Journal of Sound and Vibration (2002) **258**(2), 385–397
doi:10.1006/jsvi.5105, available online at http://www.idealibrary.com on IDEAL®

A NEW INFINITE IMPULSE RESPONSE FILTER-BASED ADAPTIVE ALGORITHM FOR ACTIVE NOISE CONTROL

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(*Received* 6 *July* 2001, *and in final form* 19 *December* 2001)

1. INTRODUCTION

Compared to the finite impulse response (FIR) filter, the infinite impulse response (IIR) filter can be used to model an ANC system better with much fewer coefficients due to its inherent zero-pole structure. Hence, the computation load reduces, and the system performance improves [1]. However, the filtered-U LMS (FULMS) algorithm [2], an IIR filter-based algorithm commonly used so far, cannot ensure global convergence [3]. In this paper, we propose a new algorithm based on an IIR filter which ensures global convergence with slightly increased computation load.

Figure 1(a) depicts a typical ANC situation where the primary noise signal $d(n)$ from the noise source is to be cancelled at the location of the error sensor. The noise source is available from a reference sensor. The idea is to find the optimal filter (controller) $W(z)$ such that the noise source signal filtered by this optimal filter and sent out at the loudspeaker interferes with the primary noise signal in such a way that at the location of the error sensor a zone of silence is generated.

The transfer functions in Figure 1(a) are defined as follows:

$G(z)$: The transfer function from the noise source to the reference sensor. $G(z) = 1$ (i.e., perfect correlation) is assumed in this paper for convenience.

$P(z)$: The transfer function of the primary path (from the noise source to the error sensor).

$S(z)$: The transfer function of the secondary path (from the output of the filter to the error sensor).

$F(z)$: The transfer function of the feedback path (from the output of the filter to the reference sensor).

The block diagram of an IIR filter-based adaptive algorithm is shown in Figure 1(b), where the IIR filter is implemented by two FIR filters. The error signal is fed back to the adaptive algorithm as a control signal in order to adaptively find the optimal coefficients for the IIR filter.

The paper is organized as follows: derivation of the proposed algorithm is presented without acoustic feedback in section 2; convergence analysis is given in section 3; the effects of acoustic feedback on the convergence of the proposed algorithm are discussed in section 4; in section 5 some simulation results are given; section 6 is the summary.

In this paper, the following mixed notation is used: if $H(z) = \sum_{k=-\infty}^{\infty} h_k z^{-k}$ then $H(z)u(n) = \sum_{k=-\infty}^{\infty} h_k u(n-k)$.

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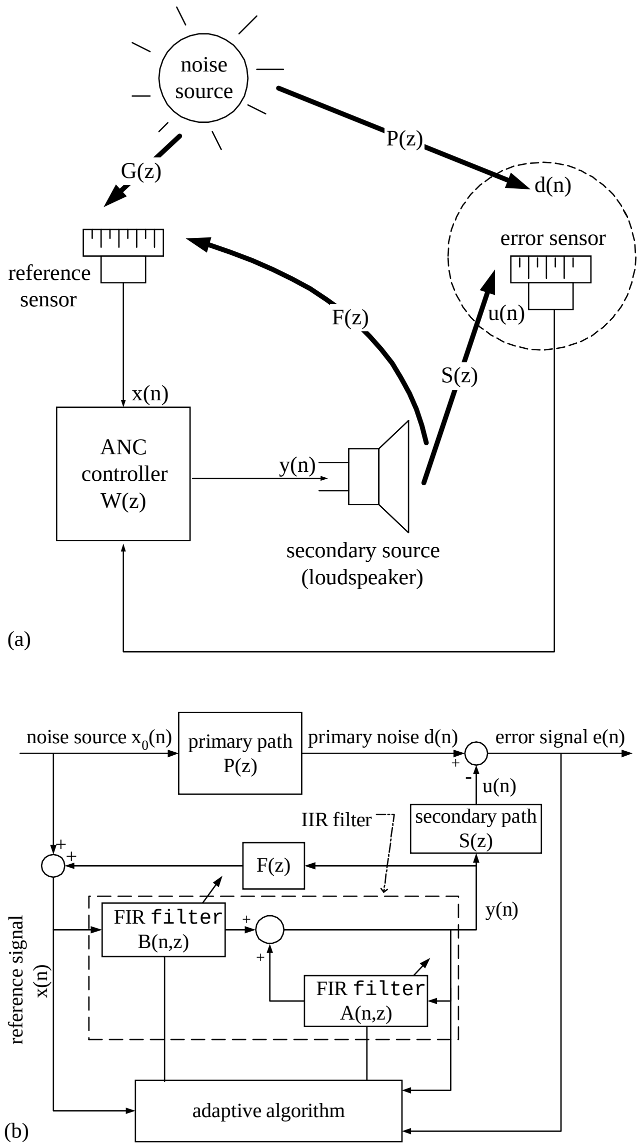

Figure 1. (a) ANC situation; (b) adaptive IIR algorithm for ANC.

2. DERIVATION OF THE NEW ALGORITHM

In this section, no acoustic feedback (i.e., $F(z) = 0$) is assumed. As shown in Figure 1(b), the error signal $e(n)$ can be expressed as

$$e(n) = d(n) - u(n) = d(n) - S(z)y(n) \tag{1}$$

or

$$e(n) = d(n) - S(z)[\mathbf{B}^{\mathrm{T}}(n)\mathbf{X}(n) + \mathbf{A}^{\mathrm{T}}(n)\mathbf{Y}(n)], \tag{2}$$

where $\mathbf{B}(n) = [b_0(n) \;\; b_1(n) \;\; \cdots \;\; b_{N-1}(n)]^{\mathrm{T}}$ represents the coefficients vector of FIR filter B, $\mathbf{A}(n) = [a_1(n) \;\; a_2(n) \;\; \cdots \;\; a_M(n)]^{\mathrm{T}}$ the coefficients vector of FIR filter A, $\mathbf{X}(n) =$

$[x(n)\ \ x(n-1)\ \ \cdots\ \ x(n-N+1)]^{\mathrm{T}}$ the input vector of FIR filter A, $\mathbf{Y}(n) = [y(n-1)\ \ y(n-2)\ \ \cdots\ \ y(n-M)]^{\mathrm{T}}$ the input vector of FIR filter B, N the order of FIR filter B, M the order of FIR filter A. With the assumption of slow adaption, equation (2) can be written as [3]

$$e(n) = d(n) - \mathbf{B}^{\mathrm{T}}(n)(S(z)\mathbf{X}(n)) - \mathbf{A}^{\mathrm{T}}(n)(S(z)\mathbf{Y}(n)) \tag{3}$$

or

$$e(n) = d(n) - \boldsymbol{\theta}^{\mathrm{T}}(n)\boldsymbol{\varphi}(n), \tag{4}$$

where

$$\boldsymbol{\theta}(n) = [b_0(n)\ \ b_1(n)\ \ \cdots\ \ b_{N-1}(n)\ \ a_1(n)\ \ \cdots\ \ a_M(n)]^{\mathrm{T}}, \tag{5}$$

$$\boldsymbol{\varphi}(n) = [S(z)\mathbf{X}^{\mathrm{T}}(n)\ \ S(z)\mathbf{Y}^{\mathrm{T}}(n)]^{\mathrm{T}}. \tag{6}$$

Defining

$$B(n,z) = b_0(n) + b_1(n)z^{-1} + b_2(n)z^{-2} + \cdots + b_{N-1}(n)z^{-(N-1)}, \tag{7}$$

$$A(n,z) = a_1(n)z^{-1} + a_2(n)z^{-2} + \cdots + a_M(n)z^{-M}, \tag{8}$$

equation (2) can be written as

$$e(n) = d(n) - B(n,z)[S(z)x(n)] - A(n,z)[S(z)y(n)]. \tag{9}$$

In the FULMS algorithm, $E\{e^2(n)\}$, the mean square error (MSE) at time n, is adopted as the cost function. Using the LMS method, $\boldsymbol{\theta}(n+1)$ can be calculated as

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) - \frac{\mu}{2}\frac{\partial e^2(n)}{\partial \boldsymbol{\theta}(n)} = \boldsymbol{\theta}(n) - \mu e(n)\frac{\partial e(n)}{\partial \boldsymbol{\theta}(n)}. \tag{10}$$

With Feintuch's assumption [4], one can obtain [3]

$$\frac{\partial e(n)}{\partial \boldsymbol{\theta}(n)} = -\boldsymbol{\varphi}(n). \tag{11}$$

Substituting equation (11) into equation (10) yields

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) + \mu e(n)\boldsymbol{\varphi}(n), \tag{12}$$

which is the expression of the FULMS algorithm. Since $\mathbf{Y}(n)$ is dependent on coefficients vector $\boldsymbol{\theta}(n)$ [1], it can be seen from equation (3) that $E\{e^2(n)\}$ is not a quadratic function of the coefficients and may have multiple local minima. Therefore, the FULMS algorithm may converge to a local minimum if the initial value of the filter coefficients is within the neighborhood of that local minimum [3].

A new cost function $E\{\xi^2(n)\}$ instead of $E\{e^2(n)\}$ is proposed in the new algorithm, where

$$\xi(n) = [1 - A(n,z)]e(n). \tag{13}$$

Let $\mathbf{E}(n) = [e(n-1)\ \ e(n-2)\ \ \cdots\ \ e(n-M)]^{\mathrm{T}}$; equation (13) can be written as

$$\xi(n) = e(n) - \mathbf{A}^{\mathrm{T}}(n)\mathbf{E}(n). \tag{14}$$

Substituting equation (3) into the above equation yields

$$\xi(n) = d(n) - \mathbf{B}^{\mathrm{T}}(n)(S(z)\mathbf{X}(n)) - \mathbf{A}^{\mathrm{T}}(n)(S(z)\mathbf{Y}(n) + \mathbf{E}(n)). \tag{15}$$

Noting that $d(n) = s(z)y(n) + e(n)$, equation (15) becomes

$$\xi(n) = d(n) - \mathbf{B}^{\mathrm{T}}(n)(S(z)\mathbf{X}(n)) - \mathbf{A}^{\mathrm{T}}(n)\mathbf{D}(n), \tag{16}$$

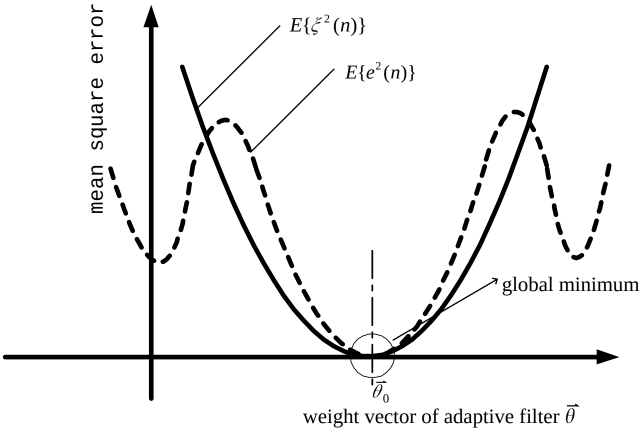

Figure 2. MSE surfaces for $E\{\xi^2(n)\}$ and $E\{e^2(n)\}$.

where $\mathbf{D}(n) = [d(n-1)\ \ d(n-2)\ \ \cdots\ \ d(n-M)]^{\mathrm{T}}$, and equation (16) can be written as

$$\xi(n) = d(n) - \boldsymbol{\theta}^{\mathrm{T}}(n)\boldsymbol{\psi}(n), \tag{17}$$

where $\boldsymbol{\psi}(n) = [S(z)\mathbf{X}^{\mathrm{T}}(n)\ \ \mathbf{D}^{\mathrm{T}}(n)]^{\mathrm{T}}$, and $\boldsymbol{\theta}(n)$ is defined as in equation (5). There are two points of importance worthy of note:

(1) Since $S(z)\mathbf{X}(n)$ and $\mathbf{D}(n)$ are both independent of $\boldsymbol{\theta}(n)$, then from equation (16) $E\{\xi^2(n)\}$ is a quadratic function of the coefficients, and thus has a unique global minimum.

(2) As shown in Appendix A, $E\{\xi^2(n)\} = 0$ if and only if $E\{e^2(n)\} = 0$. So when the coefficient vector is reaching the optimal value such that the primary $d(n)$ is perfectly cancelled [5, 2], both MSEs reach their global minimum.

Therefore, the optimization of $E\{e^2(n)\}$ can be converted into an optimization of $E\{\xi^2(n)\}$. The latter is easier to solve and global convergence can be guaranteed. Figure 2 shows some $E\{\xi^2(n)\}$ and $E\{e^2(n)\}$, where $\boldsymbol{\theta}(n)$ is simply chosen as a one-dimensional variable for the convenience of illustration.

Use the LMS method to optimize $E\{\xi^2(n)\}$; then $\boldsymbol{\theta}(n+1)$ can be calculated as

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) - \frac{\mu}{2}\frac{\partial\xi^2(n)}{\partial\boldsymbol{\theta}(n)} = \boldsymbol{\theta}(n) - \mu\xi(n)\frac{\partial\xi(n)}{\partial\boldsymbol{\theta}(n)}. \tag{18}$$

The above equation can be split into

$$b_k(n+1) = b_k(n) - \mu\xi(n)\frac{\partial\xi(n)}{\partial b_k(n)} \quad (k = 0, 1, \ldots, N-1) \tag{19}$$

and

$$a_k(n+1) = a_k(n) - \mu\xi(n)\frac{\partial\xi(n)}{\partial a_k(n)} \quad (k = 1, 2, \ldots, M). \tag{20}$$

From equation (16) it follows that

$$\begin{aligned}\frac{\partial\xi(n)}{\partial b_k(n)} &= -S(z)\left[x(n-k) + \sum_{j=1}^{M} a_j(n)\frac{\partial d(n-j)}{\partial b_k}\right]\\ &= -S(z)x(n-k) \quad (k = 0, 1, \ldots, N-1)\end{aligned} \tag{21}$$

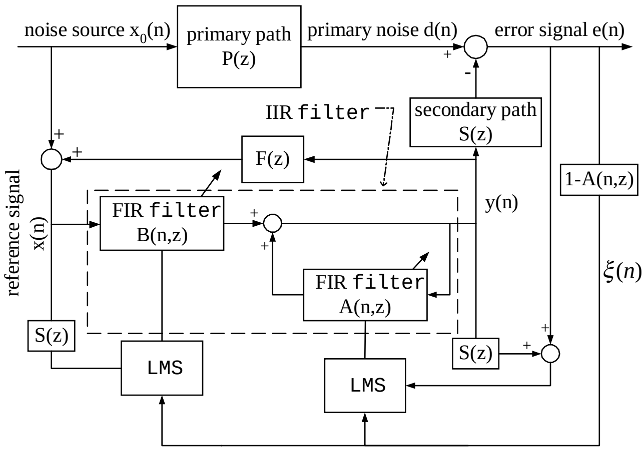

Figure 3. Block diagram of the proposed algorithm.

and

$$\frac{\partial \xi(n)}{\partial a_k(n)} = -\left[d(n-k) + \sum_{j=1}^{M} a_j \frac{\partial d(n-j)}{\partial a_k(n)}\right] = -d(n-k) \quad (k = 1, 2, \ldots, M). \tag{22}$$

Note that $d(n) = S(z)y(n) + e(n)$, and equation (22) becomes

$$\frac{\partial \xi(n)}{\partial a_k(n)} = -[S(z)y(n-k) + e(n-k)] \quad (k = 1, 2, \ldots, M). \tag{23}$$

Substituting equation (21) into equation (19) and equation (23) into equation (20) yields

$$b_k(n+1) = b_k(n) + \mu\xi(n)[S(z)x(n-k)] \quad (k = 0, 1, \ldots, N-1), \tag{24}$$

$$a_k(n+1) = a_k(n) + \mu\xi(n)[S(z)y(n-k) + e(n-k)] \quad (k = 1, 2, \ldots, M) \tag{25}$$

or equivalently

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) + \mu\xi(n)\boldsymbol{\psi}(n). \tag{26}$$

Equations (14), (24) and (25) are expressions of the new algorithm; the block diagram is shown in Figure 3.

To compare the computation complexity of the proposed algorithm with that of the FULMS algorithm, let the secondary path transfer function $S(z)$ be implemented by an FIR filter of order L. The computation complexities of the proposed algorithm and the FULMS algorithm are listed in Tables 1 and 2 respectively. It is seen from Tables 1 and 2 that the computation load of the proposed algorithm is $3M + 2N + 2L + 5$, and that for the FULMS algorithm is $2M + 2N + 2L + 3$. So the complexity of the proposed algorithm increases only slightly compared to that of the FULMS algorithm.

The proposed algorithm is different from the SHARF one, which is proposed by Larimore *et al.* [6] and has been introduced to ANC by Eriksson [2] and Kuo [3]. Using the SHARF algorithm in ANC, the coefficients of the IIR filter are updated as

$$\boldsymbol{\theta}(n+1) = \boldsymbol{\theta}(n) + \mu[C(z)e(n)][\boldsymbol{\varphi}(n)], \tag{27}$$

where $C(z)$ is a moving averaging filter. $C(z)/D(z)$ should be strictly positive real (SPR) in order to guarantee global convergence of the SHARF algorithm, where $D(z)$ represents the system denominator [7]. The main problem of the SHARF algorithm seems to be the

TABLE 1

Computational complexity of the FULMS algorithm

Operation	Mult/Add
$y(n) = \mathbf{B}^{\mathrm{T}}(n)\mathbf{X}(n) + \mathbf{A}^{\mathrm{T}}(n)\mathbf{Y}(n)$	$M + N + 1$
$S(z)\mathbf{X}(n)$	L
$S(z)\mathbf{Y}(n)$	L
$\mathbf{B}(n+1) = \mathbf{B}(n) + \mu e(n)[S(z)\mathbf{X}(n)]$	$N + 1$
$\mathbf{A}(n+1) = \mathbf{A}(n) + \mu e(n)[S(z)\mathbf{Y}(n)]$	$M + 1$
$\sum$	$2M + 2N + 2L + 3$

TABLE 2

Computational complexity of the proposed algorithm

Operation	Mult/Add
$y(n) = \mathbf{B}^{\mathrm{T}}(n)\mathbf{X}(n) + \mathbf{A}^{\mathrm{T}}(n)\mathbf{Y}(n)$	$M + N + 1$
$S(z)\mathbf{X}(n)$	L
$\mathbf{D}(n) = S(z)\mathbf{Y}(n) + \mathbf{E}(n)$	$L + 1$
$\xi(n) = e(n) - \mathbf{A}^{\mathrm{T}}(n)\mathbf{E}(n)$	$M + 1$
$\mathbf{B}(n+1) = \mathbf{B}(n) + \mu e(n)[S(z)\mathbf{X}(n)]$	$N + 1$
$\mathbf{A}(n+1) = \mathbf{A}(n) + \mu e(n)[S(z)\mathbf{D}(n)]$	$M + 1$
$\sum$	$3M + 2N + 2L + 5$

non-existence of a robust practical procedure to define the moving averaging filter $C(z)$ to satisfy the SPR condition. This is a consequence of the fact that the characteristics of $D(z)$ are unknown in practice.

3. CONVERGENCE ANALYSIS

It should be noted that just like many other IIR filter-based adaptive algorithms, the proposed one may suffer instability as some poles of the filter go out of the unit circles [3]. In the following, convergence analysis is performed with the stability being premised. All signals are assumed to be wide-sense stationary.

For given orders M and N, let $\boldsymbol{\theta}^o = [b_0^o \ b_1^o \ \cdots \ b_{N-1}^o \ a_1^o \ \cdots \ a_M^o]^{\mathrm{T}}$ represent the coefficients of the optimal IIR filter, which enable $E\{e^2(n)\}$ to reach the global minimum; then the optimal transfer function is expressed as

$$\frac{B^o(z)}{1 - A^o(z)} = \frac{b_0^o + b_1^o z^{-1} + b_2^o z^{-2} + \cdots + b_{N-1}^o z^{-(N-1)}}{1 - a_1^o z^{-1} - a_2^o z^{-2} - \cdots - a_M^o z^{-M}}. \tag{28}$$

As shown in Figure 4, $d(n)$ can be decomposed as

$$d(n) = d_m(n) + d_u(n), \tag{29}$$

where

$$d_m(n) = S(z)\frac{B^o(z)}{1 - A^o(z)}x(n), \quad d_u(n) = d(n) - d_m(n). \tag{30, 31}$$

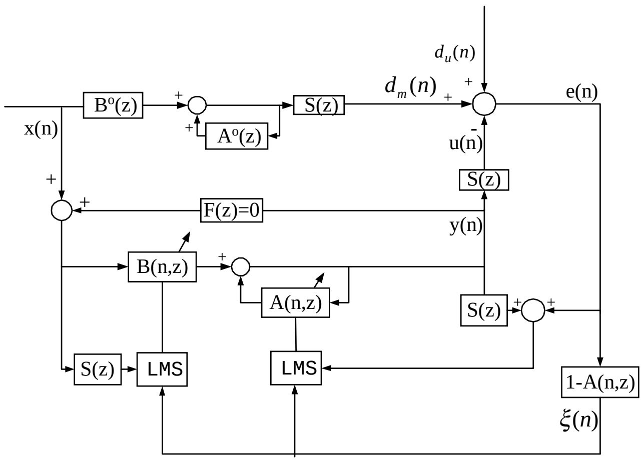

Figure 4. Block diagram of the proposed algorithm with the primary noise $d(n)$ decomposed into the modelled portion $d_m(n)$ and the unmodelled portion $d_u(n)$.

$d_m(n)$ and $d_u(n)$, in some sense, can be, respectively, regarded as the modelled and unmodeled portion of the primary noise $d(n)$ [8]. Obviously, $E\{d_u^2(n)\}$ is equal to the global minimum of $E\{e^2(n)\}$.

3.1. $d_u(n) = 0$

If the adaptive IIR filter has sufficient orders such that the unmodelled portion $d_u(n)$ is equal to zero, then equation (30) can be written as

$$d(n) = S(z)\frac{B^o(z)}{1 - A^o(z)}x(n) \tag{32a}$$

or

$$d(n) = \boldsymbol{\theta}^{o\mathrm{T}}\boldsymbol{\psi}(n). \tag{32b}$$

Using equation (32b), equation (17) can be written as

$$\xi(n) = -\tilde{\theta}^{\mathrm{T}}\boldsymbol{\psi}(n), \tag{33}$$

where $\tilde{\theta}(n) = \boldsymbol{\theta}(n) - \boldsymbol{\theta}^o$. From equations (33) and (26), it follows that

$$\tilde{\theta}(n+1) = [\mathbf{I} - \mu\boldsymbol{\psi}(n)\boldsymbol{\psi}^{\mathrm{T}}(n)]\tilde{\theta}(n). \tag{34}$$

Taking expectation at both sides of the above equation yields [3]

$$E\{\tilde{\theta}(n+1)\} = [I - \mu E\{\boldsymbol{\psi}(n)\boldsymbol{\psi}^{\mathrm{T}}\}]E\{\tilde{\theta}(n)\}. \tag{35}$$

Let λ_{max} represent the maximum eigenvalue of the matrix $E\{\boldsymbol{\psi}(n)\boldsymbol{\psi}^{\mathrm{T}}\}$; if

$$0 < \mu < \frac{2}{\lambda_{max}}, \tag{36}$$

it can be seen from equation (35) that as $n \to \infty$, then $E\{\tilde{\theta}(n)\} \to 0$, i.e., $E\{\boldsymbol{\theta}(n)\} \to \boldsymbol{\theta}^o$, which is the desired result of global convergence.

Note that

$$\lambda_{max} < tr[E\{\boldsymbol{\psi}\boldsymbol{\psi}^{\mathrm{T}}\}] = N\sigma_{sx} + M\sigma_d, \tag{37}$$

where $tr[\cdot]$ represents the trace of a matrix, σ_{sx} and σ_d represent the power of $S(z)x(n)$ and $d(n)$, respectively, and the step-size μ in equation (36) can be further restricted as [3]

$$0 < \mu < \frac{2}{N\sigma_{sx} + M\sigma_d}. \tag{38}$$

It is worthy of note that the step-size μ may be well below the bound given in equation (38) due to the assumption of slow adaption, which is common for a filtered-version algorithm such as FXLMS, FULMS [3].

3.2. $d_u(n) \neq 0$

Equation (30) can be written as

$$d_m(n) = \boldsymbol{\theta}^{o\mathrm{T}}\boldsymbol{\phi}(n), \tag{39}$$

where

$$\boldsymbol{\phi}(n) = [S(z)x(n)S(z)x(n-1) \ \cdots \ S(z)x(n-N+1) \ \cdots \ d_m(n-1) \ \cdots \ d_m(n-M)]^{\mathrm{T}}.$$

Substituting equation (39) into equation (29) yields

$$d(n) = \boldsymbol{\theta}^{o\mathrm{T}}\boldsymbol{\phi}(n) + d_u(n) = \boldsymbol{\theta}^{o\mathrm{T}}\boldsymbol{\psi}(n) + [1 - A^o(z)]d_u(n). \tag{40}$$

Defining $\tilde{\theta}(n) = \boldsymbol{\theta}(n) - \boldsymbol{\theta}^o$, from equations (40) and (17) it follows that

$$\xi(n) = -\tilde{\theta}(n)^{o\mathrm{T}}\boldsymbol{\psi}(n) + [1 - A^o(z)]d_u(n). \tag{41}$$

Using equations (41) and (26), one can get

$$\tilde{\theta}(n+1) = [\mathbf{I} - \mu\boldsymbol{\psi}(n)\boldsymbol{\psi}^{\mathrm{T}}(n)]\tilde{\theta}(n) + \mu\{[1 - A^o(z)]d_u(n)\}\boldsymbol{\psi}(n). \tag{42}$$

Taking expectation at both sides of equation (42) yields

$$E\{\tilde{\theta}(n+1)\} = [\mathbf{I} - \mu E\{\boldsymbol{\psi}(n)\boldsymbol{\psi}^{\mathrm{T}}(n)\}E\{\tilde{\theta}(n)\}] + \mu E\{\{[1 - A^o(z)]d_u(n)\}\boldsymbol{\psi}(n)\}. \tag{43}$$

Comparing equation (43) with equation (35), it can be seen that they both have the same convergence condition; however, due to the presence of the extra term $E\{\{[1 - A^o(z)]d_u(n)\}\boldsymbol{\psi}(n)\}$, $E\{\tilde{\theta}(n)\}$ in equation (43) will converge to a non-zero value. Defining $R = E\{\boldsymbol{\psi}(n)\boldsymbol{\psi}^{\mathrm{T}}(n)\}$, $r = E\{\{[1 - A^o(z)]d_u(n)\}\boldsymbol{\psi}(n)\}$, for a suitable step-size μ, it can be seen from equation (43) that as $n \to \infty$, then $E\{\tilde{\theta}(n)\} \to R^{-1}r$, so

$$E\{\boldsymbol{\theta}(\infty)\} - \boldsymbol{\theta}^o = E\{\tilde{\theta}(\infty)\} = R^{-1}r. \tag{44}$$

Equation (44) gives the expression for the bias in steady state. Therefore, when $d_u(n)$ is not equal to 0, $E\{\boldsymbol{\theta}(n)\}$ may converge to a biased result other than the optimal one $\boldsymbol{\theta}^o$. Generally, when the power of $d_u(n)$ is relatively insignificant such that the bias is small, the proposed algorithm can still be expected to have a better noise reduction than the FULMS algorithm.

4. EFFECTS OF ACOUSTIC FEEDBACK

When acoustic feedback exists, i.e., $F(z) \neq 0$, the reference signal $x(n)$, as shown in Figure 1(b), can be expressed as

$$x(n) = x_0(n) + F(z)y(n) = x_0(n) + F(z)\boldsymbol{\theta}^{\mathrm{T}}(n)\boldsymbol{\varphi}(n). \tag{45}$$

With the assumption of slow adaption, the above equation can be written as [9]

$$x(n) = x_0(n) + \boldsymbol{\theta}^{\mathrm{T}}(n)[F(z)\boldsymbol{\varphi}(n)]. \tag{46}$$

Equation (46) indicates that the reference signal $x(n)$ is dependent on $\boldsymbol{\theta}(n)$, so from equation (16), it follows that $\xi(n)$ is no longer a linear combination of elements of the coefficient vector $\boldsymbol{\theta}(n)$; thus $E\{\xi^2(n)\}$ is no longer a quadratic function. Therefore, when acoustic feedback exists, the proposed algorithm may converge to a local minimum. There are two points worth discussing:

(1) In references [3, 2], the dependence of reference signal $x(n)$ on the coefficient vector $\boldsymbol{\theta}(n)$ is ignored, and the rationale for ignoring the dependence of $x(n)$ on $\boldsymbol{\theta}(n)$ is shown in reference [10]. Under that condition, the proposed algorithm can still show global convergence, as shown in section 2.

(2) By comparing equations (3) and (16), it is seen that $\mathbf{X}(n)$ and $\mathbf{Y}(n)$ in equation (3) are both dependent on the coefficient vector $\boldsymbol{\theta}(n)$, while in equation (16) only $\mathbf{X}(n)$ is dependent on $\boldsymbol{\theta}(n)$, and $\mathbf{D}(n)$ is independent of $\boldsymbol{\theta}(n)$. $E\{\xi^2(n)\}$ is expected to have less minima than $E\{e^2(n)\}$. Therefore, the global convergence of the proposed algorithm is more probable than that of the FULMS algorithm.

5. SIMULATION RESULTS

Many simulation results show that the proposed algorithm presents good convergence properties. One example is given below.

In this example, the proposed algorithm is compared to the FULMS algorithm by using experimental transfer functions that are available in the disk attached to reference [3]. According to the transfer functions given in reference [3], M and N (orders of the adaptive IIR filter) should be, respectively, greater than or equal to 73 and 72 in order to model the system perfectly. Frequency responses of the primary path $P(z)$, secondary path $S(z)$ and feedback path $F(z)$ are shown in Figures 5, 6 and 7, respectively, where the normalized frequency is used (with Nyquist frequency equal to 1). In the simulation, initial coefficients

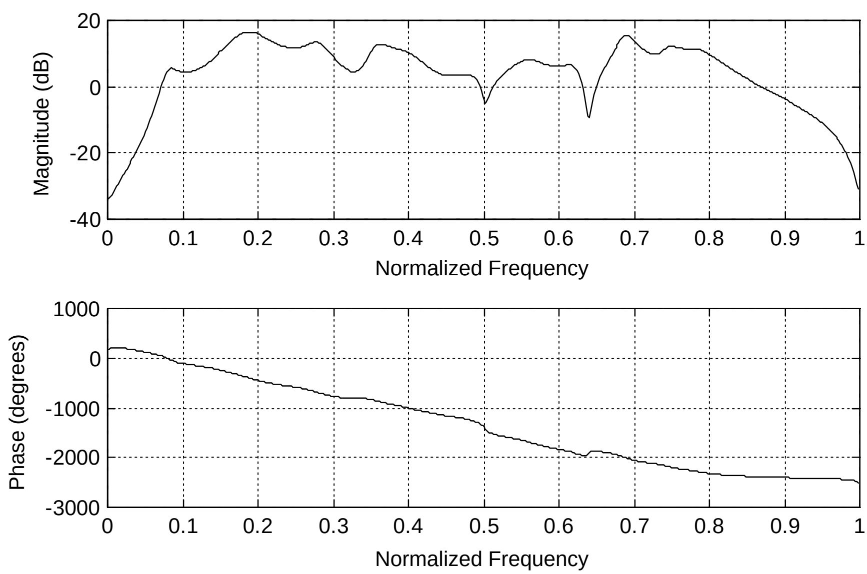

Figure 5. Frequency response of the primary path.

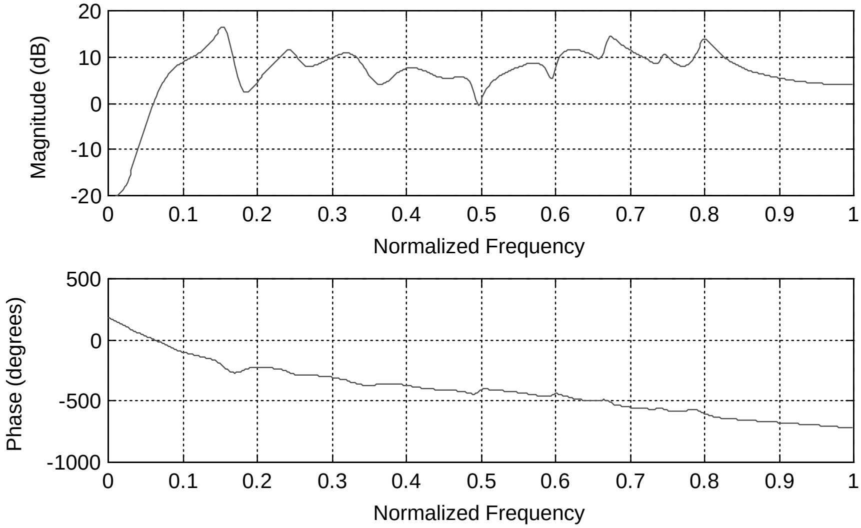

Figure 6. Frequency response of the secondary path.

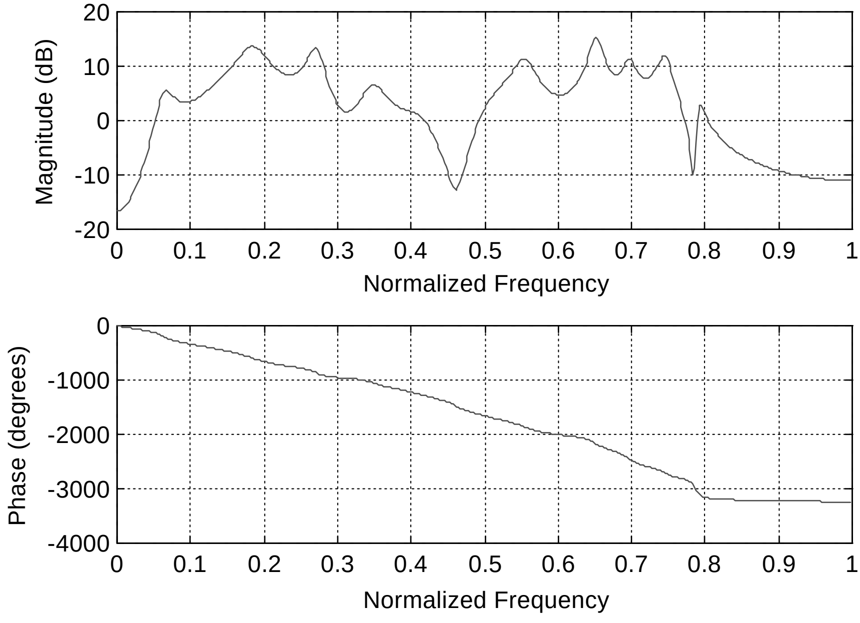

Figure 7. Frequency response of the acoustic feedback path.

are all set to 0. When the noise source is assumed to be a white noise with a variance of 1, simulations are carried out in the following four cases: (1) $M = 75, N = 75$, (2) $M = 50, N = 50$, (3) $M = 25, N = 25$, (4) $M = 13, N = 13$. It is found that the maximum acceptable step-size for the FULMS algorithm is much smaller than that for the proposed algorithm under the same condition. To cancel the ambiguous effects of step-size on the steady state performance, the same step-size is used for the two algorithms. In case 1, with step-size $\mu = 0{\cdot}5 \times 10^{-5}$, the total noise reductions in steady state are about 15 dB for the

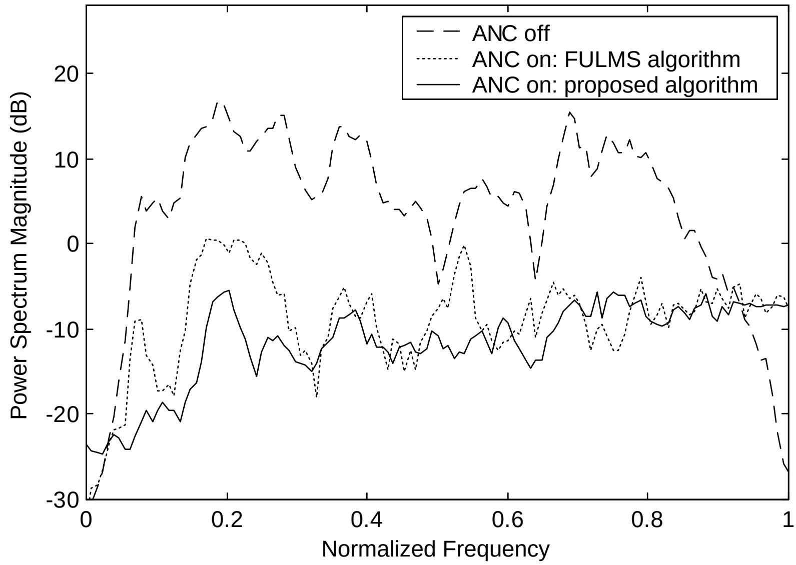

Figure 8. Power spectrum of the residual noise. Noise source: white noise; filter order: $M = 75$, $N = 75$; step-size: $\mu = 0{\cdot}5 \times 10^{-5}$.

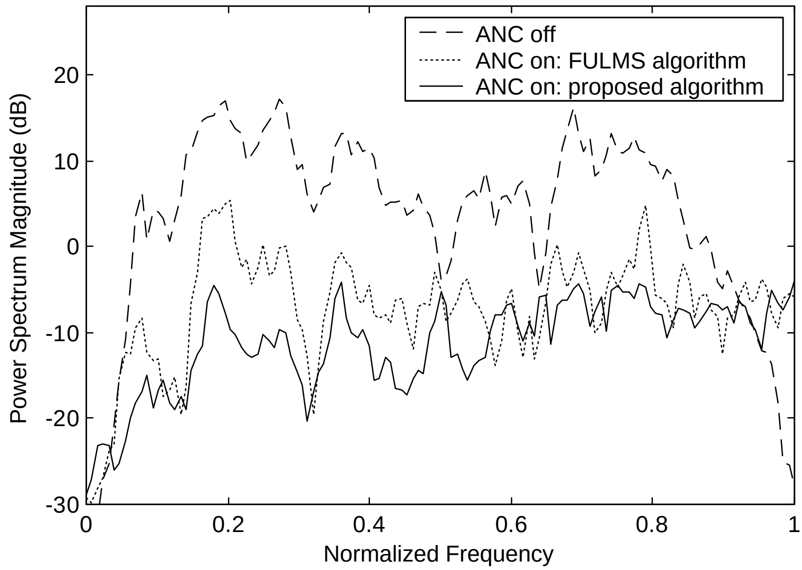

Figure 9. Power spectrum of the residual noise. noise source: white noise; filter order: $M = 50, N = 50$; step-size: $\mu = 1 \times 10^{-5}$.

FULMS algorithm and 19 dB for the proposed algorithm. In case 2 with step-size $\mu = 1 \times 10^{-5}$, and in case 3 with step-size $\mu = 1 \times 10^{-4}$, the corresponding total noise reductions are 13 and 19 dB, and 2 and 13 dB respectively. In case 4, it failed to converge by the FULMS algorithm, and 5 dB noise reduction is obtained by the proposed algorithm with step-size $\mu = 1 \times 10^{-4}$. The noise reduction effects at different frequency for the two algorithms in the first three cases are shown in Figures 8–10, from which it can be seen that the proposed algorithm has a better noise reduction than the FULMS algorithm in a wide

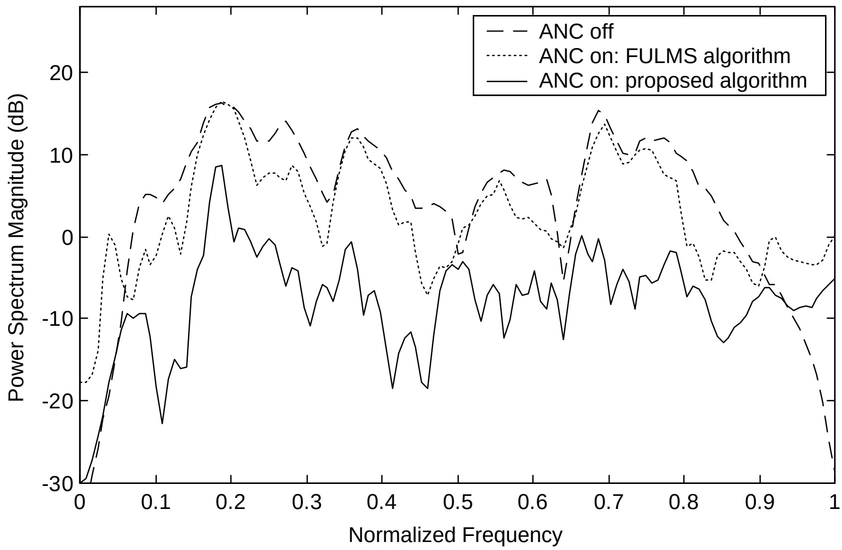

Figure 10. Power spectrum of the residual noise. Noise source: white noise; filter order: $M = 25, N = 25$; step-size: $\mu = 1 \times 10^{-4}$.

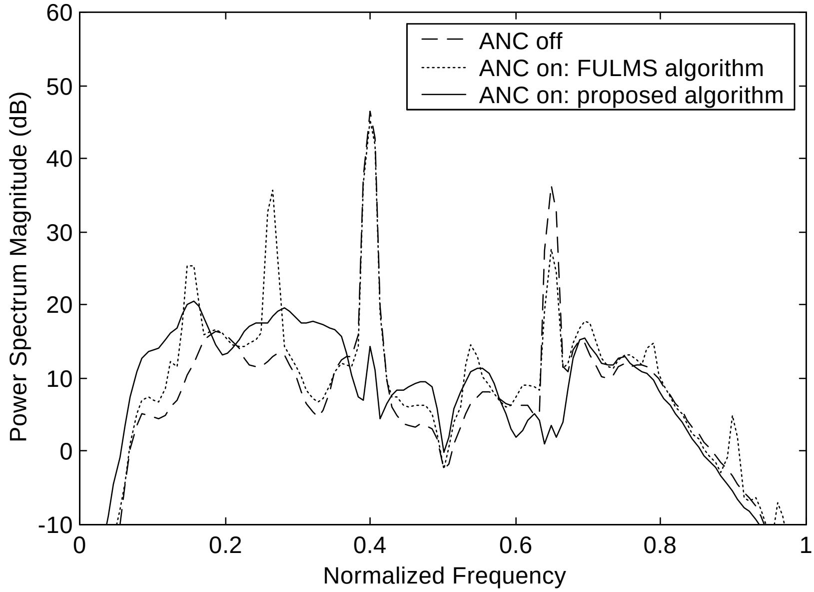

Figure 11. Power spectrum of the residual noise. Noise source: two sinusoids corrupted by a white noise; filter order: $M = 50, N = 50$; step-size: $\mu = 1 \times 10^{-4}$.

frequency range for both the sufficient and reduced order cases. It is interesting to note that the noise reduction of the FULMS algorithm decreases more rapidly than that of the proposed algorithm with the reduction of the filter order.

When the noise source is two sinusoids (with normalized frequency 0·4 and 0·65, respectively, and both amplitudes 10) corrupted by a white noise (with a variance of 1), with step-size $\mu = 1 \times 10^{-5}$ the total noise reduction for the FULMS algorithm is less than

1 dB; however, 14 dB has been gained for the proposed algorithm. The noise reduction effects at different frequency are shown in Figure 11, from which it can be seen that two sinusoids are well cancelled by the proposed algorithm. For the FULMS algorithm only one sinusoid is reduced significantly; unfortunately, the two new peaks in the low frequency counteract the reduction. Appearance of the new peaks may be a consequence of the improper poles of the adaptive IIR filter.

6. SUMMARY

Using IIR filters instead of FIR filters in the ANC system can reduce the computational complexity significantly. However, the conventional IIR filter-based FULMS algorithm cannot ensure global convergence. In this paper, we proposed a new IIR filter-based algorithm, which is shown to have a better global convergence property.

REFERENCES

1. J. J. Shynk 1989 *IEEE ASSP Magazine* **6**, 4–21. Adaptive IIR filtering.
2. L. J. Eriksson 1991 *Journal of the Acoustical Society of America* **89**, 257–265. Development of the filtered-U algorithm for active noise control.
3. S. M. Kuo and D. R. Morgan 1996 Active Noise Control System, New York: Wiley.
4. P. L. Feintuch 1977 *Proceedings of the IEEE* **64**, 1622–1624. An adaptive recursive LMS filter.
5. I.-S. Kim , H.-S. Na, K.-J. Kim and Y. Park 1994 *Journal of the Acoustical Society of America* **95**, 3379–3389.Constraint filtered-x and filtered-u least-mean-square algorithms for the active control of noise in ducts.
6. M. G. Larimore, J. R. Treichler and C. R. Johnson Jr. 1980 *IEEE Transactions on Acoustics, Speech, and Signal Processing* **ASSP-28**, 428–440. SHARF: an algorithm for adapting IIR digital filters.
7. C. R. Johnson Jr, M. G. Larimore, J. R. Treichler and B. D. O. Anderson 1981 *IEEE Transactions on Acoustics, Speech, and Signal Processing* **ASSP-29**, 659–669. SHARF convergence properties.
8. C. R. Johnson, Jr and B. D. O. Anderson 1981 *IEEE Transactions on Acoustics, Speech, and Signal Processing* **ASSP-29**, 1212–1215. Sufficient excitation and stable reduced-order adaptive IIR filter.
9. D. H. Crawford and R. W. Stewart 1997 *Journal of the Acoustical Society of America* **101**, 2097–2103. Adaptive IIR filtered-v algorithm for active noise control.
10. L. J. Eriksson, T. A. Lank and M. C. Allie 1999 *Proceedings of Active* 99, *Fort Lauderdale, FL,* 2–4 *December* 1999, 949–960. On-line secondary path modeling for FIR and IIR adaptive control in the presence of acoustic feedback.
11. P. A. Regalia 1995 Adaptive IIR Filtering. New York: Marcel Dekker.

APPENDIX A: THE PROOF OF "$E\{\xi^2(n)\}=0$ IF AND ONLY IF $E\{e^2(n)\}=0$"

Let $E(\mathrm{e}^{\mathrm{j}\omega})$ represent the power spectral density function of $e(n)$, with the assumption of slow adaption; it follows from equation (13) that [11]

$$\begin{aligned} E\{\xi^2(n)\} &= E\{[(1-A(n,z)e(n)]^2\} \\ &= \frac{1}{2\pi}\int_0^{2\pi}|1-A(n,\mathrm{e}^{\mathrm{j}\omega})|^2 E(\mathrm{e}^{\mathrm{j}\omega})\,\mathrm{d}\omega. \end{aligned} \tag{A1}$$

Since all the poles of $1-A(n,z)$ are within the unit circle (if only the IIR filter is stable), it is obvious that $\forall\omega\in[0,2\pi)$, $|1-A(n,\mathrm{e}^{\mathrm{j}\omega})|>0$. Note that $E\{e^2(n)\}=(1/2\pi)\int_0^{2\pi}E(\mathrm{e}^{\mathrm{j}\omega})\,\mathrm{d}\omega$, and it follows from equation (A1) that

$$E\{\xi^2(n)\}=0 \Leftrightarrow \int_0^{2\pi}|E(\mathrm{e}^{\mathrm{j}\omega})|^2\,\mathrm{d}\omega=0 \Leftrightarrow E\{e^2(n)\}=0. \tag{A2}$$