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ELECTRIC CIRCUIT DESIGN AND TESTING OF INTEGRATED DISTRIBUTED STRUCTRONIC SYSTEMS

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Modelling distributed parameter systems (DPS) by electric circuits and fabricating the complicated equivalent circuits to evaluate system responses poses many challenging research issues for many years. Electrical modelling and analysis of distributed sensing/control of smart structures and distributed structronic systems are even scarce. This paper is to present a technique to model distributed structronic control systems with electric circuits and to evaluate control behaviors with the fabricated equivalent circuits. Electrical analogies and analysis of distributed structronic systems is proposed and dynamics and control of beam/sensor/actuator systems are investigated. To determine the equivalent circuits and system parameters, higher order partial derivatives are simplified using the finite difference method; partial differential equations (PDE) are transformed to finite difference equations and further represented by electronic components and circuits. To provide better signal management and stability, active electrical circuit systems are designed and fabricated. Electrical signals from the distributed system circuits (i.e., soft and hard) are compared with results obtained by the classical theoretical, finite element, and experimental techniques.

1. INTRODUCTION

Mechanical components (e.g., mass, spring, and damper) of discrete mechanical systems and structures modelled by ordinary differential equations (ODE) can usually find their equivalent electrical components (e.g., capacitor, inductor, and resistor) in electrical systems; discrete mechanical or structural systems can be modelled by electric circuits and analyzed by electrical analogies. For simple discrete systems, the electric circuit modelling and analysis is relatively straightforward. For distributed systems represented by partial differential equations (PDE), generic electronic components (i.e., resistors, capacitors, inductors, and transformers) are used to represent standard elastic components simulating the elastic beams [1]. However, for multi-field distributed (parameter) structronic (structure + electronic) systems, their electrical analogies are not so easily defined.

Shah *et al.* [2] applied the finite element method (FEM) to discretize a piezoelectric laminated beam and then proposed to use very large-scale-integration (VLSI) chips, with the coefficients determined by the FEM, to simulate the beam dynamic and control responses. Hardware designs and implementation of complete distributed structronic control systems integrating structural elasticity, distributed sensing and distributed control were not reported. Although using inductors, capacitors, transformers, and resistors in modelling of distributed control systems seems trivial conceptually; however, when estimating system parameters the transformers performing divisions and multiplications in

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the difference equation have to deal with signals at natural frequencies of the distributed system. At low frequencies, keeping a steady ratio for both current and voltage at the transformer's two sides (i.e., input and output) is difficult. Besides, nominal values of capacitors and inductors might be too large, beyond the available standard components, to realize "true" equivalent circuits of the original distributed systems. Furthermore, fabricating designated VLSI chips for specific structures also seems impractical in real applications. Accordingly, new improved electrical modelling techniques for distributed structronic systems need to be explored.

Smart structures and structronic systems are considered as one of the key technologies in the 21st century [3–6]. Among the commonly used smart materials, piezoelectric materials have been widely used as sensors and actuators in sensing and control of structronic systems [2, 6–10]. This paper is to report a technique to improve the electrical modelling and analysis of distributed structronic systems with coupled distributed sensors and actuators. Partial differential equations of an Euler–Bernoulli beam coupled with distributed piezoelectric sensors and actuators—a structronic beam system—are presented first, followed by electrical modeling and design of active electric circuits/components based on the finite difference discretization. Note that the structronic beam system is designed to control the damping and frequency of the beam. Circuit signals and finite difference solutions are compared with theoretical solutions. Soft and hard electric circuit models of the distributed beam/sensor/actuator/control system are setup to validate the new technique. Electrical signals and data are compared with theoretical, simulation, finite element, and experimental results.

2. DISTRIBUTED BEAM/SENSOR/ACTUATOR STRUCTRONIC SYSTEM

There are three fundamental tools, namely (1) analytical, (2) numerical, and (3) experimental, commonly used in many research and development work. Closed-form analytical solutions of structronic control systems represented by PDEs can be achieved only for simple geometries and boundary conditions. For complicated systems with distributed parameters, deriving closed-form solutions is difficult and impractical, even impossible for many cases. Physical models fabricated to validate the analytical or numerical solutions are often expensive and limited to laboratory facilities and experimental constrains. Numerical techniques, e.g., the FEM and the finite difference method (FDM), can be employed, in practice, to analyze the distributed structronic systems. The FEM is a geometric discretization method, which discretizes the geometry and formulating the system ODEs based on characteristics of divided finite "elements". However, the FDM is a mathematical discretization method, which requires the original PDE model, and difference equations are derived based on the original PDEs, boundary conditions, and number of "differences" (similar to elements in FEM). In this section, electrical modelling of a distributed structure/sensor/actuator system derived from the finite difference discretization is presented first. Boundary control and active circuit design are presented next.

2.1. ELECTRICAL MODELLING OF A STRUCTRONIC BEAM SYSTEM

An elastic Euler–Bernoulli beam is among the simplest distributed systems typically modelled by a fourth order PDE:

$$YI\frac{\partial^4 u_3}{\partial x^4} + \rho A\ddot{u}_3 = bF_3, \tag{1}$$

where Y is Young's modulus, $I = bh^3/12$ is the area moment of inertia, b is the beam width, h is the beam thickness, u_3 denotes the transverse displacement, ρ is the mass density, $A = bh$ is the cross-section area, and F_3 is the transverse excitation. Consider a generic cantilever Euler–Bernoulli beam sandwiched between two thin piezoelectric layers serving as distributed sensor and actuator, respectively (Figure 1), i.e., a distributed structronic cantilever beam system. The top piezoelectric layer is the distributed sensor and the bottom layer is the distributed actuator. The sensing signal acquired from the distributed sensor is amplified and feedback to the distributed actuator actively counteractes the beam oscillation.

The mathematical model of the structronic (beam/sensor/actuator) system now becomes

$$YI\frac{\partial^4 u_3}{\partial x^4} + \rho A\ddot{u}_3 - \frac{b\partial^2(M_{11}^a)}{\partial x^2} = bF_3, \tag{2}$$

where $M_{11}^a = \tilde{\xi}[\partial u_3/\partial x]\big|_0^L$ is the control momentm, $\tilde{\xi} = -Gh^s r_1^a d_{31} Y_p r_1^s h_{31}/L$ is a constant determined by a number of geometric and material parameters [6], G is the feedback gain, h^s is the sensor thickness, r_1^a is the actuator distance, d_{31} and h_{31} are the piezoelectric constants, Y_p is Young's modulus of piezoelectric layers, r_1^s is the sensor distance, and L is the beam length. It is assumed that the resistance of the surface electrodes is neglected so that the voltage is uniformly distributed. The control moment M_{11}^a is a function of two end-slopes of the distributed actuator and is not a function of spatial co-ordinates. Accordingly, the system equation of motion reduces to the original form with a controllable boundary condition at the free end for the fully distributed sensor/actuator configuration. Thus, since the beam's boundary conditions are fixed at $x = 0$ and free at $x = L$ and the actuator layer is fully distributed, the distributed control action becomes an equivalent boundary control action, i.e., a control moment acting at the free end [11, 6].

Based on the definition of differentiation, derivatives can be represented by consecutive responses u_i separated by a finite difference Δx, i.e., $\mathrm{d}u/\mathrm{d}x = \lim_{\Delta x \to 0}(\Delta u/\Delta x) = \lim_{\Delta x \to 0}[u(x + \Delta x) - u(x)]/\Delta x$.

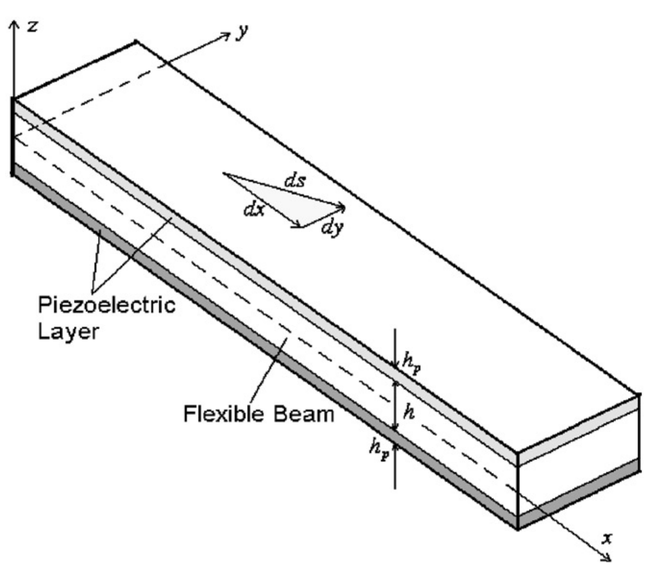

Figure 1. Distributed beam/sensor/actuator structronic system.

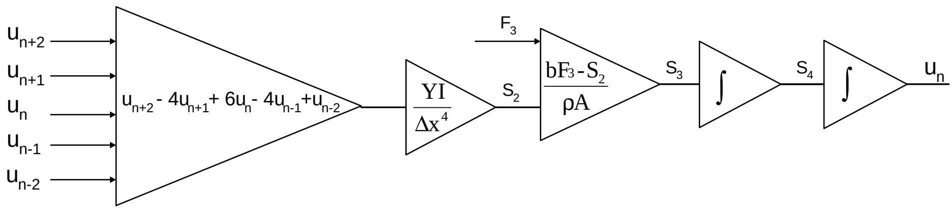

Figure 2. Equivalent circuit for displacement calculation.

There are backward differences, forward differences and central differences, and the central difference usually leads to better accuracy, where the central difference is defined as $du/dx = \lim_{\Delta x \to 0}(2\Delta u/2\Delta x) = \lim_{\Delta x \to 0}[u(x+\Delta x) - u(x-\Delta x)]/2\Delta x$. Higher order differences can be derived based on the first order difference [12]. Accordingly, for the distributed beam structronic system, one usually takes a uniform difference Δx along the beam length L, i.e. $\Delta x = L/m$ where m is the total number of differences. Assume u_n is the displacement of the nth node, thus, $\Delta u = u_{n+1} - u_n$. The fourth order differences in the continuous beam equation can be derived and the generic finite difference beam equation, defined at the nth node, becomes

$$YI\left[\frac{u_{n+2} - 4u_{n+1} + 6u_n - 4u_{n-1} + u_{n-2}}{\Delta x^4}\right] + \rho A\frac{\partial^2 u_n}{\partial t^2} = bF_3. \tag{3}$$

Furthermore, the nodal displacement can be calculated and the equivalent electric circuit be fabricated using the active and passive electrical components accordingly (Figure 2).

Conceptually, a smaller difference Δx would lead to better resolution and accuracy in the circuit modelling and analysis. However, in reality, directly dividing YI by $(\Delta x)^4$ would yield a signal too large to handle in a circuit. Thus, the circuit is modified to avoid the signal divergence. Thus, the original difference equation is modified to $YI(u_{n+2} - 4u_{n+1} + 6u_n - 4u_{n-1} + u_{n-2})/\Delta x^3 + \Delta x \rho A(\partial^2 u_n/\partial t^2) = \Delta x b F_3$. Furthermore, transformers used for signal amplification usually have a deficiency in low-frequences amplification. In order to maintain uniform amplification of wide-band frequency, operational amplifiers (op-amp's) [13] are selected to implement the equivalent electric circuit of the distributed structronic beam/sensor/actuator system. Figure 3 illustrates the final circuit schematic consisting of a number of operational amplifiers and resistors. Nominal values of these resistors and capacitors are determined by material, geometry, and control parameters summarized in Appendix A. Representative signals S_i at various key locations are noted in both Figures 2 and 3. Boundary conditions, boundary control, and their equivalent electric circuits derived from the finite difference discretization are discussed next, followed by feedback control of the beam/sensor/actuator system.

2.2. BOUNDARY CONDITIONS AND EQUIVALENT BOUNDARY CONTROL

A cantilever beam coupled with distributed piezoelectric sensor/actuator layers, i.e., the distributed structronic beam system, was defined above. As discussed previously, since the distributed sensor and actuator are both fully distributed over the entire beam length, the resulting control action, i.e., the counteracting control moment, congregates at the free end. Thus, the original distributed control problem becomes a boundary control problem. For the distributed controlled beam, boundary conditions at the fixed end ($x = 0$ or $n = 0$) are the same as the original elastic beam; however, boundary conditions at the free end

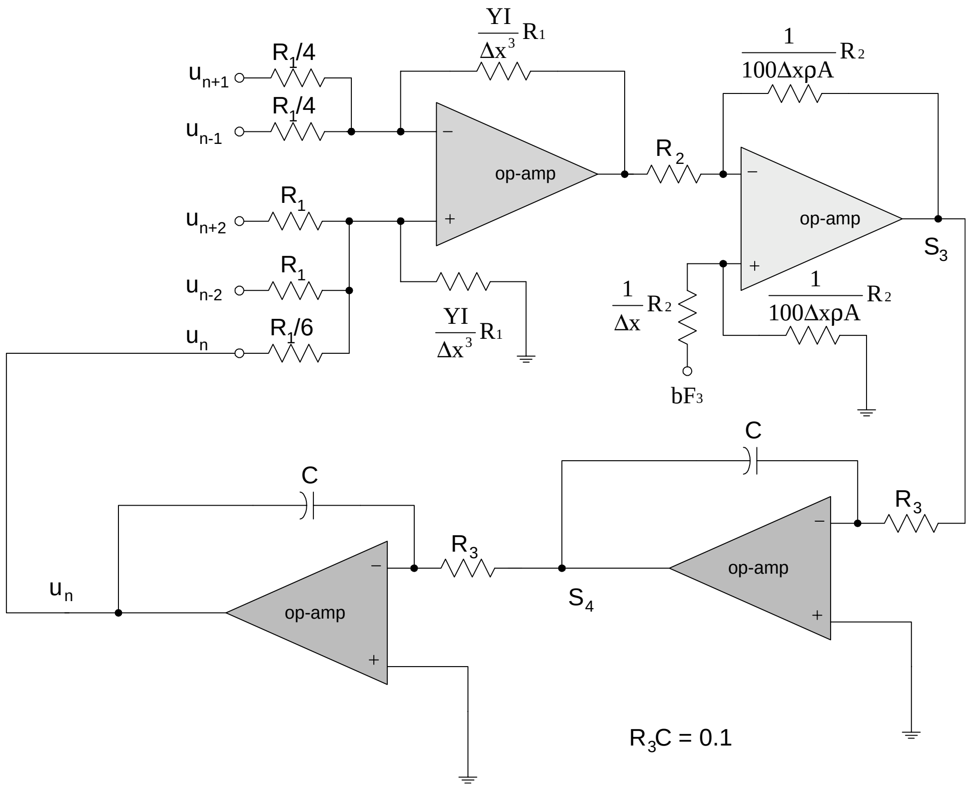

Figure 3. Circuit schematic of the distributed beam/sensor/actuator structronic system.

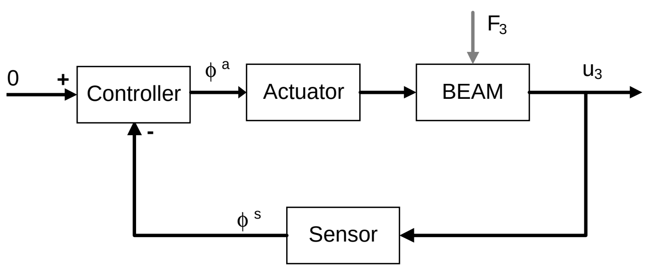

Figure 4. Control system block diagram.

($x = L$ or $n = m$) are different, corresponding to the control algorithms. (Note that the beam is divided into m differences and n denotes the node number.) The overall beam with distributed sensor and actuator layers can be represented in a block diagram, Figure 4, in which the controller designates the control algorithms; ϕ^a is the actuator signal; and ϕ^s is the sensing signal. Original elastic and derived control boundary conditions are presented in this section. Note that the fixed-end boundary conditions of all cases are identical. Thus, only the free-end boundary conditions are defined for the control cases. The complete feedback control circuit schematic of the distributed beam structronic system is defined afterwards.

2.2.1. *Elastic boundary conditions (without control)*

- Fixed end ($x = 0$, $n = 0$): displacement $u_0 = 0$; slope $\partial u_0/\partial x = 0$, then $u_{-1} = u_1$.
- Free end ($x = L$, $n = m$, m is number of elements divided, u_m is the transverse displacement at the free end): moment $\partial^2 u_m/\partial x^2 = 0$, then $u_{m+1} = 2u_m - u_{m-1}$; shear force $(\partial^3 u_m/\partial x^3) = 0$, then $u_{m+2} = 4u_m - 4u_{m-1} + u_{m-2}$, or $u_{m+2} = 2u_{m+1} - 2u_{m-1} + u_{m-2}$.

2.2.2. *Displacement feedback control*

Assume that the control voltage is proportional to the sensing signal, i.e., $\phi^a = G\phi^s$, where ϕ^a is the actuator signal; ϕ^s is the sensing signal; and G is the gain factor. Boundary conditions at the free end ($x = L$) are defined as follows:

Moment: $M_{11}^*(L) = LbM_{11}^a(L)$, where $M_{11}^a(L) = \tilde{\xi}(\partial u_3(L)/\partial x) = \tilde{\xi}(\partial u_m/\partial x)$ and $\tilde{\xi} = -(Gh^s r_1^a d_{31} Y_p r_1^s h_{31}/L)$. Thus, using moment definition yields $-YI(\partial^2 u_m/\partial x^2) = LbM_{11}^a(L) = Lb\tilde{\xi}(\partial u_m/\partial x)$. Since the difference equation of the double derivative is $\partial^2 u_m/\partial x^2$

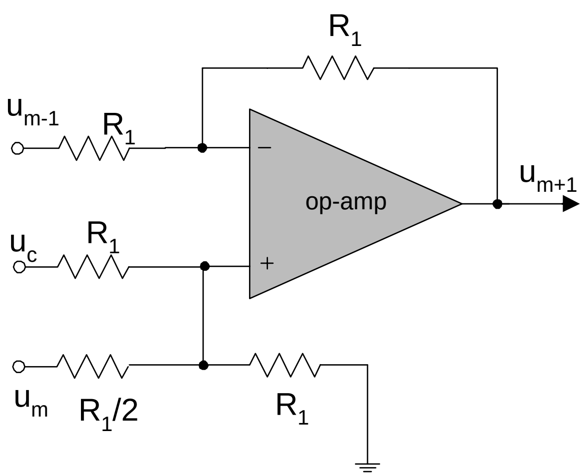

Figure 5. Circuit schematic of the boundary moment control.

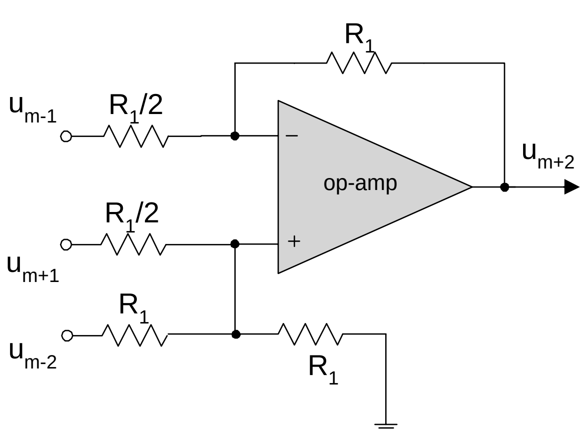

Figure 6. Circuit schematic of the boundary shear force.

$= (u_{m+1} - 2u_m + u_{m-1})/\Delta x^2 = -Lb(M_{11}^a(L)/YI)$, thus, the displacement $u_{m+1} = -Lb(M_{11}^a(L)\Delta x^2/YI) + 2u_m - u_{m-1} = u_c + 2u_m - u_{m-1}$ and $u_c = -Lb(M_{11}^a(L)\Delta x^2/YI)$.

This circuit is realized in Figure 5 where u_c is determined by control algorithms, including the other algorithms discussed in this section.

Shear force $\partial^3 u_m/\partial x^3 = 0$, then $u_{m+2} = 2u_{m+1} - 2u_{m-1} + u_{m-2}$. This circuit diagram representing the shear force is illustrated in Figure 6.

2.2.3. *Velocity feedback control*

The control signal is now defined as $\phi^a = G\partial\phi^s/\partial t$. Free-end boundary conditions are as follows:

Moment: $M_{11}^*(L) = LbM_{11}^a(L)$ and $M_{11}^a(L) = r_1^a d_{31} Y_p \phi^a$ or $M_{11}^a(L) = -\tilde{\xi}\partial^{\left[(\partial u_3/\partial x)_0^L\right]}/\partial t = -\tilde{\xi}\partial^{[\partial u_3(L)/\partial x]}/\partial t = -\tilde{\xi}\partial^{(\partial u_m/\partial x)}/\partial t$ [6, 11]. Thus, $-YI(\partial^2 u_m/\partial x^2) = LbM_{11}^a(L) =$

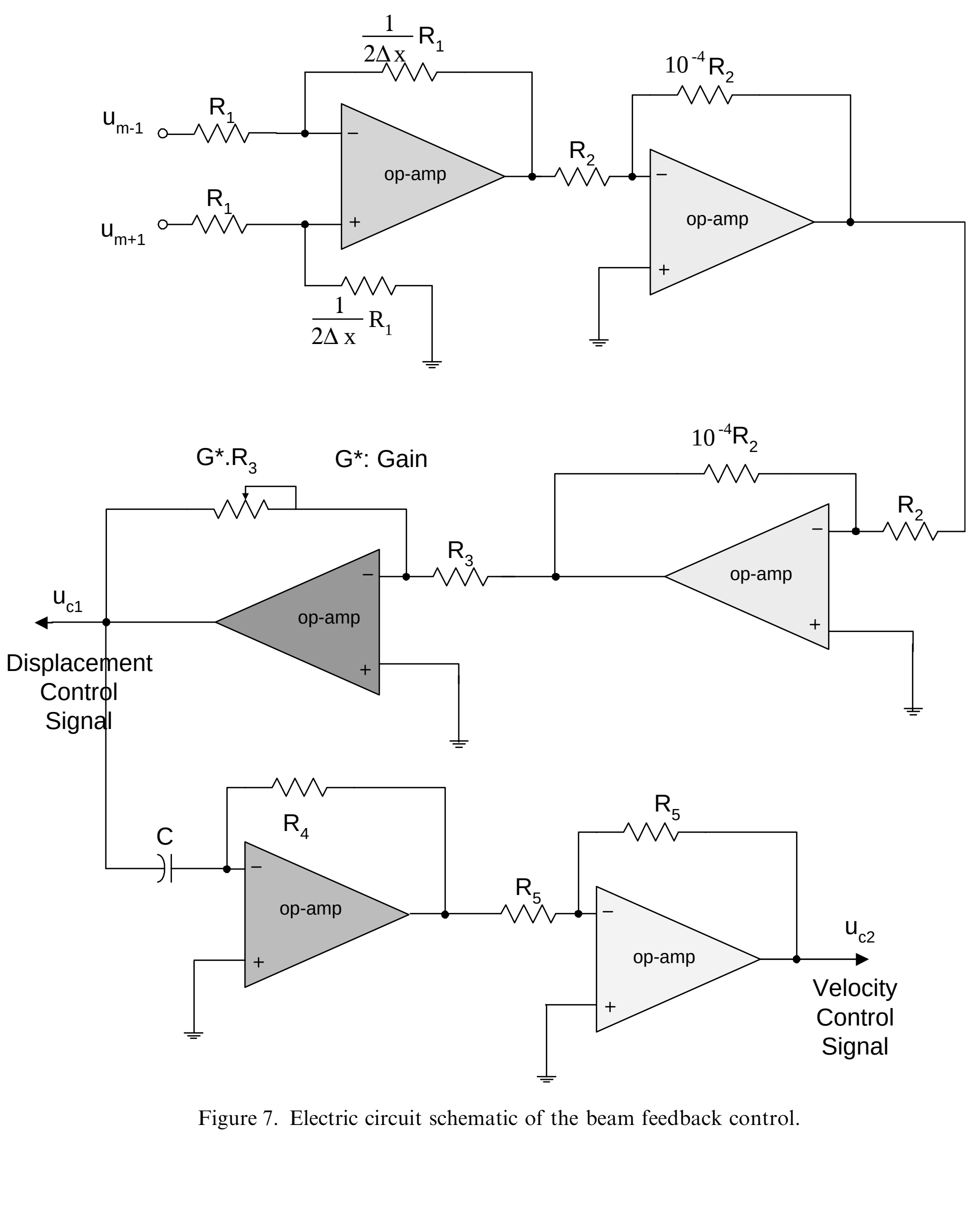

Figure 7. Electric circuit schematic of the beam feedback control.

$-Lb\tilde{\xi}\partial^{(\partial u_m/\partial x)}/\partial t$ and the displacement is defined by $u_{m+1} = Lb(M_{11}^{a}(L)\Delta x^2/YI) + 2u_m - u_{m-1} = -u_c + 2u_m - u_{m-1}$ where u_c is the control signal.

Shear force: $\partial^3 u_m/\partial x^3 = 0$, then $u_{m+2} = 2u_{m+1} - 2u_{m-1} + u_{m-2}$.

The complete feedback control circuit schematic of the distributed structronic beam system is illustrated in Figure 7. Note that u_{c1} and u_{c2} correspond to the displacement and the velocity feedback signals respectively.

Note that the op-amp set-ups given in Figures 5 and 6 have the advantage to minimize the current offsets in the circuit [14]. Accordingly, these set-ups representing the boundary conditions and boundary control improve the overall accuracy of the structronic beam model with minimal bias-current error.

3. CIRCUIT FABRICATION, TESTING, AND ANALYSIS

Based on the procedures presented previously, an active electric circuit for the cantilever Euler–Bernoulli beam with distributed sensor and actuator layers is fabricated and shown in Figure 8. (Material and geometric properties of the beam/sensor/actuator system are

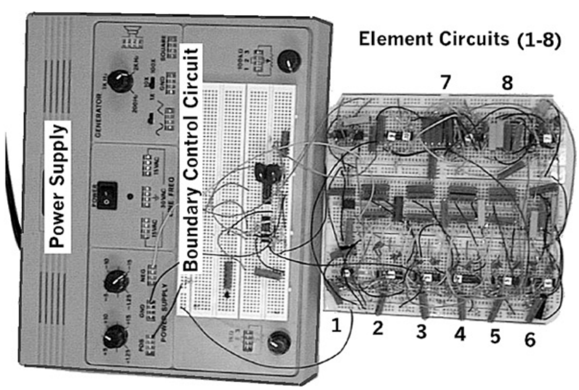

Figure 8. The electric circuit of the distributed beam structronic system.

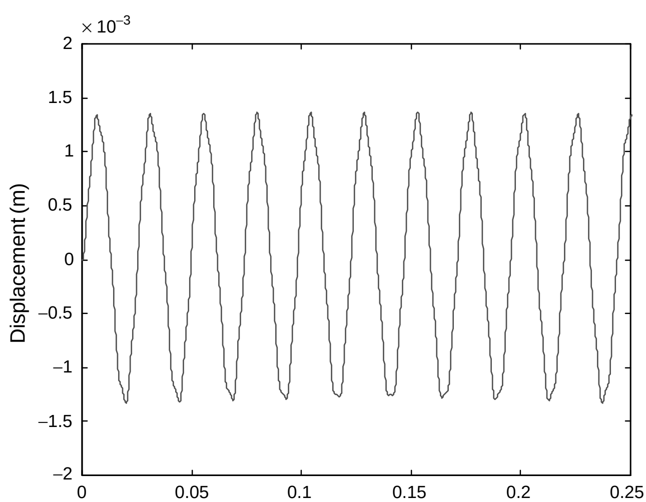

Figure 9. Free response of the cantilevered structronic system (displacement feed back control gain = 0).

summarized in Appendix A.) The beam is discretized into eight elements and consequently there are eight component circuits numbered from 1 to 8 representing eight differences of the beam system; each component circuit is similar to Figure 3. The boundary control circuit is represented at the top breadboard mounted on the power supply and it is used to calculate u_9 and u_{10} (i.e., u_{c1} and u_{c2}) derived from the feedback control. u_9 and u_{10} denote the nodes outside the boundary and they are used to define boundary conditions. Thus, the feedback control circuit provides the control signal changing the boundary condition in the equivalent boundary control of the cantilevered structronic system. The identical circuit is also set-up and calculated using SIMULINK to validate the experimental circuit results. Figure 9 shows the original free response, without any control effect. The frequency shown is the natural frequency of the structronic beam system. Theoretically, it is different from the natural frequency of the elastic cantilever beam,

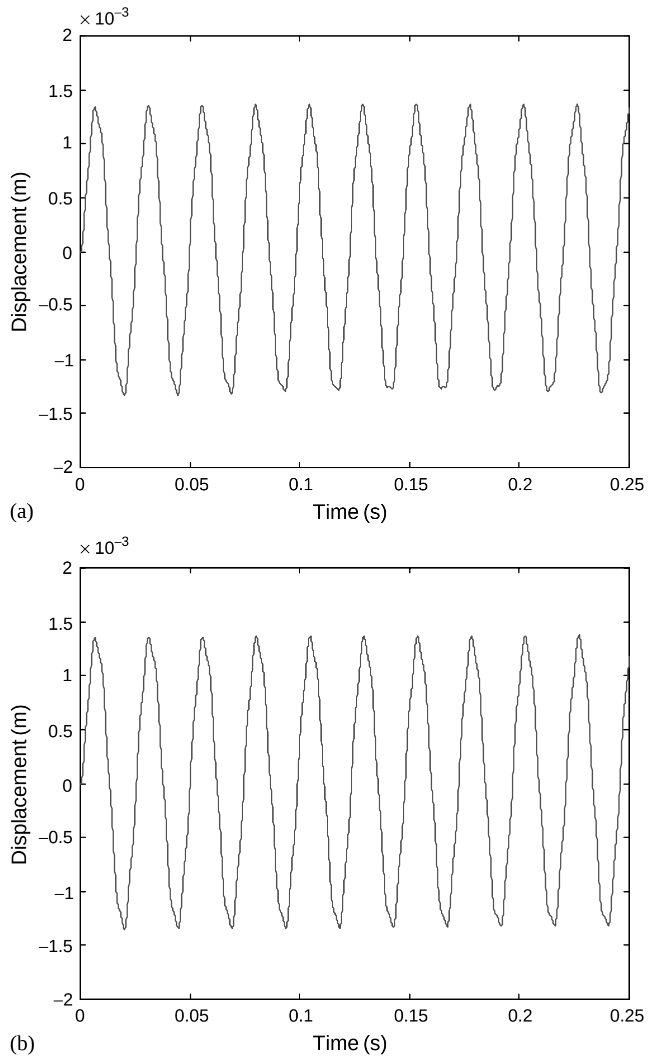

Figure 10. Displacement feedback responses of the cantilevered structronic system: (a) gain = 100 and (b) gain = 500.

due to the laminated piezoelectric materials. However, the piezoelectric layers are really thin and light-weighted compared with the elastic beam. Thus, their influence on natural frequency is minimal and hence neglected. Figures 10(a,b) show the displacement feedback responses and Figures 11(a–d) illustrate the velocity feedback responses with different control gains.

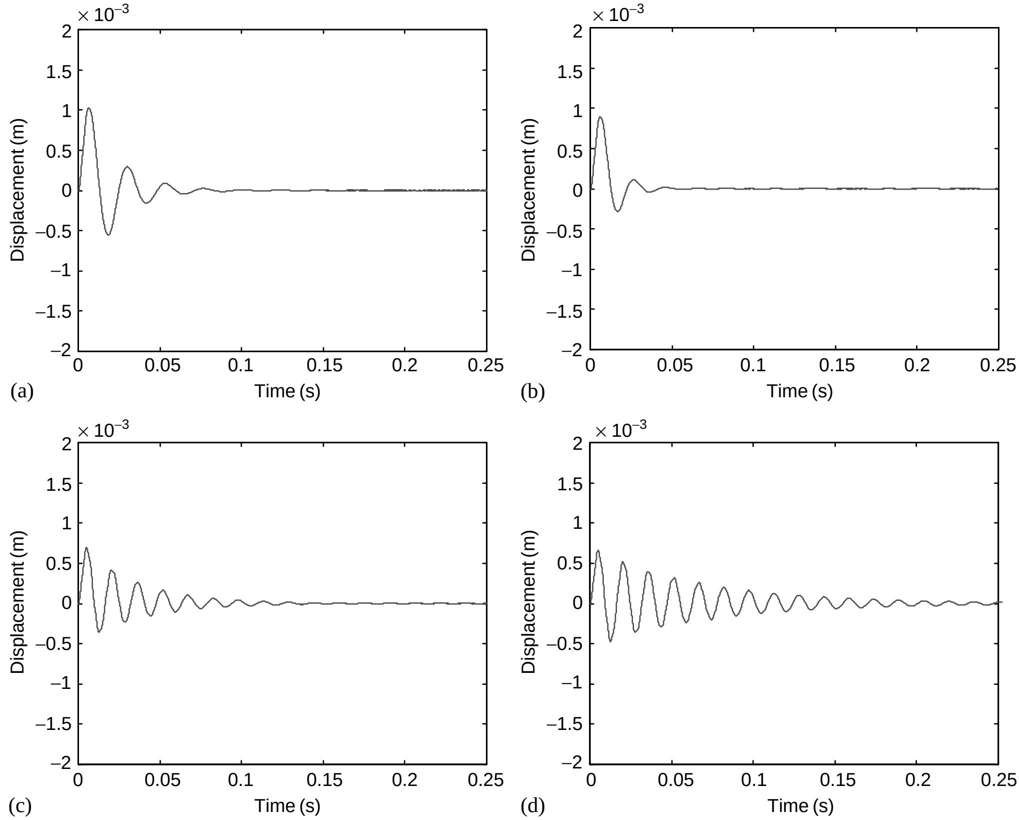

Figure 11. Velocity feedback responses of the cantilevered structronic system, (a) gain = 50; (b) gain = 100; (c) gain = 500 and (d) gain = 1000.

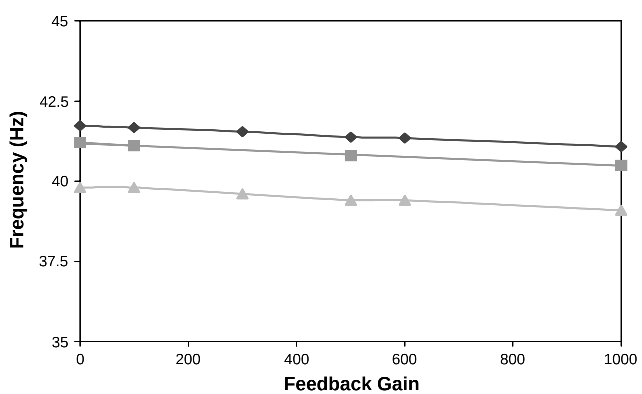

Figure 12. Frequency variation in the displacement feedback control: ◆, theory; ■, simulink; ▲, circuit.

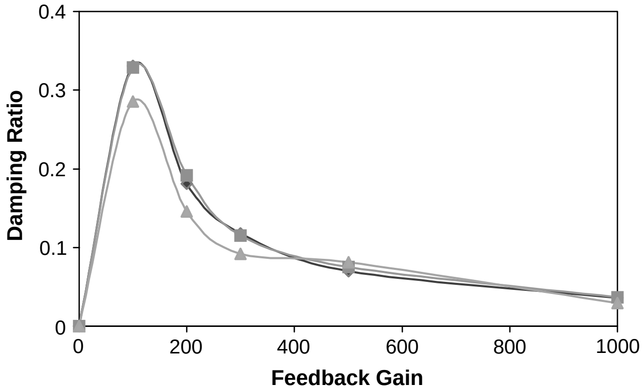

Figure 13. Damping ratio variation in the velocity feedback control: ◆, theory; ■, simulink; ▲, circuit.

The inferred frequency and damping ratio changes with respect to various control gains are calculated and plotted in Figures 12 and 13. The displacement feedback control influences the system oscillation frequency, which can be used to avoid resonance. On the other hand, the velocity feedback control can manipulate system damping. Errors of the circuit signals compared with the theoretical and simulation results are also calculated. These data suggest that the circuit signal measurements compare well with the theoretical and simulation results for the displacement feedback. However, the variations in the velocity feedback are relatively significant, due to the differentiation circuit in the boundary feedback circuit, although the theoretical results compare well with the simulation results. (Note that comparisons with other techniques, e.g., the FEM and experimental results, can also be inferred [11, 6].)

4. CONCLUSIONS

Smart structures and structronic (structure + electronic) systems are considered as one of the key technologies in the 21st century. Conventional techniques used in modelling and analysis of structronic systems encompasses (1) theoretical analysis, (2) numerical analysis, and (3) laboratory experiments. This research is to investigate the fourth modelling and analysis technique based on the electrical analogy, i.e., using active electronic circuits and components to model system characteristics and sensing/control effects of distributed structronic systems governed by PDEs. Circuit design of distributed elastic systems was proposed decades ago. However, hardware implementations of distributed parameter systems (DPSs) were rather limited, although implementation of discrete systems was considered trivial. In order to model the PDE consisting of structural elasticity, sensing (the direct piezoelectric effect), and control (the converse piezoelectric effect) by electric circuits and to fabricate the equivalent hardware, the PDE was discretized using the finite difference technique. Distributed control effectiveness of distributed structronic systems—elastic continua coupled with distributed sensors and actuators—was evaluated using the electric circuit modelling technique. Theoretical results were compared favorably with the experimental data of a hardware electric circuit of the distributed structronic system. Note that the op-amp set-ups can minimize the current offsets in the circuit and thus can significantly improve the accuracy of the structronic beam model with the minimal bias-current error. This suggests that the electrical circuit modelling technique serves as a viable alternative tool for advanced modelling and analysis of complicated distributed structronic control systems. Active circuit signals comply with dynamic and control characteristics of the distributed structronic control systems.

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APPENDIX A: MATERIAL AND GEOMETRIC PROPERTIES OF THE BEAM/SENSOR/ ACTUATOR SYSTEM

Material and geometric parameters of the structronic beam system are summarized in Tables A1 and A2, [6,11].

Table A1

Properties for the plexiglas beam [11]

Y (Young's modulus)	$3{\cdot}1028 \times 10^9\,\mathrm{N/m^2}$
ρ (mass density)	$1190{\cdot}0\,\mathrm{kg/m^3}$
h (thickness)	$1{\cdot}6 \times 10^{-3}$ m
b (width)	$0{\cdot}01$ m
L (length)	$0{\cdot}1$ m
μ (the Poisson ratio)	$0{\cdot}3$
I (area moment of inertia)	$bh^3/12 = 3{\cdot}4133 \times 10^{-12}\,\mathrm{m^4}$

Table A2

Properties of polyvinylidene fluoride (PVDF) sensor/actuator layers

Y_p (Young's modulus)	$2{\cdot}00 \times 10^9\,\mathrm{N/m^2}$
ρ_p (mass density)	$1800{\cdot}0\,\mathrm{kg/m^3}$
h^s (thickness of sensor)	40 μm
h^a (thickness of actuator)	40 μm
r_1^s (sensor distance)	$(h^s + h)/2 = 8{\cdot}2 \times 10^{-4}$ m
r_1^a (actuator distance)	$(h^a + h)/2 = 8{\cdot}2 \times 10^{-4}$ m
d_{31} (piezoelectric constant)	$2{\cdot}3 \times 10^{-11}$ (m/m)/(v/m)
h_{31} (piezoelectric constant)	$g_{31} \times Y_p = 2{\cdot}16 \times 10^{-1} \times 2{\cdot}0 \times 10^9 = 4{\cdot}32 \times 10^8$(m/m)/(v/m)