h[m,n]$ is further refined to stem from a stochastic function $H[m,n]$, such that

$$\mathrm{E}\{H[m,n]\} = h[m,n]. \tag{4}$$

3.2.2. *Inverse input–output relationship*

The objective is to recover the best estimator $\hat{P}_N$ of P_N from the noisy measurement Y_N. It can be shown that the general form of the inverse input–output relationship is [6]:

$$\hat{P}_N[m] = \sum_{n=0}^{N-1} g[m,n]\, Y_N[n], \quad m = 0, \ldots, N-1, \tag{5}$$

where $g[m,n]$ is a non-causal Green's function with period N. The identification of $g[m,n]$ is typically achieved by minimizing the mean square error between $\hat{P}_N[m]$ and $P_N[m]$:

$$g_{opt}[m,n] = \arg\min_{g} \sum_{m=0}^{N-1} \mathrm{E}\left\{ \left| \hat{P}_N[m] - P_N[m] \right|^2 \right\}. \tag{6}$$

Although the minimization can be done either in the time(angle) or in the frequency domain, it is preferred herein to carry it out in the frequency domain as the closed form solution will then give more theoretical insight.

Let $\tilde{\mathbf{U}}_N$ and $\tilde{\mathbf{V}}_N$ be the discrete Fourier Transform vectors computed on two arbitrary random sequences $U_N[m]$ and $V_N[m]$, $m = 0, \ldots, N-1$. Then define the cross-correlation spectral matrix $\mathbf{S}_{U_N V_N}$ between spectral measures $\tilde{\mathbf{U}}_N$ and $\tilde{\mathbf{V}}_N$ as the ensemble average of the outer product

$$\mathbf{S}_{U_N V_N} = \mathrm{E}\{\tilde{\mathbf{V}}_N \tilde{\mathbf{U}}_N^{\dagger}\}, \tag{7}$$

where † stands for the transposition and conjugation operator. Note that definition (7) is theoretically valid since the time(angle) sequences $U_N[m]$ and $V_N[m]$ are of finite duration on the cycle. Let also $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{G}}$ be the matrices associated with the double discrete Fourier transforms of $h[m,n]$ and $g[m,n]$. Then, with these notations, equations (1) and (5) become:

$$\tilde{\mathbf{Y}}_N = \tilde{\mathbf{H}}\tilde{\mathbf{P}}_N + \tilde{\mathbf{N}}_N, \qquad \hat{\tilde{\mathbf{P}}}_N = \tilde{\mathbf{G}}\tilde{\mathbf{Y}}_N. \tag{8}$$

The solution to this set of equations can be shown to be

$$\tilde{\mathbf{G}}_{opt} = \mathbf{S}_{Y_N P_N} \mathbf{S}_{Y_N Y_N}^{-1} \tag{9}$$

$$= \mathbf{S}_{P_N P_N} E\{\tilde{\mathbf{H}}\} \left(E\{\tilde{\mathbf{H}} \mathbf{S}_{P_N P_N} \tilde{\mathbf{H}}\} + \mathbf{S}_{N_N N_N} \right)^{-1}. \tag{10}$$

The closed-form solution (9) highlights the following two important points.

(1) *The inverse input/output relationship described by the filter $\tilde{\mathbf{G}}_{opt}$ is invariant if and only if $\tilde{\mathbf{G}}_{opt}$ is a diagonal matrix*. This requires three combined conditions:
   (a) $\tilde{\mathbf{H}}$ is a diagonal matrix, i.e., it relates to an invariant filter,
   (b) $\tilde{\mathbf{H}}$ is purely deterministic, i.e., $H[m,n] \equiv h[m,n]$,
   (c) $\mathbf{S}_{B_N B_N}$ is a null matrix ($\mathbf{S}_{B_N B_N}$ cannot be diagonal since it is taken for a finite length random process), i.e., there is no additive noise on the measurement.

(2) *The inverse input/output relationship described by the filter $\tilde{\mathbf{G}}_{opt}$ is periodically varying under all other cases.*

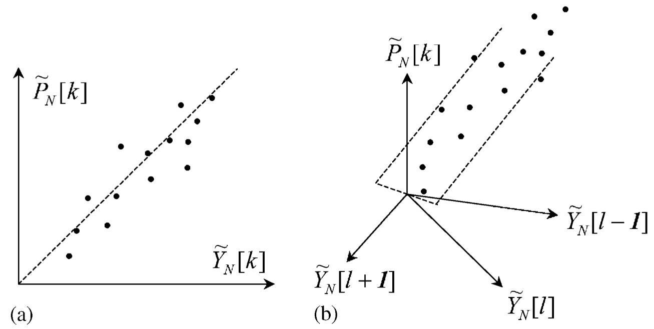

Figure 4. Comparison between the (a) invariant and (b) time-varying regression. Whilst the first one assumes the data to cluster around a line, the second assumes them to lie in a hyperplane.

The second statement is a non-trivial result demonstrating that in most cases the input–output relationship will be *periodically varying*. Although this is obviously true if the direct input–output relationship is itself periodically varying, equation (9) states it is not a necessary condition: it suffices that there exists some cyclostationary additive noise or some structural variability of the input–output relationship for the inverse filter to become periodic. Moreover, the periodicity of $g_{opt}[m, n]$ means that a spectral component $\tilde{P}_N[k]$ of the pressure trace at a given frequency line k will be reconstructed from a weighted average of the frequency components $\tilde{Y}_N[l]$ of the measured vibration signature taken at frequency lines $l = 0, \ldots, N-1$. Obviously such a regression gives an advantage over the classical invariant filter where only $\tilde{Y}_N[l]$, $l = k$ is used to regress on $\tilde{P}_N[k]$. This is well illustrated in Figure 4 which compares the time-varying regression with the invariant one.

Accordingly, it is hoped that the low-frequency components of the pressure signal could be reconstructed from the higher-frequency components of the vibration signal, that is $\tilde{P}_N[k]$ from some $\tilde{Y}_N[l]$ with $l > k$. This effect should circumvent one of the deconvolution problem mentioned in section 2.2.1.

Finally, it is worth noting that the optimal inverse filter is not the same as the inverse of the direct filter $\tilde{\mathbf{H}}$. Indeed, from equation (9), one only has $\tilde{\mathbf{G}}_{opt} = \tilde{\mathbf{H}}^{-1}$ if there is neither additive noise nor structural variability. Hence, the optimal inverse filter $[\tilde{\mathbf{G}}_{opt}]_{kl}$ will tend to be equal to the ideal *equalizer* $[\tilde{\mathbf{H}}^{-1}]_{kl}$ at those pairs of frequency lines (k, l) where the corrupting noise is insignificant.† On the other hand, $[\tilde{\mathbf{G}}_{opt}]_{kl}$ will tend to be an ideal *rejector*, i.e., a filter with zero gain at those pairs of frequency lines where the noise is predominant. Because of the bi-dimensional nature of the filter, higher rejection is ultimately achieved compared to the invariant case. Actually, noise can be theoretically attenuated even if its time(angle) and frequency supports overlap with those of the pressure forces.

3.3. DEALING WITH DETERMINISTIC ADDITIVE NOISE

Derivation of equation (9) holds true only if the additive noise N_N is supposed to be uncorrelated with the pressure trace P_N, i.e., if the cross-correlation spectral matrix $\mathbf{S}_{P_N N_N} = 0$. This is a fair assumption provided N_N is a non-deterministic process. However, when N_N contains some deterministic contributions—e.g. from periodic phenomena such

† $[M]_{kl}$ denotes the element at the kth row and lth column in matrix M.

as piston slap and inertia forces—then in general $\mathbf{S}_{P_N N_N} \neq 0$. Strictly speaking, to measure the actual *statistical* correlation between processes Y_N and P_N, one must remove their deterministic parts before starting the identification process. As explained in Part I of this paper, this means working on the new centred process defined as

$$Y_N[m] - \mathrm{E}\{Y_N[m]\} \tag{11}$$

and

$$P_N[m] - \mathrm{E}\{P_N[m]\}, \tag{12}$$

where $\mathrm{E}\{Y_N[m]\}$ and $\mathrm{E}\{N_N[m]\}$ are estimated by synchronous averaging [1].

Another way which may help to understand why this should be done is to realize that the regression hyperplane associated with filter $g_{opt}[m,n]$ must be corrected from its offset value so that it is truly affine as shown in Figure 5.

The point discussed in this section is in accordance with the fact that periodic and second order cyclostationary components should be processed independently as argued in the first part of this paper. Alternatively, this means working with *covariance* functions or matrices where *correlation* functions or matrices are usually used. In the authors' opinion, this point has not been sufficiently stressed in previous studies.

3.4. PRACTICAL IMPLEMENTATION

Equation (9) gave some theoretical insight into the structure of the inverse filter to be applied to the vibration signal in order to estimate the pressure trace. From an identification viewpoint, there are more convenient forms to be used. One common feature of all periodically varying systems is that they require identifying an increased number of degrees of freedom compared to their invariant counterparts. Therefore direct form realizations that enable the choice of a reduced number of degrees of freedom should be preferred.

We shall briefly discuss two candidates. Readers are referred for instance to reference [15] for more detail on the design of periodically varying filters.

3.4.1. *Block structure*

This form is defined by the Fourier dual of equation (9). Then, in the time(angle) domain,

$$\mathbf{G}_{opt} = \mathbf{K}_{Y_N P_N} \mathbf{K}^{-1}_{Y_N Y_N}, \tag{13}$$

where (following the discussion of section 3.3) $\mathbf{K}_{U_N V_N}$ stands for the *cross-covariance* matrix of processes $U_N[n]$ and $V_N[n]$, $n = 0, \ldots, N-1$ and $[\mathbf{G}_{opt}]_{mn} = g_{opt}[m,n]$.

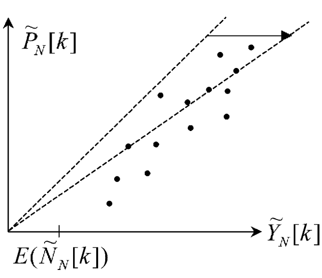

Figure 5. Illustration of the effect of deterministic additive noise which biases the regression if not taken into account.

Compared to equation (9), equation (13) directly identifies $g_{opt}[m,n]$ without the need of first transforming into the Fourier domain. The reduction of the number of degrees of freedom is conveniently achieved by replacing $\mathbf{K}^{-1}_{U_N V_N}$ by its pseudo-inverse of reduced rank [6].

3.4.2. *Filter-bank structure*

The idea is to replace the periodically varying single-input/single-output problem by an invariant multiple-input/single-output problem. This is achieved by expanding the periodic filter $g_{opt}[m,n]$ into a Fourier series:

$$g_{opt}[m+n,m] = \sum_{k=-K}^{K} g^{k}_{opt}[n]\mathrm{e}^{-\mathrm{j}2\pi km/N}, \tag{14}$$

where $g^{k}_{opt}[m]$ now acts as an invariant filter. Equation (5) hence becomes:

$$\hat{P}_N[m] = \sum_{k=-K}^{K}\sum_{n=0}^{N-1} g^{k}_{opt}[m-n]\Big(Y_N[n]\mathrm{e}^{-\mathrm{j}2\pi kn/N}\Big). \tag{15}$$

The filter-bank structure is illustrated in Figure 6. Here, the number of degrees of freedom is explicitly set by the number K of coefficients in the Fourier series. Identification of the set of $\{g^{k}_{opt}[m]\}$, $k=-K,\ldots,K$ follows from classical multi-channel procedures.

In practice, it was found that the choice of one or another implementation forms depends on the tested engine. In all cases, achieving the smallest number of degrees of freedom should be preferred.

3.4.3. *Estimation issue*

Up to here, the discussion was concerned with the implementation and identification of the periodically varying inverse filter, but no indication was given on how to estimate it. For instance, what kind of estimate should be used for the covariance matrices $\mathbf{K}_{Y_N P_N}$ and $\mathbf{K}_{Y_N Y_N}$ in equation (13) or equivalent statistics involved in the filter-bank structure? Indeed, the estimation issue is easily amenable under the assumption of cyclostationarity.

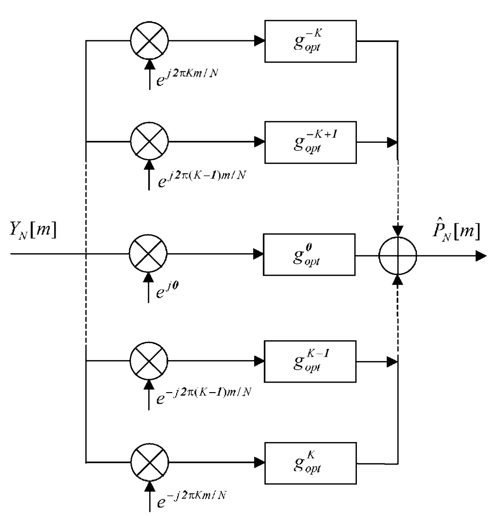

Figure 6. Filter-bank structure for the periodically varying inverse filter.

Basically, the idea is to replace ensemble averaging by *cycle averaging* so that estimation of the expectation operator is possible. The formal definition of this estimator was introduced in Part I of this paper. For instance, a consistent estimate of $\mathbf{K}_{Y_N P_N}$ is given by

$$\left[\hat{\mathbf{K}}_{Y_N P_N}\right]_{mn} = \langle p[n] y^*[m] \rangle_N^I - \langle p[n] \rangle_N^I \langle y^*[m] \rangle_N^I, \tag{16}$$

where $\langle \cdot \rangle_N^I$ means averaging over I cycles of length N on any measurement $y[n]$ and $p[n]$ of processes $Y[n]$ and $P[n]$ [1].

Application of the material introduced up to now tightly relies on this mathematical trick.

4. EXPERIMENTS

This last section aims at demonstrating the feasibility of the proposed deconvolution scheme and assessing its performance. Not only are actual results displayed, but practical issues that condition their validity are discussed as well.

4.1. METHODOLOGY

From the above discussion, it is clear that the reconstruction of the pressure trace involves two independent phases: the *identification phase* which comes up with a model of the inverse input–output relationship, and a *deconvolution phase* which uses this identified inverse model to give an estimate of the pressure traces.

Whilst the identification phase is typically performed in the laboratory, on a test engine, the deconvolution phase is performed in industry, on different engines. Therefore it is important to make the inverse filter *robust* enough so that it can apply to a large number of engines and not only to that used in the laboratory. This is a problem of statistical regularization which places a trade-off between learning an exact inverse filter on a given engine and keeping it general enough. One usual way for dealing with regularization is to split the recorded data into three independent sets.

The first set—the *learning set*—is dedicated to identifying the inverse filter whereas the second one—the *validation set*—is for tuning the number of degrees of freedom (the smaller the number of degrees of freedom, the more robust the model). In general, the smallest prediction error will then occur for an optimal finite number of degrees of freedom on the validation set much smaller than what one would obtain on the testing set. Finally, the third set—the *test set*—is dedicated to assessing the performance of the identified model.

Ideally, data in each set should be gathered from different engines to make them as independent as possible. If this cannot be done for practical reasons, it is still important to gather them from different experiments, for instance after changing the transducers or their locations.

4.2. FREQUENCY BAND OF INTEREST

The pressure trace has to be recovered in a frequency band large enough for malfunctions to be detected. A lower bound value may for example be given by the ability to detect knocks which are known to entail a pressure gradient $(\Delta P)_M$ higher than 0.5 MPa/deg. By applying Bernstein's inequality which relates the frequency bandwidth B of a signal to its maximum derivative value and by denoting Ω the engine speed and P_M the maximum expected pressure value, one finds:

$$B \leqslant \frac{\Omega (\Delta P)_M}{2\pi P_M}. \tag{17}$$

For example, by putting $P_M = 5$ Mpa, one finds a bandwidth $B \simeq 6\Omega$ which is rather low (e.g., $B \simeq 100$ Hz with $\Omega = 1000$ r.p.m.). In practice, we found it very conservative to work with B within $1 \sim 3$ kHz, i.e., a frequency band that should allow detection of most of the possible malfunctions.

4.3. ANALYSIS OF RESULTS

This section presents some results of experiments run on a test rig at the University of New South Wales (Australia). The engine under test was a 4 cylinder 4-stroke Volvo diesel, on which the first cylinder was equipped with a pressure transducer and two accelerometers on its head bolts. The sampling frequency was set to 23·8 kHz and the engine speed to 900 r.p.m. A number of data recordings were performed for different load torques, along with the recording of a one pulse per cycle signal. This enabled post-synchronization of the data with respect to the top dead centre, so that after decimation by a factor 4 the pressure and the accelerometer signals were coded with a mean value of 512 time samples on each revolution. This is illustrated in Figure 7. It was mentioned in section 3·1 that a simpler procedure would be to directly angle sample the vibration signal as explained in Part I of this paper.

Different experiments allowed the data to be divided into a *learning set*, a *validation set* and a *test set*. The data from the first accelerometer were used to set up the learning set and, after repeating the experiment on another day, to set up the validation set. Finally, the data for the test set were gathered from the second accelerometer mounted 5 cm away from the first one, in order to simulate measurements made on a different engine.

4.3.1. *Coherence analysis*

Before starting the identification scheme, the learning set was used to verify some of the assumptions made on the data. Figure 8 displays the *ordinary* coherence function γ^2_{PY} along with the *multiple* coherence function M^2_{PY} between the pressure trace and its resulting vibration signature. These functions are defined in reference [6] for cyclostationary processes and should be viewed as correlation coefficients that indicate at which frequency lines an invariant or a periodically varying relationship should perform well. Basically, an ordinary coherence function with values close to 1 will indicate a strong *linear invariant* relationship, whereas a multiple coherence function with values close to 1 will rather indicate a strong *linear periodically varying* relationship. Here again, for the coherence functions to be meaningful, it is customary to apply them to the centred processes, i.e., after extraction of the synchronous averages as explained in section 3.3 and in reference [1].

From Figure 8 it is therefore already predictable that the invariant linear filter will yield poor reconstruction of the pressure trace in the low frequencies (<2500 Hz), where unfortunately it has most of its energy. Alternatively, the linear periodically varying filter fills in the gap there so that increased performance can already be expected.

Thus, the coherence analysis supports the prospect of using a periodically varying inverse filter with evident demonstration that the difficult issue of reconstructing low-frequency components from a higher-frequency signal is alleviated.

4.3.2. *Invariant inverse filter approach*

For the sake of comparison, reconstruction of the pressure trace was first performed using the optimal invariant filter estimated on $I = 192$ cycles of the learning set. It is worth saying that in this case the accelerometer signals had to be pre-processed by applying a

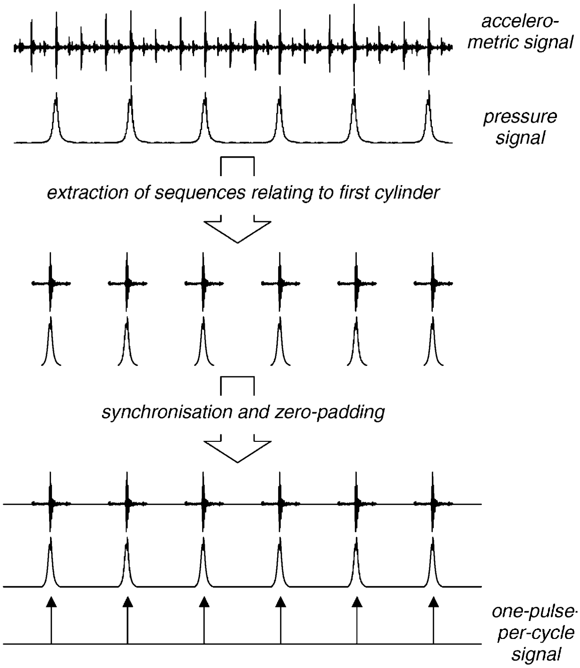

Figure 7. Conditioning of the recorded signals before processing. Both acceleration and pressure signals from the learning set are needed for the identification phase, while only the acceleration signal from the test set intervenes in the decovolution phase.

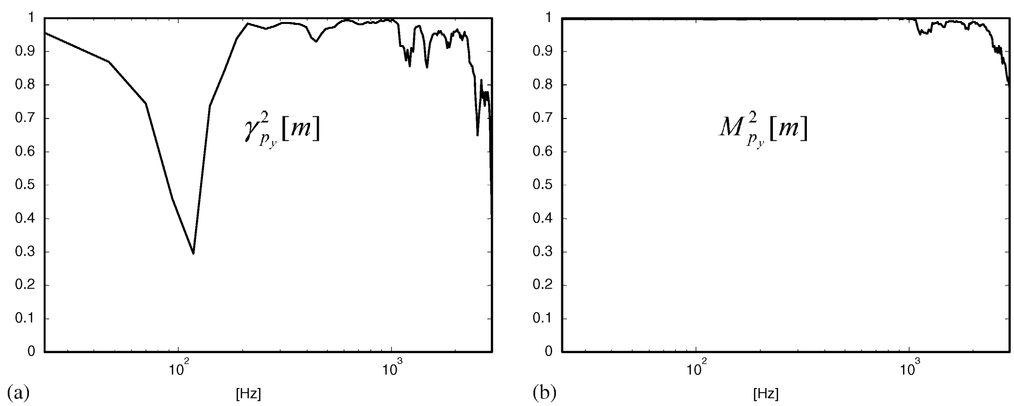

Figure 8. (a) Ordinary and (b) multiple coherence functions estimated on pressure $I = 192$ cycles. The sequences were centred before estimation by extracting their synchronous averages.

weighting function in order to isolate the vibration signatures resulting from the different combustion events and also to make the inverse filter more robust by decreasing its number of degree of freedom. Here a Laplace–Gauss weighting function was used and showed similar efficiency as the suggested one-sided exponential function of reference [11]. The results obtained on the test set are displayed in Figure 9 for two different operating points (5 and 35 daN indicated torque). Estimates of the pressure curves from accelerometers 1 and 2 are compared with the actual pressure measurement in the dotted

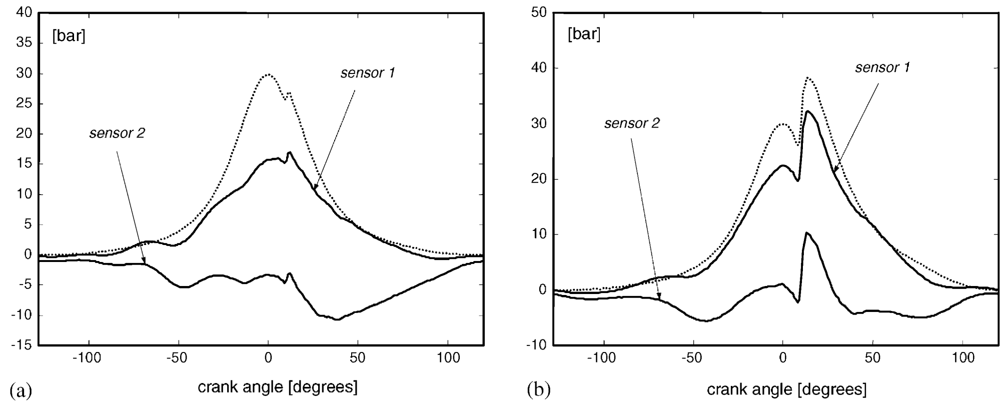

Figure 9. Invariant inverse filter: comparison of the estimated (full lines) and measured (dotted line) pressure traces on the test set. (a) Indicated torque = 5 daN. (b) Indicated torque = 35 daN.

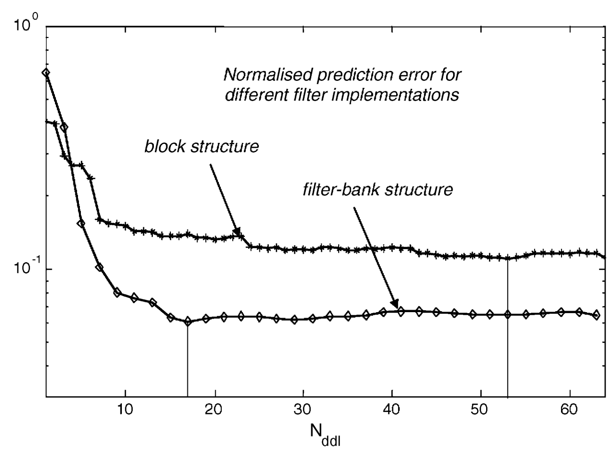

Figure 10. Evolution of the normalized prediction error (*r.m.s.* value) on the validation set w.r.t. to the number of degrees of freedom. The minimum is attained for $N_{ddl} = 53$ for the block structure and $N_{ddl} = 17$, i.e, $K = 8$ for the filter-bank structure.

line. Remember that sensor 1 was used for identifying the inverse filter and therefore is expected to yield better performances than sensor 2 which was located 5 cm away.

Both estimates demonstrate the poor ability of the linear invariant inverse filter to restitute low-frequency components. This expected effect is obviously more striking for sensor 2.

4.3.3. *Periodically varying inverse filter approach*

The optimal inverse periodically varying filter was estimated on $I = 192$ cycles of the learning set. In this case, the filter-bank structure was found to yield the lowest prediction error on the validation set with $2K + 1 = 17$ Fourier coefficients. Therefore the periodically varying filter could be compared with 17 invariant filters proceeding in parallel. Figure 10 shows the evolution of the normalized prediction error in r.m.s. value with respect to the number of degrees of freedom on the validation set for the two tested structures.

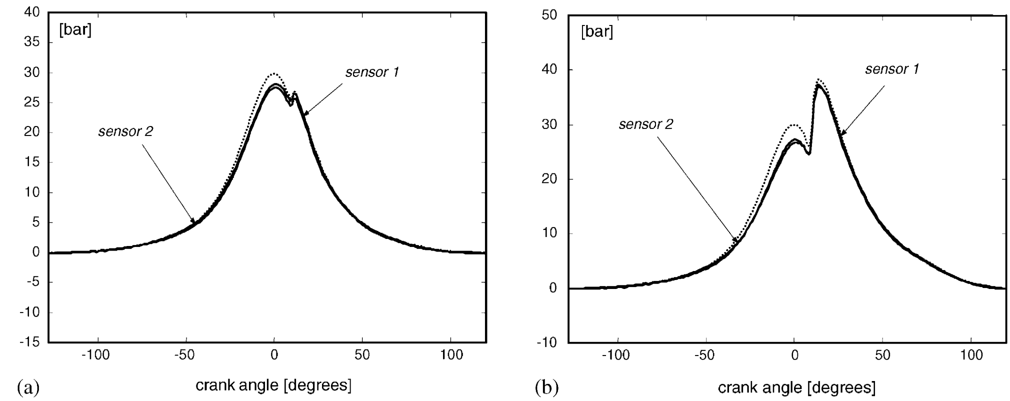

Figure 11. Periodically varying inverse filter: comparison of the estimated (full lines) and measured (dotted line) pressure traces on the test set. (a) Indicated torque = 5 daN. (b) Indicated torque = 35 daN.

Incidentally, it was checked that virtually no weighting function was necessary to isolate the vibration signatures resulting from the combustions, because the periodically varying filter had a good ability to reject unwanted additive noise as explained in section 3·2·2.

Results for the reconstruction of the pressure traces in the test set are displayed in Figure 11. As expected, the reconstruction is better achieved than before, especially when considering low-frequency components. The periodically varying approach seems to solve one of the ill-posed issues associated with the deconvolution of the pressure trace thanks to a more complex design of the optimal inverse filter. This point was further confirmed by verifying that the pressure traces could be reasonably reconstructed even after setting to zero the frequency components of the vibration signatures in the band [0;600 Hz], thus demonstrating the robustness of the deconvolution scheme with respect to this issue.

Finally, note that both estimates from sensor 1 and 2 are very similar. Here again, this demonstrates the superior robustness of the periodically varying filter with respect to structural variability as discussed in section 3.2.2.

5. CONCLUSION

The cylinder pressure traces in internal combustion engines is recognized to be an indicator of value, most often to be used for controlling a set of operating parameters, but for diagnostic purposes as well. Despite its relevance, its actual use in condition monitoring has been inhibited by the difficulty of measuring it.

This paper has addressed the feasibility of reconstructing the pressure trace from non-intrusive vibration measurements by means of inverse filtering. This is a difficult problem that is recognized to be ill-posed, at least for two reasons:

(1) the pressure trace has most of its energy in low frequencies (<500 Hz) where it is poorly conveyed as vibration energy to the engine block rigidity,
(2) the measured vibration signatures on the engine block are corrupted by non-negligible additive noise such as piston slap and inertial forces.

The contribution of the paper was to recast the deconvolution problem into the theoretical framework of cyclostationary processes which actually encompasses previous studies and leads to new breakthroughs. As a matter of fact, a major result was to show that the optimal inverse filter is to be sought in a periodically varying form instead of an invariant form as usually done. This result holds true under reasonable conditions: in fact, as long as there is evidence of either (1) some additive (cyclostationary) noise, (2) some random structural variability or (3) periodic variations of the input–output relationship between the pressure trace and the vibration signature.

Basically, accepting a periodically varying inverse filter means that a component of the pressure trace at a given frequency can be reconstructed from a linear combination of the components of the vibration signature taken over a range of different frequencies. It was shown that exploitation of this spectral redundancy helps to counteract the effect of corrupting noise and enables reconstruction of low-frequency components from higher-frequency ones, thus alleviating the ill-posed nature of the deconvolution problem.

Finally guidelines for the implementation and identification of the inverse periodically varying filter were given. They largely rely on the concept of cyclic statistics introduced in the first part of this paper.

Parts I and II of this paper have both addressed the prospects of condition monitoring IC engines from structural vibration measurements. The authors hope they have succeeded in demonstrating that a number of recognized difficulties associated with this issue can be efficiently tackled by taking advantage of the cyclostationarity of the involved processes.

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