Journal of Sound and Vibration (2002) **258**(2), 345–357
doi:10.1006/jsvi.5188, available online at http://www.idealibrary.com on IDEAL®

GLOBAL CONTROL OF STRUCTURAL VIBRATION USING MULTIPLE-TUNED TUNABLE VIBRATION NEUTRALIZERS

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(*Received* 10 *January* 2001, *and in final form* 11 *March* 2002)

Tunable vibration absorbers are used to control vibration due to time-varying harmonic disturbances. Either vibration which is local to the neutralizer, or global vibration of the host structure can be chosen as the quantity to be suppressed. In this paper, the latter is the subject of investigation, but using multiple neutralizers rather than a single device. It is shown that by positioning these devices carefully, the global vibration of a structure (as characterized by its kinetic energy) can be effectively reduced at each single frequency in the frequency range of interest, and is comparable to the performance of active control. A methodology on how to correctly position the devices, an on how to determine their optimum mass is suggested.

1. INTRODUCTION

Since its inception by Ormondroyd and Den Harthog [1] the vibration neutralizer (absorber) has been used to suppress vibrations in many applications, examples of which are described by Hunt [2]. The device has mainly been used to reduce vibration at troublesome resonance frequencies, but more recently it has been used to attenuate the response of structures at single forcing frequencies [3]. In this latter case, the optimum parameters of the neutralizer are quite different to those described in reference [1]. A relatively new innovation has been to make the neutralizer adaptive so that it can track changes in the forcing frequency [4]. The stiffness of the neutralizer is adjusted, so that its natural frequency is always coincident (as far as possible) with the forcing frequency. Several methods to tune the natural frequency of the neutralizer have been described, for example references [5, 6].

Whilst some applications involve the use of a neutralizer to control vibration of a structure at the point where it is attached, other applications require the neutralizer to control sound radiation from a structure and hence global rather than vibration local to the device [7–11]. A fundamental study into the optimum parameters and positioning of a single neutralizer to control global vibration of a beam was studied by Brennan and

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Dayou [12], and this paper is an extension of this work. An investigation into the effectiveness of multiple-tuned tunable vibration neutralizers in reducing the global response of a structure is described. Procedures to determine their positions and optimum characteristics in terms of mass and damping ratios are suggested. It should be noted that single-frequency excitation is considered throughout the paper.

The paper is arranged in five sections. Following this introduction, section 2 describes the formulation that couples the structure to multiple neutralizers, together with some simulations to illustrate their effect on the vibration of the structure. Neutralizer location and optimization are discussed in section 3, and a comparison between the use of multiple-tuned tunable vibration neutralizers and active control is presented in section 4 before the paper is concluded with some conclusions in section 5.

2. PROBLEM FORMULATION

A model that describes the behavior of a structure with multiple vibration neutralizers attached has been formulated by Fuller and Maillard [13] for a cylindrical structure and by Brennan and Dayou [12] for a general structure. This model is briefly described here for convenience, but the reader is referred to reference [12] for details. Figure 1 shows a structure with J neutralizers attached. The displacement of the host structure at any point can be written in terms of a finite number of M modes as

$$w(x) = \Phi^{\mathrm{T}}(x)\mathbf{q}, \tag{1}$$

where $w(x)$ is the displacement at location x, $\Phi(x)$ is the M-length vector of the normalized mode shapes and $\mathbf{q}$ is the M-length vector of modal displacement amplitudes. Superscript T denotes the transpose of the vector and $e^{j\omega t}$ the time dependence is suppressed for clarity. The vector of modal amplitudes, $\mathbf{q}$ is given by

$$\mathbf{q} = \mathbf{A}[\boldsymbol{g} + \mathbf{\Phi}\boldsymbol{f}], \tag{2}$$

where $\mathbf{A}$ is the $M \times M$ diagonal matrix of complex modal amplitudes, $\mathbf{g}$ is the M-length vector of primary forces acting on the host structure which means it can accommodate multiple sources, (in Figure 1 only one primary force f_p is shown) $\mathbf{\Phi}$ is the $M \times J$ matrix of modal amplitudes of the host structure where the entry ϕ_{mj} is the mth modal amplitude at the jth neutralizer position and $\mathbf{f}$ is the J-length vector of forces applied to the structure by the neutralizers. This vector of forces can be written in terms of the dynamic stiffness matrix of the neutralizers, i.e., $\mathbf{f} = -\mathbf{K}\mathbf{w}(x_j)$, where $\mathbf{K}$ is a diagonal $J \times J$ dynamic stiffness matrix of the neutralizers and $\mathbf{w}(x_j)$ is the J-length displacement vector of the host structure evaluated at the location of the j-th neutralizer. The dynamic stiffness of the j-th

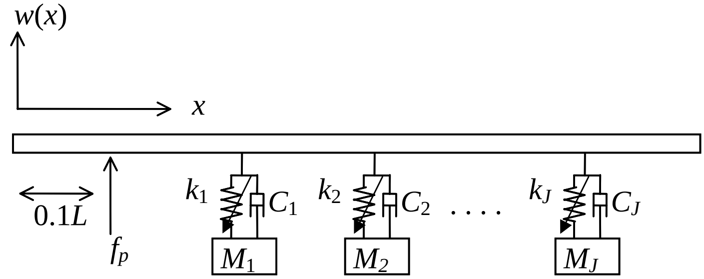

Figure 1. Multiple tunable vibration neutralizers attaced on a general structure.

neutralizer is given by [4]

$$K_j = -m_j\omega^2\left[\frac{1 + \mathrm{i}2\zeta_j\alpha_j}{1 - \alpha_j^2 + \mathrm{i}2\zeta_j\alpha_j}\right], \tag{3}$$

where m_j, k_j and ζ_j are the neutralizer's mass, stiffness and damping ratio respectively. $\alpha_j = \omega/\omega_j$ is the tuning ratio of the jth neutralizer where $\omega_j = (k_j/m_j)^{1/2}$ is the neutralizer's natural frequency. Using equation (1), the vector **f** can also be written as

$$\mathbf{f} = -\mathbf{K}\mathbf{\Phi}^{\mathrm{T}}\mathbf{q}. \tag{4}$$

Combining equations (2) and (4) gives the modal amplitude of the host structure with multiple neutralizers attached,

$$\mathbf{q} = \left[\mathbf{I} + \mathbf{A}\mathbf{\Phi}\mathrm{K}\mathbf{\Phi}^{\mathrm{T}}\right]^{-1}\mathbf{A}\mathbf{g}. \tag{5}$$

Equation (5) can be used to calculate the time-averaged kinetic energy of the host structure, which is given by [14]

$$KE = \frac{M_s\omega^2}{4}\mathbf{q}^{\mathrm{H}}\mathbf{q}, \tag{6}$$

where superscript H denotes the Hermitian transpose and M_s is the mass of the host structure.

To illustrate how the application of multiple neutralizers affects the kinetic energy of the host structure some simulations are presented. For simplicity, a simply supported steel beam is considered as a host structure with a dimension of $1\,\mathrm{m} \times 0{\cdot}0381\,\mathrm{m} \times 0{\cdot}00635\,\mathrm{m}$. It was shown in reference [12] that a key parameter in the use of a neutralizer to control global vibration is the ratio μ/ζ, where μ is the mass ratio of the neutralizer to the host structure and ζ is the neutralizer damping ratio. μ_j/ζ_j for each of the j neutralizers in the simulation presented below is fixed at 40 with $\zeta_j = 0{\cdot}001$. For convenience, a single primary force is used to investigate the use of multiple neutralizers, although the theory presented in this paper is applicable for multiple primary sources. The force is positioned at $0{\cdot}1L$, where L is the length of the beam.

It has been shown previously by the authors [12], that the performance of a single-tuned neutralizer is comparable with feed-forward active control in some frequency ranges despite increasing the kinetic energy of the structure in other frequency ranges. The frequencies at which an increase in the kinetic energy occurs are close to resonance frequencies of the coupled system of the host structure and neutralizer. Applying multiple-tuned neutralizers is found to have a similar effect. However, at frequencies away from the new resonances, application of more than a single-tuned neutralizer results in a greater reductions in kinetic energy compared to when only one neutralizer is used. This is clearly shown in Figure 2, which shows the kinetic energy of the beam when one, two and three neutralizers are attached over a frequency range with the neutralizers tuned, so that $\alpha = 1$ *at each frequency*. In Figure 2(a), it can be seen that when a single-tuned neutralizer is applied at $x = L/2$, no reduction is observed in the kinetic energy of the beam at the second natural frequency, as this point coincides with the nodal point of the beam. However, a reduction of about 50 dB at this frequency (second beam natural frequency) is observed in Figure 2(a), when the second neutralizer is attached at $x = L/3$. There is a further reduction of about 10 dB when the third neutralizer is attached at $x = L/4$ as shown in Figure 2(b). It can be seen in Figure 2(c), that if the third-tuned neutralizer is attached at $x = L/5$ instead of $x = L/4$, an additional reduction of about 15 dB instead of 10 dB can be achieved at this frequency.

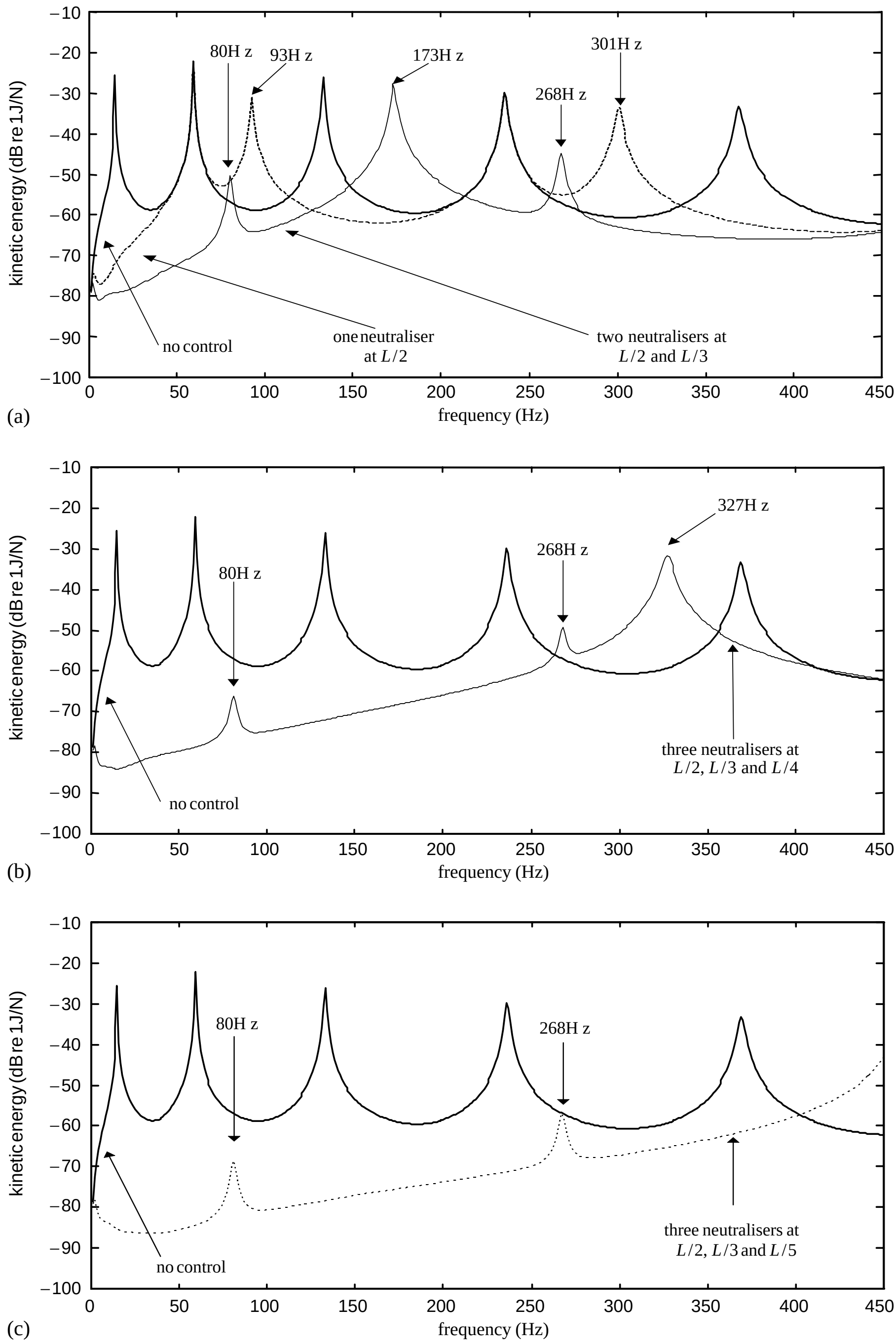

Figure 2. Kinetic energy of a beam with multiple-tuned neutralizers. μ_j/ζ_j for all neutralizers is fixed at 40 with $\zeta_j = 0{\cdot}001$, the primary force is applied at $0{\cdot}1L$. (a) Tuned neutralizers at $L/2$, and at $L/2$ and $L/3$ respectively. (b) Three tuned neutralizers at $L/2$, $L/3$ and $L/4$. (c) Three tuned neutralizers at $L/2$, $L/3$ and $L/5$.

Further examination of Figure 2 shows that the application of an additional tuned neutralizer shifts the new resonance to another frequency. Application of the second neutralizer at $x = L/3$, for example, shifts the new resonances from 93 and 301 Hz to 80, 173 Hz and 268 Hz. Similar results are observed when the third neutralizer is applied. This shows that the last neutralizer attached plays an important role in determining the frequency at which the new resonances occurs. This is clearly shown in Figures 2(b) and 2(c), where application of the third neutralizer at $x = L/4$ results in new resonances at 80, 268 and 327 Hz, but when the third neutralizer is moved to $x = L/5$, the new resonances occur at 80 and 268 Hz, and the third new resonance is moved outside the frequency range of interest.

From the observations discussed above, there are two important questions to be answered:

(1) How to determine the locations for the multiple-tuned neutralizers, so that the new resonance will not occur in the frequency range of interest?
(2) What is the value for μ_j/ζ_j for each neutralizer to be selected?

The following section discusses these issues. However, since the purpose in this investigation is to reduce the kinetic energy of the structure over a range of frequencies, optimization of the neutralizer locations is not carried out as each frequency may require a different optimum location.

3. LOCATING AND OPTIMIZING THE NEUTRALIZERS

3.1. CHOOSING THE NEUTRALIZER LOCATIONS

The procedure formulated here is aimed at shifting the new resonance frequencies outside the frequency range of interest, so that there is no increase in the kinetic energy of the host structure within this range.

The following procedure is to be followed when using J tuned neutralizers.

(1) Choose the location for attaching the (J-1)th, (J-2)th,...., and first neutralizers on the host structure. No specific guideline to be followed on selecting the locations for these neutralizers.
(2) Choose a location on the structure for attaching the Jth neutralizer.
(3) "Test" this location to determine if a new resonance occurs in the frequency range of interest. If this happens, then find another location and do the "test" again until no new resonances are found in the frequency range of interest.
(4) Find the optimum mass and damping ratios for each neutralizer (see later in this section).

The "test" to determine the appropriate location for the Jth neutralizer can be done analytically but will probably involve measurements on a real structure. The analytical determination is as follows. The modal amplitudes of the simply supported beam with J neutralizers attached to it is given by equation (5). Setting $\mathbf{g}$ to zero gives the free vibration of the beam, which after rearrangement gives

$$\left[\mathbf{I} + \mathbf{A\Phi K \Phi}^{\mathrm{T}}\right]\mathbf{q} = 0 \tag{7}$$

which means that

$$\mathbf{\Phi}^{\mathrm{T}}\mathrm{A}\mathbf{\Phi}\mathrm{K} = -\mathbf{I} \tag{8}$$

or

$$\mathbf{\Phi}^{\mathrm{T}}\mathbf{A\Phi} = -\mathbf{K}^{-1}. \tag{9}$$

Since all neutralizers are tuned, the moduli of the elements in $\mathbf{K}^{-1}$ are very small. Therefore, at the new resonances, the determinant of $|\mathbf{\Phi}^{\mathrm{T}}\mathbf{A}\mathbf{\Phi}|$ must be very small and hence the new resonances occur when

$$\det\left[\mathbf{\Phi}^{\mathrm{T}}\mathbf{A}\mathbf{\Phi}\right] = \min. \tag{10}$$

Equation (10) may have several minima over the frequency range of interest which means that several new resonances may occur.

The diagonal terms of $\left[\mathbf{\Phi}^{\mathrm{T}}\mathbf{A}\mathbf{\Phi}\right]$ are the point receptances of the host structure at each neutralizer location and the off-diagonal terms are the transfer receptances between neutralizer locations. Since these receptances can be measured, the procedure described above can be implemented practically. This can be done by forming a receptance matrix from the measurements of the point and transfer receptance of the host structure at the neutralizer locations. The new resonances occur at the minima of the determinant of the modulus of the receptance matrix.

If the determinant of the modulus of the receptance matrix has a minimum in the frequency range of interest, a new location for the final neutralizer has to be selected. For this new location, the point and transfer receptances are again measured until the appropriate location is found where the determinant of the modulus of the receptance matrix has no minimum in the frequency range of interest. There may, however, be a situation where the new resonance cannot be shifted outside of the frequency range of interest by moving only one neutralizer to a new location. In this case, another neutralizer may need to be placed at a new location.

As an example, the determinants of the moduli of the receptance matrices for the configurations considered in Figure 2 are plotted in Figure 3. The minima are the new resonances and the maxima are the old resonance frequencies of the beam. It should be noted that for the case when a single-tuned neutralizer is located at $L/2$, the natural frequency when the neutralizer location coincides with the nodal point is not shown. Comparison of Figures 2 and 3 confirms that the minima in Figure 3 coincide with the new resonances evident in Figure 2.

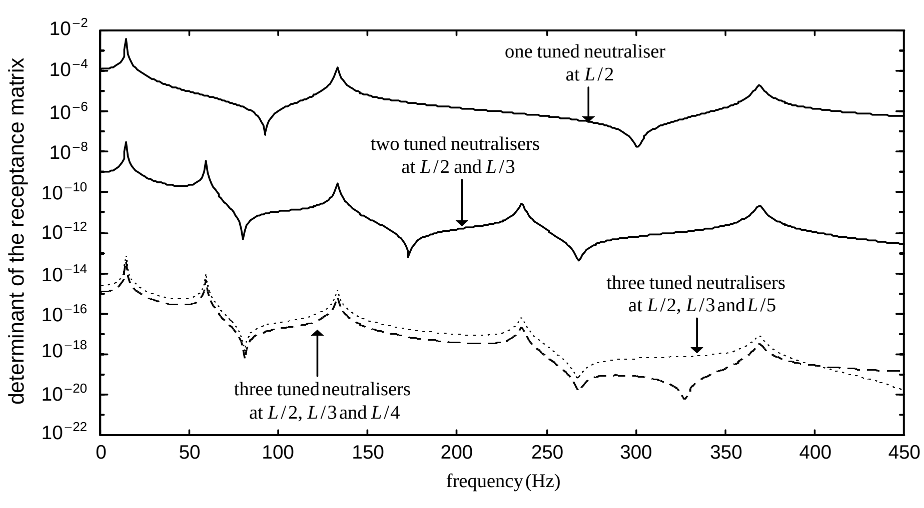

Figure 3. Determination of the new resonance frequencies when multiple-tuned neutralizers are fitted on the beam. The minima are the new resonances.

The effect of changing the position of the last neutralizer to be attached is clearly seen in Figures 2(b) and 2(c). When the neutralizer, initially located at $L/4$ (Figure 2(b)), is shifted to $L/5$ (Figure 2(c)), the resonance frequency at 327 Hz is moved to a much higher frequency, outside the frequency range of interest. This corresponds to step 3 in the procedure given above.

Although the optimization of the neutralizers location is outside of the scope of this paper, it is worthwhile to mention that for a disturbance in the form of point forces, it is best that the neutralizers be applied at the location of these forces. This in principle will completely neutralize the vibration of the host structure as discussed by the authors in reference [12]. If access to these locations are not possible, or if the disturbance is in distributed form, one may have to choose locations where no new resonance frequency is generated using the above procedures, which may be considered as their optimum locations.

3.2. DETERMINING OPTIMUM NEUTRALIZER MASS AND DAMPING RATIOS

It has been shown by the authors [12] that if a single-tuned neutralizer with optimum mass and damping ratios is fitted, then the structure is effectively pinned at the neutralizer location. The objective here is to determine the mass of the neutralizer that pins the structure at the neutralizer location when more than one neutralizer is fitted; this gives the 'optimum' neutralizer mass.

The equation that describes the behavior of the structure when more then one neutralizer is attached contains a fully populated matrix, and is difficult to simplify. Therefore, an analytical formulation to determine the optimum neutralizer masses when more than one neutralizer is used is not presented here, but rather a numerical method is used. The optimum mass of each neutralizer is determined one by one after their appropriate locations have been identified as described above. The following procedure is suggested.

(1) Pin the structure at the 2nd, 3rd,..., jth,..., and Jth neutralizer locations. This can be done by first setting μ_2/ζ_2, μ_3/ζ_3, ..., μ_j/ζ_j, and μ_J/ζ_J, for each neutralizer to be very large. Note that μ_j is the ratio between the jth neutralizer mass to the host structure mass and ζ_j is the damping ratio of the jth neutralizer.
(2) Calculate the kinetic energy of the structure as a function of μ_1/ζ_1. The optimum μ_1/ζ_1 is at the point where the kinetic energy of the structure has converged [12].
(3) Set the dynamic stiffness of the first neutralizer to be optimum and then calculate the kinetic energy of the structure as a function of μ_2/ζ_2, with μ_j/ζ_j of the 3rd,..., jth,..., and Jth neutralizers set to be very large. Again, the optimum value of μ_2/ζ_2 is taken when the kinetic energy has converged to a minimum.
(4) Repeat step 3 for each additional neutralizer.

The pinned condition discussed in the above procedure can be clearly seen in Figure 4; when μ_j/ζ_j of the neutralizers is set to be very high, the displacement at the neutralizer application points is reduced to zero and no further changes can be achieved in the modal displacement amplitude or kinetic energy of the beam even if the masses of the neutralizers are increased.

Figure 5(a) shows how the optimum mass and damping ratios of the tuned neutralizers fitted to the simply supported beam at $L/2$, $L/3$ and $L/4$ were determined using the procedure suggested above at the second natural frequency of the beam. The pinned conditions at the neutralizer attachment points were achieved by setting the mass ratio

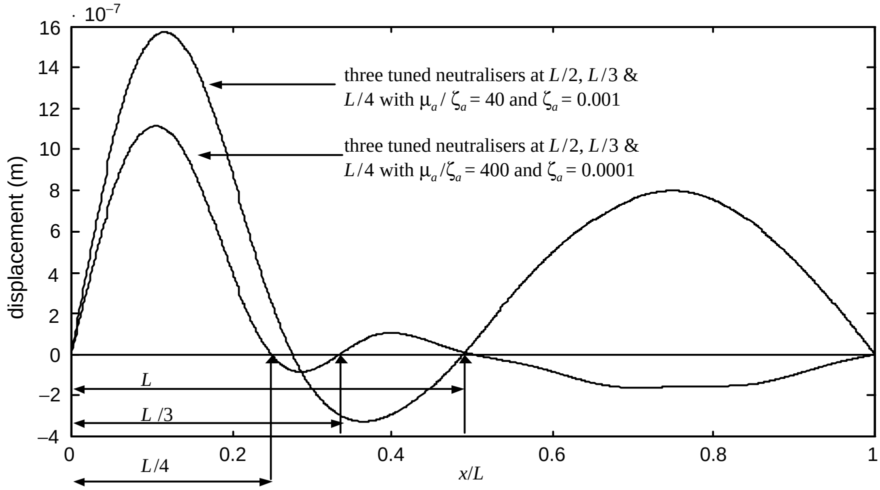

Figure 4. The displacement of the beam at its second natural frequency with three neutralizers attached when μ_j/ζ_j is set quite low and very high. Primary force applied at 0·1L.

$\mu_j = 10$ and $\zeta_j = 0.001$ for all neutralizers. Similar results are shown in Figure 5(b), but at the third natural frequency of the beam. It can be seen that the optimum mass and damping ratios of the neutralizers are lower in this case. This is because the force from the neutralizer increases with frequency and is thus larger at higher frequencies.

Figure 5(a) shows that when the beam is pinned at $x = L/2$ and $L/3$, the maximum reduction in its kinetic energy using the third neutralizer is achieved when $\mu_3/\zeta_3 \approx 200$. However, when $\mu_3/\zeta_3 = 20$, the reduction is only about 3dB less than when it is equal to 200. This means that increasing the third neutralizer mass by a factor of 10 will only reduce the kinetic energy of the beam by a factor of 2. However, the same level of reduction can be achieved (reduction by a factor of 2) as μ_3/ζ_3 increases from 10 to 20. This shows that as the kinetic energy of the structure approaches its minimum value, the additional reduction that can be achieved gets smaller. Since adding excessive mass is not desirable, it is important to know what the minimum achievable kinetic energy of a structure is, so that as this value is approached it can be decided if further reduction is worth the additional neutralizer mass.

3.3. DETERMINING THE MINIMUM ACHIEVABLE KINETIC ENERGY OF THE HOST STRUCTURE

The minimum achievable kinetic energy of the host structure when using multiple-tuned neutralizers can be determined as follows. The vector of the modal amplitudes of the host structure with a single neutralizer attached is given in equation (5), which can be rewritten as [15]

$$\mathbf{q}^{(1)} = \mathbf{A}^{(1)}\mathbf{g}, \tag{11}$$

where the superscript (1) shows that only one neutralizer is attached, and

$$\mathbf{A}^{(1)} = \left[\mathbf{I} - \frac{K_1}{D}\mathbf{A}\Phi(x_1)\Phi^{\mathrm{T}}(x_1)\right]\mathbf{A}, \tag{12}$$

where $D = 1 + K_1\Phi^{\mathrm{T}}(x_1)\mathbf{A}\Phi(x_1)$. When the second neutralizer is attached, the vector of modal amplitudes can be written in terms of the modified vector of complex modal

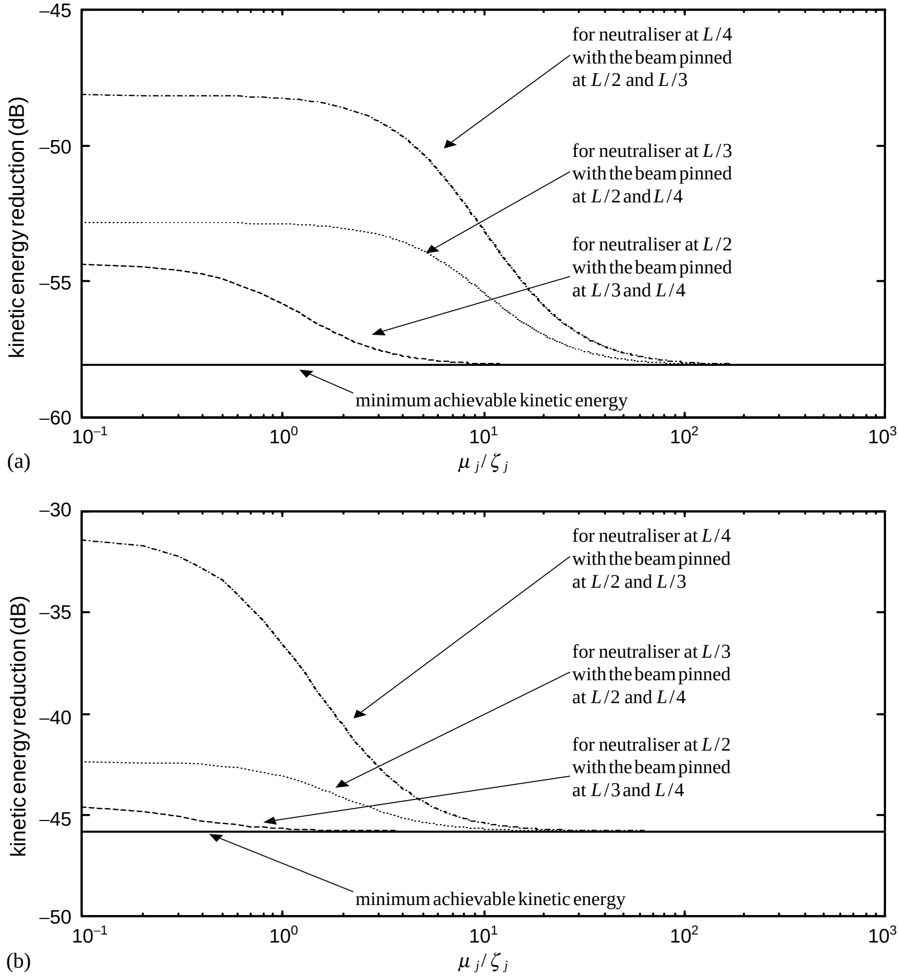

Figure 5. Determination of the optimum mass and damping ratios of the multiple tuned neutralizers at two natural frequencies of the beam. The primary force is applied at $0{\cdot}1L$ and ζ_j for all neutralizers is 0·001. (a) Second natural frequency of the beam. (b) Third natural frequency of the beam.

amplitudes with a single neutralizer attached ($\mathbf{A}^{(1)}$) and the generalized total forces from the primary and the second neutralizer as

$$\mathbf{q}^{(2)} = \mathbf{A}^{(1)}\left[\mathbf{g} - \Phi(x_2)K_2\Phi^{\mathrm{T}}(x_2)\mathbf{q}^{(2)}\right], \tag{13}$$

where $[\mathbf{g} - \Phi(x_2)K_2\Phi^{\mathrm{T}}(x_2)\mathbf{q}^{(2)}]$ is termed the generalized total force from the primary force and from the second neutralizer. The superscript (2) shows that there are two neutralizers attached to the host structure. Rearranging this equation gives

$$\mathbf{q}^{(2)} = \left[\mathbf{I} + \mathbf{A}^{(1)}\Phi(x_2)K_2\Phi^{\mathrm{T}}(x_2)\right]^{-1}\mathbf{A}^{(1)}\mathbf{g}, \tag{14}$$

The matrix $[\mathbf{A}^{(1)}\Phi(x_2)K_2\Phi^{T}(x_2)]$ has a rank of one and therefore the same procedure outlined in reference [12] can be applied which after rearrangement results in

$$\mathbf{q}^{(2)} = \left[\mathbf{I} - \frac{K_2}{D^{(1)}}\mathbf{A}^{(1)}\Phi(x_2)\Phi^{\mathrm{T}}(x_2)\right]\mathbf{A}^{(1)}\mathbf{g}, \tag{15}$$

where $D^{(1)} = 1 + K_2\Phi^{\mathrm{T}}(x_2)\mathbf{A}^{(1)}\Phi(x_2)$. Following the above steps, the vector of the modal amplitudes when J neutralizers are used can be written as

$$\mathbf{q}^{(J)} = \mathbf{A}^{(J)}\mathbf{g}, \tag{16}$$

where

$$\mathbf{A}^{(J)} = \left[\mathbf{I} - \frac{K_J}{D^{(J-1)}}\mathbf{A}^{(J-1)}\Phi(x_J)\Phi^{\mathrm{T}}(x_J)\right]\mathbf{A}^{(J-1)} \tag{17}$$

and

$$D^{(J-1)} = 1 + K_J\Phi^{\mathrm{T}}(x_J)\mathbf{A}^{(J-1)}\Phi(x_J). \tag{18}$$

It can be seen that when only one neutralizer is attached, that is when $J=1$, the index of the term $\mathbf{A}$ in equation (16) becomes 1 and therefore the vector of the modal amplitudes is simply the same as given in equation (12). It should be noted that the steps taken from Equations (16) to (18) must be carried out successively for each neutralizer.

For the Jth neutralizer, the neutralizer mass term is found only in the ratio of $K_J/D^{(J-1)}$ in equation (17). Therefore, when this ratio has converged, the kinetic energy of the beam also converges. This can only be true when $|K_J\Phi^{\mathrm{T}}(x_J)\mathbf{A}^{(J-1)}\Phi(x_J)| \gg 1$, so that

$$D^{(J-1)} \approx K_J\Phi^{\mathrm{T}}(x_J)\mathbf{A}^{(J-1)}\Phi(x_J). \tag{19}$$

As a result, vector of the modal amplitudes of the structure can now be written as

$$\mathbf{q}^{(J)} = \left[\mathbf{I} - \frac{1}{\Phi^{\mathrm{T}}(x_J)\mathbf{A}^{(J-1)}\Phi(x_J)}\mathbf{A}^{(J-1)}\Phi(x_2)\Phi^{\mathrm{T}}(x_2)\right]\mathbf{A}^{(J-1)}\mathbf{g}, \tag{20}$$

where the $(J-1)$th vector of complex modal amplitudes is given by

$$\mathbf{A}^{(J-1)} = \left[\mathbf{I} - \frac{1}{\Phi^{\mathrm{T}}(x_{J-1})\mathbf{A}^{(J-2)}\Phi(x_{J-1})}\mathbf{A}^{(J-2)}\Phi(x_{J-1})\Phi^{\mathrm{T}}(x_{J-1})\right]\mathbf{A}^{(J-2)}. \tag{21}$$

Substituting equation (20) into equation (6) for the kinetic energy of the beam gives the minimum achievable kinetic energy with J-tuned neutralizers attached to the host structure. Again, it should be noted that steps taken from equations (16) to (18) must be carried out successively for each neutralizer.

Figure 6 shows the minimum achievable kinetic energy of the beam when the application of the tuned neutralizers causes pinned conditions at their attachment points $L/2$, $L/3$ and $L/4$, and at $L/2$, $L/3$ and $L/5$. It can be seen that the minimum achievable kinetic energy is also a function of the location of the Jth neutralizer. This shows that proper selection of the neutralizer attachment points not only determines the frequencies where the new resonances occur, but also the reduction in the kinetic energy that can be achieved.

If a theoretical model of the structure is not available then a model has to be constructed using measured data, which could be done, for example, using experimental modal analysis. The procedure described above could then be used.

One of the important issues that may arise here is the number of neutralizers to be used. This in principal is the choice of the user. This paper has set guidelines on where to apply these devices and what are the appropriate parameters of the neutralizers. With this information in hand, the minimum achievable kinetic energy can be then predicted. If

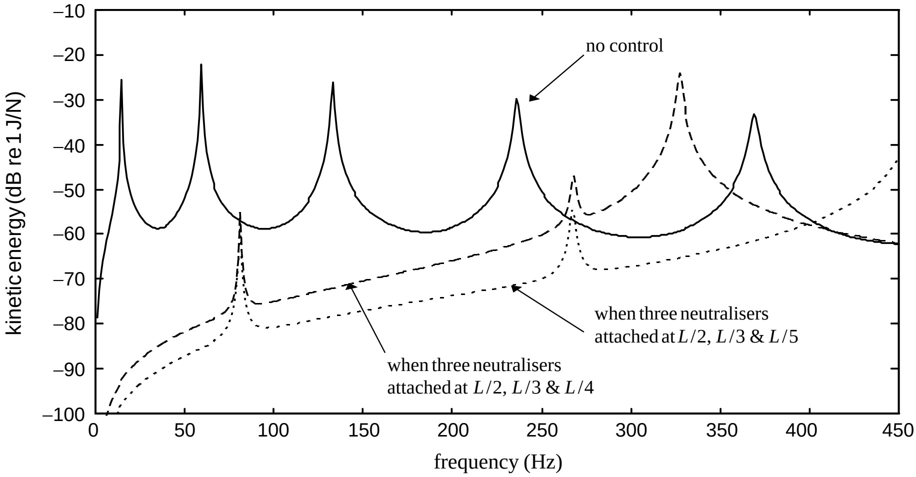

Figure 6. Minimum achievable kinetic energy of the beam when it is pinned at $L/2$, $L/3$ and $L/4$ and at $L/2$, $L/3$ & $L/5$ with the application of multiple-tuned neutralizers. A harmonic point primary force is applied at $0{\cdot}1L$.

further reduction of kinetic energy is required, then the number of the neutralizers can be increased accordingly until the requirement is achieved.

4. COMPARISON WITH ACTIVE CONTROL

To evaluate the performance of the multiple-tuned neutralizers, the kinetic energy of the host structure when using optimal feedforward active control is used as a benchmark. This active control technique, which illustrates the performance limits of any active control system for a given set of actuators [14] is also assumed to be the performance limit of control using tunable vibration neutralizers. It involves replacing the neutralizers with idealized forces, and the optimal active control solution is the one where the appropriate magnitude and phase of each force is chosen to minimize the kinetic energy of the host structure at each frequency.

The issue here is to determine how effective multiple-tuned tunable vibration neutralizers are in reducing the global vibration of the host structure, and how they compare with active control. Figure 7(a) shows a comparison in kinetic energy of the simply supported beam, when the control devices are located at $L/2$, $L/3$ and $L/4$. The tuned neutralizers are considered to operate at their maximum capabilities, that is when the minimum kinetic energy of the host structure is achieved as discussed in section 3. It can be seen from this figure that except at 80 Hz, the maximum reduction for frequencies below 150 Hz when using tuned vibration neutralizers is within 5 dB of that when active control devices at the same locations are used. Above this frequency, the use of tuned multiple vibration neutralizers is much less effective. The increase in kinetic energy at 250 Hz and above is observed because there are two new resonance frequencies in this range and they are quite close to each other.

When one of the tuned neutralizers, positioned at $L/4$, is repositioned at $L/5$, the frequency range where the effectiveness of the neutralizers comparable to active control increases. It can be seen in Figure 7(b) that except at 80 Hz, the difference is less then 3 dB for frequencies below 250 Hz.This shows that the location of a neutralizer plays an important role in determining its effectiveness.

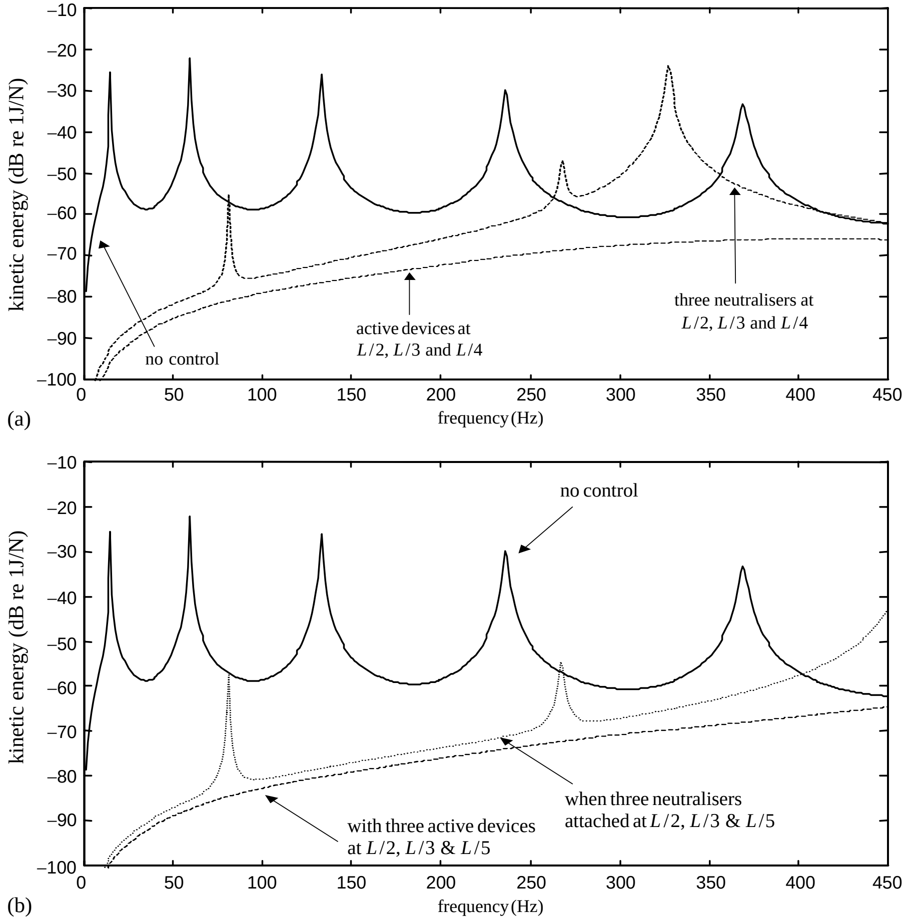

Figure 7. Minimum achievable kinetic energy of the beam when it is pinned at $L/2$, $L/3$ & $L/4$ and at $L/2$, $L/3$ & $L/5$ compared with optimal feed-forward active control technique with active devices places at the same locations. A harmonic point primary force is applied at $0{\cdot}1L$. (a) Control devices located at $L/2$, $L/3$ and $L/4$. (b) Control devices located at $L/2$, $L/3$ and $L/5$.

5. CONCLUSIONS

The use of multiple-tuned vibration neutralizers to control the global response of a structure has been investigated in this paper. It was found that although the application of such devices may increase the kinetic energy of the host structure at some frequencies, there is a way to prevent this from occurring within the frequency of interest. This method involves the careful positioning of the neutralizers, which is also important in optimizing their performance. A way of checking for reasonable neutralizer positions in practice by taking receptance measurements on the host structure, has also been described. A method to determine the optimum mass for the neutralizers has been proposed, along with a simple way to determine the minimum achievable kinetic energy of the host structure for a given neutralizer array. The attenuation in kinetic energy of the host structure at the tuned frequencies of the neutralizers is a non-linear function of the mass ratio between the individual neutralizers and the host structure. Much larger neutralizer masses are required to achieve reduction in the host-structure kinetic energy as the minimum achievable kinetic energy is approached.

Formulations in this paper have been kept in their general form. Therefore, although simulations have been carried out on a simply supported beam as an example, the procedures described in this paper can be directly used to any type of structure. With this great flexibility, the proposed method can for example, be used to determine where to apply the multiple-tuned tunable neutralizers on a passengers aircraft. This is to reduce the induced vibration from its propeller to the passenger's cabin, which as a result may reduce the interior noise of the aircraft.

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