Journal of Sound and Vibration (2002) **258**(1), 191–198
doi:10.1006/jsvi.5091, available online at http://www.idealibrary.com on IDEAL®

INVESTIGATION ON THE WHIRLING MOTION OF FULL ANNULAR ROTOR RUB

Y. S. Choi

School of Mechanical Engineering, Sung Kyun Kwan University, Suwon, Kyunggi-do 440-746, *Korea.*
E-mail: yschoi@yurim.skku.ac.kr

(*Received* 1 *November* 2001)

1. INTRODUCTION

The performance of rotating machinery such as a motor or a gas turbine of a jet engine can be maximized by reducing the clearance between the rotor and stator. During the whirling motion of the rotor, the rotor can contact the stator. The contact between the rotor and stator during the whirling motion is called rubbing. Rubbing phenomenon occurs in various ways depending on the design parameters and operation conditions of rotating machinery. Largely, rubbing phenomenon is divided into two types, full annular rub and partial rub. Partial rub is the intermittent contact between the rotor and stator. In contrast, full annular rub is characterized by continuous contact during the entire whirling motion. The whirling direction may be the same as the rotor spin direction, i.e., forward whirling, or opposite, i.e., backward whirling. In the case of forward whirling, the whirling frequency is usually the same as the rotor speed, called forward synchronous whirling. Forward whirl always accompanies slipping between the rotor and stator. On the other hand, backward whirl occurs with slipping or no slipping, depending on the rotor speed. If there is no slipping at the contact surface, the rotor just rolls on the inner surface of the stator, called backward rolling. If slipping exists, it is called backward slipping. As the rotor speed increases or decreases, different rubbing phenomena take place depending on the rotor speed.

Once rubbing occurs during the operation of rotating machinery, the runouts of the rotor and stator become large and the operation can potentially become destructive. Because of this disastrous phenomenon in rotating machinery, many researchers have been interested in the safe and reliable design of rotating machinery. Black [1] can be mentioned as a notable contributor to this problem. He tried to explain the physics of rub using the polar receptance of the rotor and stator on a radial symmetric rotor. Ehrich [2] made a simple model and calculated the responses of the rotor and stator numerically. Muszynska [3] did an extensive study on this rubbing problem. Muszynska tried numerical simulations and experiments for various cases. Choi [4] treated this problem as a piecewise linear vibration problem, and wrote an algorithm to calculate the steady state solution using the FFT technique. The result showed that super- and sub-harmonic responses could be found due to rubbing. Crandall and Lingener [5] showed a very specific pattern of whirling frequency during rubbing experimentally, and developed a theory to explain backward rolling and backward slipping. Choi [6] demonstrated rubbing phenomenon which has different modes, depending on whether the operation speed is increasing or decreasing, and explained the onset of backward whirling and the disappearance of backward slipping using the receptance of the rotor and stator. However, no other clear physical explanation for the onset of the whirling motions during rubbing has been done.

0022-460X/02/$35.00

In this study, an experimental apparatus was made, and various rubbing phenomena were demonstrated from the experimental apparatus. Numerical simulations were also performed for the investigation of the physics of full annular rub. This study shows that the effects of the friction coefficient and the eccentricity of the rotor can explain the onset of backward rolling and backward slipping as the rotor speed increases or decreases.

2. EQUATION OF MOTION

Figure 1 shows an analytical model of rotating machinery for the investigation of rotor rub. The radius of the rotor is a, and the clearance between the rotor and stator is δ. R and S are the center of the rotor and stator, and O and G are the geometric center of both end bearings and the center of the rotor respectively. If the direction of whirling motion of the rotor is the same as the direction of rotor rotation, the whirling motion is called forward whirl, and for the opposite direction of whirling motion, backward whirl. Forward whirl always accompanies slipping motion at the contact point between the rotor and stator. Backward whirl can be backward rolling without slipping or with slipping on the contact surface. For the case of backward rolling, the relative velocity at the contact points between the rotor and stator is zero, and the whirling speed of the rotor is given by

$$\dot{\phi} = -\frac{a}{\delta}\omega. \tag{1}$$

If backward slipping occurs, the whirling speed will be smaller than that of equation (1). $\dot{\phi}$ is the whirling speed, and ω is the spin rate of the rotor, i.e., the rotating speed. From the model of Figure 1, the equations of the rotor and stator are as shown in equation (2) in the x and y directions:

$$m_r\ddot{x}_r + c_r\dot{x}_r + k_r x_r + R_x = m_r e\omega^2 \sin\omega t, \tag{2a}$$

$$m_r\ddot{y}_r + c_r\dot{y}_r + k_r x_r + R_y = m_r e\omega^2 \cos\omega t, \tag{2b}$$

$$m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s - R_x = 0, \qquad m_s\ddot{y}_s + c_s\dot{y}_s + k_s y_s - R_y = 0, \tag{2c, d}$$

where m_r and m_s are the mass of the rotor and stator respectively. c_r, c_s, k_r, and k_s are the damping and the stiffness coefficients of the rotor and stator respectively. R_x and R_y are

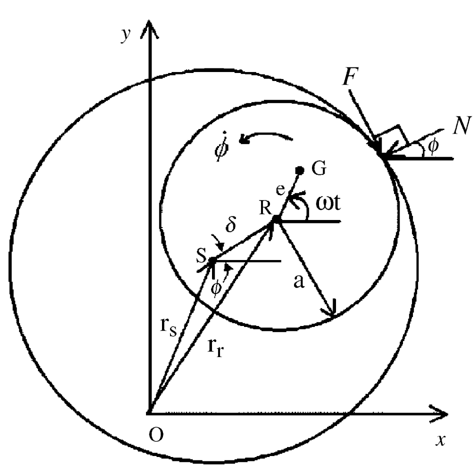

Figure 1. Analytical model of a rubbing rotor.

the reaction forces in the x and the y direction respectively. The assumption of full annular rub results in the following kinematic relation between the displacements of the rotor and stator from Figure 1:

$$x_r = x_s + \delta \sin\phi, \quad y_r = y_s + \delta \cos\phi. \tag{3}$$

Adding equations (2a) and (2c) eliminates R_x, and similarly, R_y can be eliminated from equations (2b) and (2d). The differentiation of equation (3) under the assumption of constant whirl speed obtains $\dot{x}_r$, $\dot{y}_r$, $\ddot{x}_r$ and $\ddot{y}_r$. By combining these results, the following equations of the stator motion for the x and y directions are obtained:

$$\begin{aligned}
&(m_r + m_s)\ddot{x}_s - m_r\delta\dot{\phi}^2 \sin\phi + (c_r + c_s)\dot{x}_s + c_r\delta\dot{\phi}\cos\phi + (k_r + k_s)x_s \\
&+ k_r\delta \sin\phi = m_r e\omega^2 \sin\omega t, \\
&(m_r + m_s)\ddot{y}_s - m_r\delta\dot{\phi}^2 \cos\phi + (c_r + c_s)\dot{y}_s - c_r\delta\dot{\phi}\sin\phi + (k_r + k_s)x_s \\
&+ k_r\delta \cos\phi = m_r e\omega^2 \cos\omega t.
\end{aligned} \tag{4}$$

For synchronous forward whirl $\dot{\phi}$ will be equal to the rotor speed ω, and for backward rolling, equation (1) can be applied. Equation (4) can be solved numerically using the Runge–Kutta method under the assumption of pre-determined whirling type by setting the whirling frequency $\dot{\phi}$. At each time step, the velocity and displacement of the stator can be known from the numerical integration. The reaction forces R_x and R_y can also be derived from the manipulation of equation (2).

The normal force N and friction force F can also be calculated using the results of R_x and R_y. If the normal force and friction force are calculated as given above, the friction coefficient between the rotor and stator can be obtained. The pre-determined whirling frequencies and arbitrary initial conditions for the numerical integration can be an interrogation point.

3. EXPERIMENT

An experimental apparatus was designed and made for the realization of the rubbing phenomenon in laboratory scale as shown in Figure 2. The experimental apparatus is

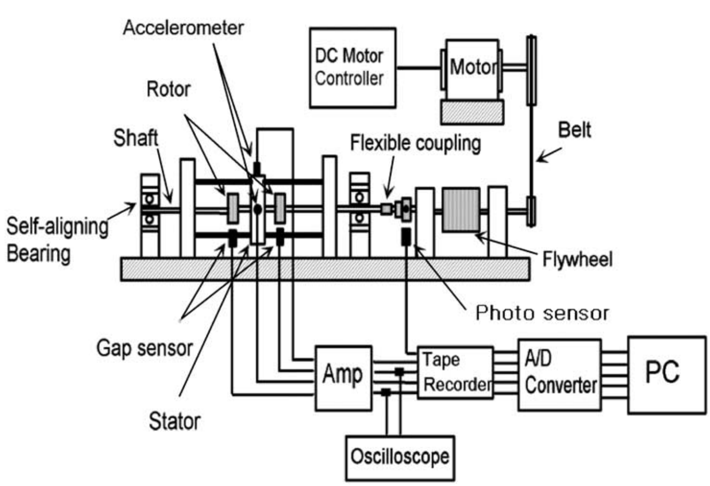

Figure 2. Schematic of the experimental apparatus.

TABLE 1

Parameters of the experimental apparatus

Parameters	Values
Shaft radius (mm)	4·0
Clearance (mm)	0·45
Rotor mass (kg)	0·944
Rotor natural frequency (Hz)	23
Rotor loss factor	0·06
Stator mass (kg)	0·874
Stator natural frequency (Hz)	76
Stator loss factor	0·12
Eccentricity (kg mm)	6·13

driven by a DC motor of 1·5 kW with a controller that makes the change of speed of the motor. A belt driver is used to prevent the transmission of the vibration that occurs in the motor. The diameter ratio between the motor and the rotor is 3:1 for the increase of the rotor speed. A flywheel is used to minimize the fluctuation of the rotor speed. And the effect of bending moment and the misalignment of shaft is minimized using flexible coupling and self-aligning bearings at both sides of the rotor supports. The two stators are made from an aluminum and an acryl plate of 5 mm thickness, with a central hole that is slightly larger than the diameter of the rotor to provide a clearance between the rotor and stator. The stator is supported by four simple supported beams of piano wires, which builds the stator stiffness. The simple supported beam is more stable than a cantilever beam during the operation of the experimental apparatus. The whirling motion of the rotor is detected using gap sensors for the x and y directions respectively. The stator motion is monitored by two accelerometers for the x and y directions respectively. The rotor speed is also monitored using a photo-sensor. All data are recorded on a magnetic tape, then converted to digital data by an A/D converter and stored on a PC. Table 1 shows the specifications of the experimental apparatus that are measured directly or by an impact test. All the parameters are the same between aluminum and acryl stator except the clearance size. For aluminum stator it is 0·45 mm and 0·5 mm for acryl. Aluminum and acryl are used for the differentiation of the friction coefficient between the rotor and stator. The rotation speed varies from 120 to 1200 r.p.m.

Figure 3 shows the whirling frequencies of the rotor as the rotor speed increases or decreases for the case of the aluminum stator. Most cases are forward whirl, where the whirling frequencies are equal to the rotating frequency. In order to obtain a backward whirling motion, special cares are necessary, i.e., less misalignment of the shaft, uniform clearance, and dry friction condition between the rotor and stator, which can be obtained through a long operation by drying out the remaining lubricant at the contact point. When increasing the rotor speed in the aluminum stator case, backward rolling occurs in the range between 2·6 and 5·5 Hz, from 5·5 Hz backward slipping occurs until 16·7 Hz, and then backward rolling appears again. The forward and backward can be detected by watching the orbit through an oscilloscope or drawing the orbit for the stored data using a computer. During decreasing the rotor speed, backward rolling continues until 11·7 Hz, which is a different transition speed compared to the previous case, and backward slipping occurs until 5·5 Hz, where the rotor goes back to backward rolling until 2·6 Hz, which are the same transition speeds as in the previous case. In the range below 2·6 Hz, there is no rubbing phenomenon since the whirling amplitude is less than the magnitude of the

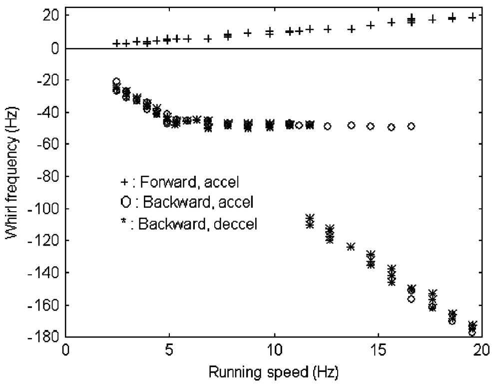

Figure 3. Whirling frequency versus rotor running speed from the experiment using an aluminum stator.

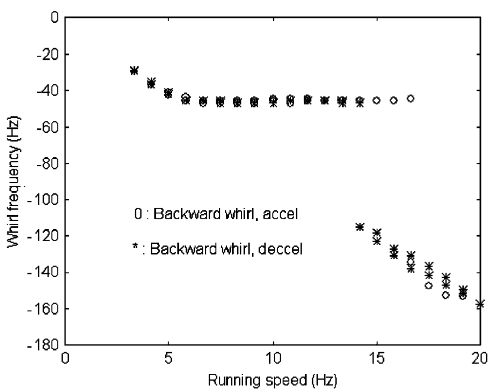

Figure 4. Whirling frequency versus rotor running speed from the experiment using an acryl stator.

clearance. Figure 4 shows the results obtained for the acryl stator using the same design parameters except the clearance size. Backward rolling occurs in the range between 3 and 5·8 Hz, while backward slipping occurs in the range between 5·8 to 16·7 Hz when the rotor speed is increasing, and in the 14–5·8 Hz range when the rotor speed is decreasing. The orbits of the whirling rotors for the case of the aluminum stator are shown in Figure 5 at a rotor speed of 12·7 Hz and the transient state from backward rolling to backward slipping. The results show that backward slipping results in a large amplitude compared to other cases of backward rolling or forward whirling.

4. NUMERICAL ANALYSIS AND DISCUSSION

A class of planar kinetic problems involves the rolling of a cylinder on a rough planar surface. The cylinder can roll or slide. If the friction force between the cylinder and surface is less than the maximum static friction force, the cylinder rolls without slipping. When the cylinder slides, the friction force becomes the kinetic friction force. A rubbing rotor within a small clearance between the rotor and stator presents the same kind of kinetics problem

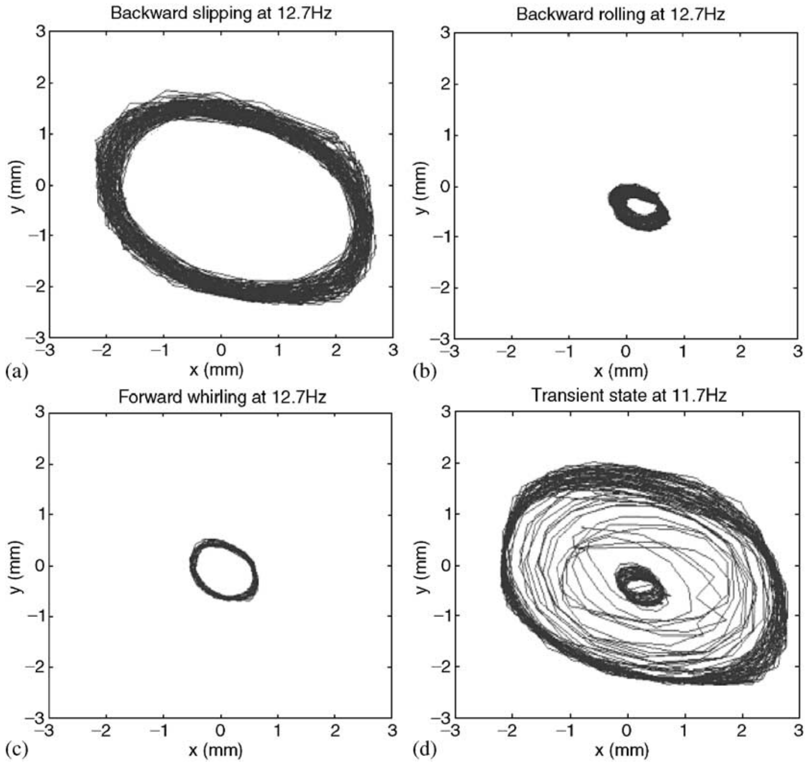

Figure 5. Whirling motions of the rotor during (a) backward slipping, (b) backward rolling, (c) forward whirl at 12·7 Hz and (d) transient state from backward rolling to backward slipping with decreasing rotor speed at 11·7 Hz

on a circular surface. The rotor can just roll or slip depending on the friction coefficient between the rotor and stator on the inner surface of the stator.

Equation (4) with the parameters of Table 1 is solved numerically using the Runge–Kutta method for the cases of forward whirl, backward rolling, and backward slipping for both increasing and decreasing rotor speeds. For the analysis of backward rolling, the whirling speed $\dot{\phi}$ is set according to equation (1), and the friction coefficient is calculated using equation (6). If the calculated friction coefficient is larger than the static friction coefficient, the calculation becomes meaningless, which means the assumed backward rolling does not exist at the rotor speed. For the analysis of backward slipping, the whirling speed is assumed to be the whirling speed at which the friction coefficient is equal to the static friction coefficient, and the normal force is calculated using equation (6). The negative normal force is physically meaningless. It indicates that backward slipping cannot be occurred at the rotor speed where the calculated normal force becomes negative. The analysis of backward whirling considers only the case where the whirling amplitude is larger than the clearance distance, since otherwise the full annular rub phenomenon does not occur.

The friction coefficient between the rotor and stator cannot be known *a priori*. The variation of necessary friction coefficient versus rotor speed is calculated numerically using equations (4)–(6) for the case of backward rolling with aluminum or acryl stator as shown in Figures 6 and 7 respectively. The comparison of necessary friction coefficient and

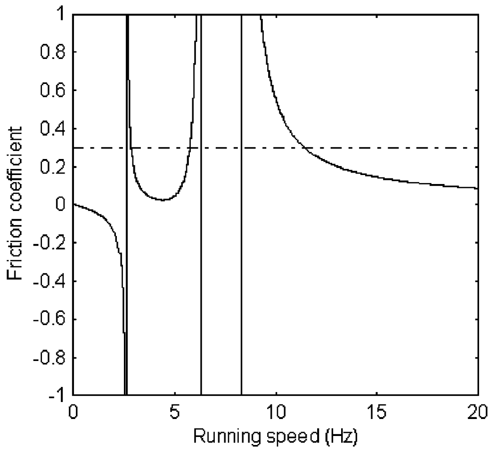

Figure 6. Friction coefficient versus rotor speed for aluminum stator.

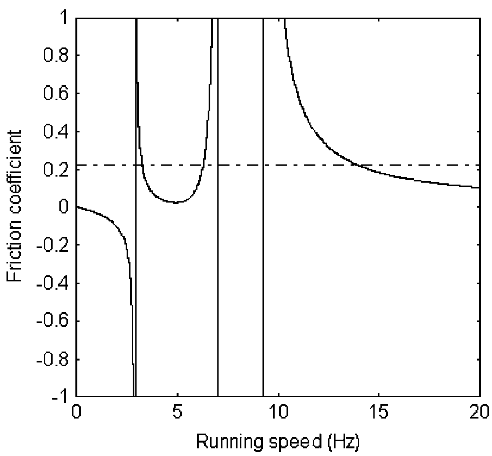

Figure 7. Friction coefficient versus rotor speed for acryl stator.

experimental results shows that the static friction coefficient should be 0·30 for aluminum stator and 0·22 for acryl stator respectively. For this comparison, three points of the onset of backward rolling should coincide with the experimental results. Fortunately, the above reasoning shows good agreement with experimental results.

The above analysis explains the onset of backward rolling as the rotor speed increases, and the end of backward rolling as the rotor speed decreases. But the disappearance of backward slipping during increasing rotor speed has not been explained. A further numerical analysis for the case of backward rolling during increasing the rotor speed including the eccentricity obtains the variation of the normal force, as shown in Figure 8. At a speed of 16·7 Hz, the normal force starts to drop to negative. It means that the assumption of backward slipping is not acceptable any more. But the assumption of backward rolling is satisfied. This shows that the eccentricity can decide where backward rolling disappears.

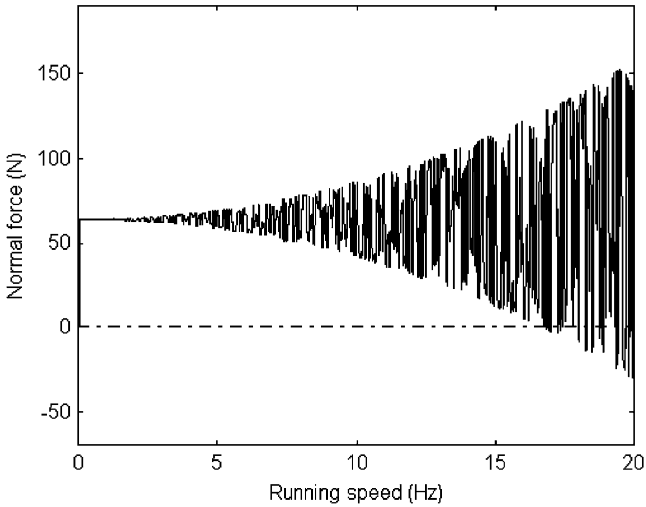

Figure 8. Required normal force due to the eccentricity during backward slipping.

5. CONCLUSION

In order to explain the onset and disappearance of the backward whirling motion of full annular rotor rub, an experiment and the numerical analysis for the experiment were performed. Special cares during the experiment are necessary to obtain backward whirling motions. The conditions of the onset of backward rolling and backward slipping could be predicted on the basis of the experimental results, and theoretical analysis is performed for the experimental findings. There are good agreements between the experimental and the numerical results, and the results show that the friction coefficient between the rotor and stator is the major parameter to decide the onset and disappearance of backward rolling and the eccentricity of the rotor decides the final operating speed of backward slipping when the rotor speed increases.

Conclusively, a rubbing rotor within a small clearance between the rotor and stator presents the same kind of kinetics problem on a circular surface. The rotor can just roll or slip depending on the friction coefficient between the rotor and stator on the inner surface of the stator.

REFERENCES

1. H. F. Black 1968 *Journal of Mechanical Engineering Science* **10**, 1–12. Interaction of a whirling rotor with a vibrating stator across a clearance annulus.
2. F. F. Erich and J. J. O'Connor 1966 *American Society of Mechanical Engineers Paper WA/MD*-8. Stator whirl with rotor in bearing clearance.
3. A. Muszynska 1984 ImechE **C281/84**, 327–335. Partial lateral rotor to stator rubs.
4. Y. S. Choi and S. T. Noah 1987 *Journal of Vibration, Acoustics, Stress and Reliability in Design* **169**, 255–261. Nonlinear steady state response of a rotor-support system.
5. S. H. Crandall, A. Ligener and W. Zhang 1990 *Proceedings of the 12th International Conference on Nonlinear Oscillations, Cracow*, September 27. Backward whirl due to rotor–stator contact.
6. Y. S. Choi 1994 *KSME Journal* **8**, 404–413. Dynamics of rotor rub in annular clearance with experimental evaluation.