Journal of Sound and Vibration (2002) **258**(3), 549–561
doi:10.1006/jsvi.5275, available online at http://www.idealibrary.com on IDEAL®

SUBJECTIVE PREFERENCE FOR SOUND SOURCES LOCATED ON THE STAGE AND IN THE ORCHESTRA PIT OF AN OPERA HOUSE

S. Sato and H. Sakai

Graduate School of Science and Technology, Kobe University, Rokkodai, Nada, Kobe 657-8501, *Japan.*
E-mail: s_sato@mac.com

AND

N. Prodi

Dipartimento di Ingegneria, Universitá di Ferrara, Via Saragat, I-44100 *Ferrara, Italy*

(*Accepted* 30 *May* 2002)

The present study investigates whether the subjective preference theory can be applied to the sound field in an opera house. Paired-comparison tests were conducted to obtain scale values of subjective preference. As the source locations of the music on the stage and in the orchestra pit were moved, listeners were asked to give their acoustical preference. The acoustical factors at each listening position were obtained from the interaural cross-correlation function and binaural impulse responses measured at each listening position. The relationship between the scale values of subjective preference and orthogonal acoustical factors (LL, $IACC$, τ_{IACC}, Δt_1 for the pit source, Δt_1 for the stage, T_{sub} for the pit source, and T_{sub} for the stage source) was determined by using factor analysis, which shows that the preference theory is applicable. Total scores obtained from factor analysis and measured scale values are in good agreement.

1. INTRODUCTION

Investigations on historical opera houses with the aim of preserving cultural heritage are being carried out [1]. They are useful for rebuilding old theatres which have already disappeared, for the preservation of existing theatres, and for education. Investigators need to determine the objective acoustical parameters that can fully describe the acoustic sound field in an opera house and determine measurement procedures for these parameters. A typical opera house is distinguished from a concert hall by its large-volume stagehouse, orchestra pit, and box seats. Acoustics of opera houses have been evaluated by using knowledge obtained from surveys of concert hall [2, 3]. The measurement procedures, including the set-up of the theatre, the positions of sound sources and receivers, and measurement devices to obtain impulse responses as raw acoustical data, have been described in proposed guidelines [1]. Acoustical parameters to be measured for the evaluation of the sound field should also be determined.

Among the subjective attributes of sound fields, subjective preference is considered to provide an overall impression of the sound field. The theory of subjective preference allows

0022-460X/02/$35.00

a sound field to be evaluated in terms of the following four orthogonal acoustical factors [4]: listening level (*LL*), initial time-delay gap between the direct sound and the first reflection (Δt_1), subsequent reverberation time (T_{sub}), and magnitude of the interaural cross-correlation function (*IACC*), all of which describe the sound signals arriving at each ear. These factors were identified from the systematic investigation of sound fields by using computer simulation and listening tests (paired-comparison tests) [5]. The subjective preference theory has been validated by tests in concert halls [6, 7]. However, this theory has not been confirmed for an opera house. The present study investigates the relationship between the subjective preference of the sound field and the orthogonal acoustical factors at each seat, on the basis of the theory of subjective preference for sound fields. Subjective preference is evaluated according to scale values obtained by paired comparison. Whether the theory of subjective preference can be applied to the sound field of an opera house was investigated.

2. SUBJECTIVE PREFERENCE JUDGMENTS

2.1. SOURCE SIGNAL

Romanza "Tormento" by P. Tosti was used as the source signal. The vocal (soprano) and piano accompaniment were channelled separately. The duration of the signal was 16 s. In the theory of subjective preference, the source signal is characterized in terms of its autocorrelation function (ACF). The effective duration τ_e of the long-time ACF, defined as the time delay at which the envelope of the normalized ACF becomes -10 dB from its initial value, determines the most preferred delay time for early reflections and the optimum subsequent reverberation time [4]. When signals are from music containing large fluctuations in tempo, these optimum values are more accurately expressed by the minimum value of the effective duration $(\tau_e)_{min}$ of the running ACF of the source signal [8, 9]. The source signal with $(\tau_e)_{min}$ is the most active part, and the listener is sensitive to that part in terms of the changes in the temporal acoustical factors.

In an anechoic chamber, the vocal and piano signals reproduced by two loudspeakers were picked up by a microphone. The distance between the loudspeaker and the microphone was 1·0 m. The effective durations of the running ACFs were calculated after passing the signal through an A-weighted network. The running integration interval for the ACF, 2 *T*, was 2·0 s, and the running step was 100 ms. This interval was chosen on the basis of the results of several previous investigations [8–10] (Figure 1). The $(\tau_e)_{min}$ value for the signal is 16 ms.

2.2. PROCEDURE

The opera house used in the experiments was the Teatro Comunale in Ferrara, Italy. The plan of the theatre is shown in Figure 2, where the positions of sound sources and listeners are also indicated as explained later. It has a truncated elliptical plan and consists of 800 seats (two thirds of them in the five tiers of boxes) with a hall of 5000 m^2 and a stagehouse of 8500 m^3. The stage did not contain any scenery. Curtains were not lowered at the back of the stage. There were no musical instruments or chairs in the orchestra pit. The pit rail is made of a hard wooden board and is installed between the stall and the orchestra pit. Its height is 2·08 m from the pit floor. The top of the pit rail is in line with the stage.

Two loudspeakers reproducing the vocal signal were located on the stage (one just under the proscenium and the other 2·5 m behind it); two loudspeakers for reproducing

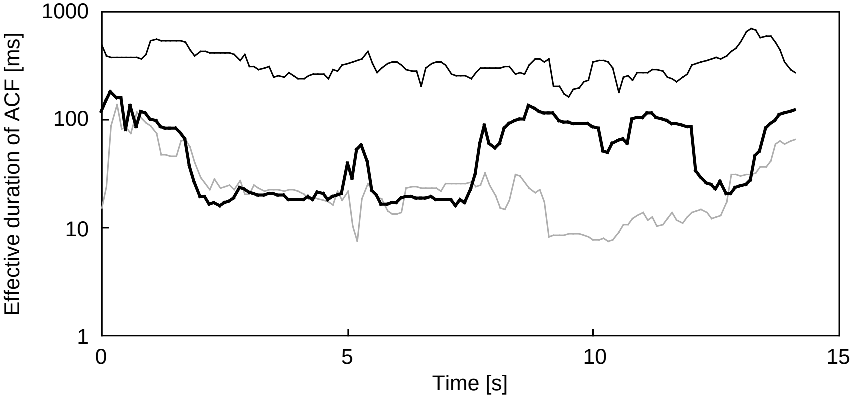

Figure 1. Measured τ_e of the running ACF of the source signal used in the experiment with a 100 ms interval as a function of the time. The running integration interval of ACF $2T$ was 2·0 s. (——), Piano; (———), vocal; (——), mixed.

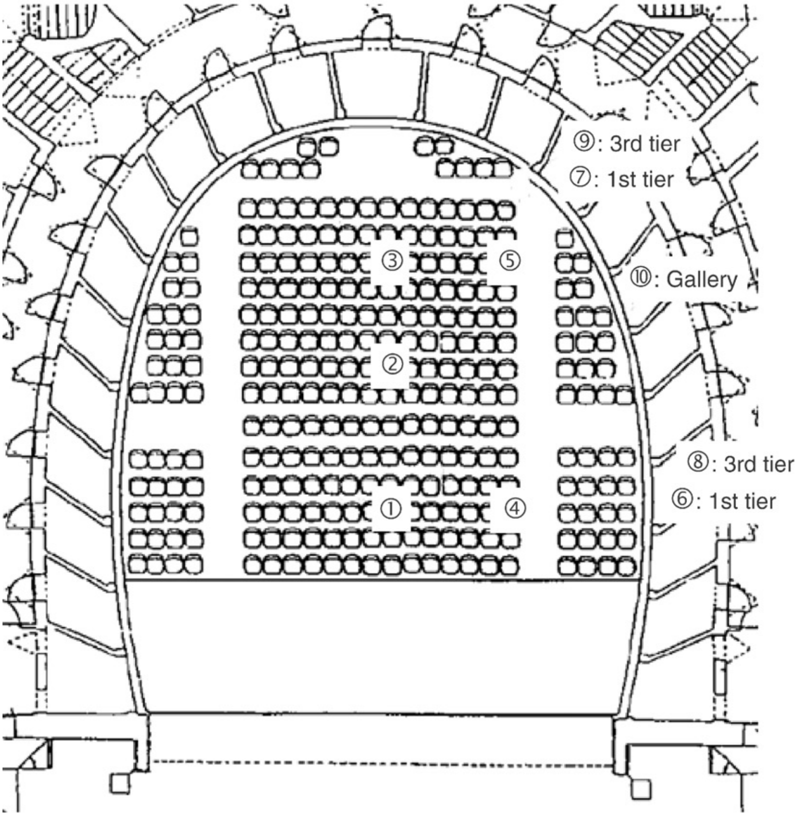

Figure 2. Plan of Teatro Comunale in Ferrara. The numbers in the circle indicate listener locations.

the piano signal were placed in the orchestra pit (one in front of the conductor's box and the other under the overhang). The heights of the loudspeakers on the stage and in the orchestra pit were 1·5 and 1·2 m above the floor level respectively (Figure 3). These heights simulated a singer on the stage and a seated musician in the orchestra pit respectively.

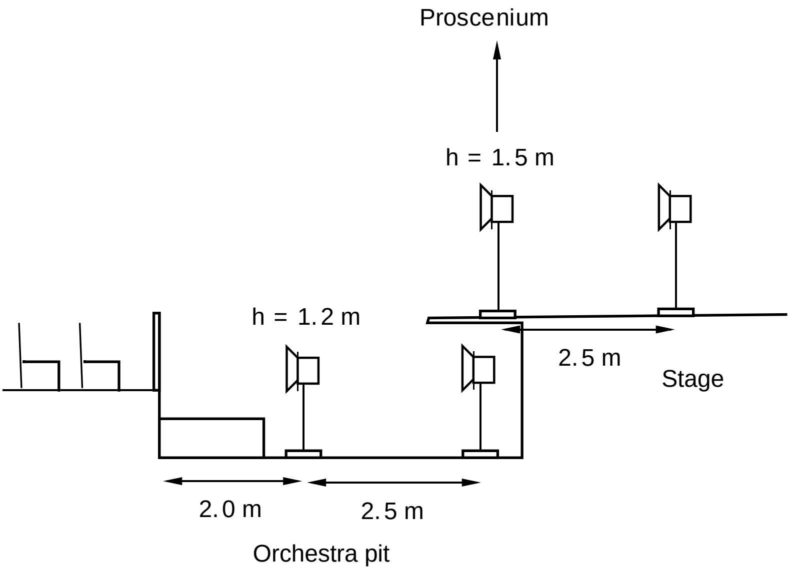

Figure 3. Positions of the loudspeakers on the stage and in the orchestra pit.

Table 1

Conditions of the loudspeaker positions in the paired-comparison tests

	Condition 1	Condition 2	Condition 3	Condition 4
Orchestra pit	Front	Front	Rear	Rear
Stage	Front	Rear	Front	Rear

2.3. PAIRED-COMPARISON TESTS

To obtain reliable subjective responses in an existing sound field, the paired-comparison method was used. This method is simple enough for non-skilled listeners to judge. To help to exclude other physical factors such as visual and tactile senses, the source locations on the stage and in the orchestra pit were switched. Paired-comparison tests using four sound sources in various combinations (Table 1) were conducted. The duration of the music signal was 16 s and the silent interval between the stimuli was 2 s. Each pair of sound fields was separated by an interval of 4 s. The tests were performed for all combinations in pairs, i.e., six pairs ($N(N-1)/2$, where $N=4$) of stimuli for a single session. The pairs were arranged in random order.

Forty-seven listeners participated in the experiments. Twenty-one of them were students of the musical department and 27 listeners were students of the Faculty of Engineering. The listeners were divided into 10 groups and were seated at specific seats (Figure 2). Five groups sat in the stalls and the other five groups sat in the boxes or gallery. As the source locations of the music were moved, listeners were asked to give their acoustical preferences. They were advised to judge every pair (not to leave a blank), to face the centre of the stage, and not to copy the answer of other persons. They were also asked to write down their name, age, sex, and musical experiences (period and instruments) on their answer sheets. Prior to the experimental sessions, a practice session was conducted by presenting three pairs of stimuli. The experimental session was repeated five times, each time the listeners changed their seats. It took about 4 min for a single session and about 30 min in total including the time for changing seats, and 25 or 26 listeners in total responded at each listener's position.

2.4. RESULTS OF SUBJECTIVE TESTS

Tests of consistency were used to investigate whether the listeners could discriminate between the sound fields presented; for example, whether a listener prefers sound field A to B, B to C, and C to A. The number of listeners who showed a significant ability to discriminate preferences was 20 in the stalls and 17 in the boxes or gallery. The test of agreement indicated that there was a significant ($p < 0{\cdot}05$) degree of agreement among the listeners. Scale values of preference were obtained by applying the law of comparative judgment and reconfirmed by quality of fit [11, 12]. According to the listener's musical experience, the data were grouped into categories and were analyzed, but no significant difference among the group was found. Figure 4 shows the scale values of preference for each listening position. The results show that the listeners in the stall preferred the frontal source position on the stage (conditions 1 and 3). The range of scale values in the boxes was smaller than that of one stalls.

3. ACOUSTICAL MEASUREMENTS AT EACH LISTENING POSITION

3.1. PROCEDURE

Acoustical measurements were conducted to obtain the acoustical factors at each listening position under the four conditions of the preference tests. The four sound sources were placed in the same positions as the former measurements and at the same height in the pit and on the stage. Settings of the hall were the same as those used in the subjective preference judgments except that there were no listeners. Definitions and calculation procedures of the acoustical factors are described in Appendix A.

To obtain the spatial factors extracted from the interaural cross-correlation function (IACF), the musical signals used in the subjective tests were reproduced using the same loudspeaker configurations as in the subjective tests and were recorded at each listening position to DAT through two condenser microphones at the entrances of both ears of the receiver facing the centre of the stage. In the measurements in the boxes, the receiver's head was located on the plane of the front of the box. Recorded signals were passed through an A-weighting network, and the IACFs were then obtained without frequency and time separations. Four orthogonal factors, namely, LL, the $IACC$, interaural time delay (τ_{IACC}), and the width of the interaural cross-correlation function (W_{IACC}), were extracted from the IACF.

To obtain the temporal factors extracted from the impulse responses, a log sine sweep with a duration of 15 s was used. The measurements were done with the aid of a switch to deliver the signal independently to each loudspeaker. For each measurement point where the receiver was placed, four measures were collected according to each sound source. From the impulse responses, Δt_1 and T_{sub} were calculated. Since the signal for each loudspeaker was measured separately, one value for the source in the orchestra pit and one for the stage under each condition were obtained.

3.2. MEASURED RESULTS

Maximum LL was observed under condition 1 at seat 4 in the stall. The range of the values of IACC was between 0·09 (condition 2 at seat 5 in the stall) to 0·65 (condition 3 at

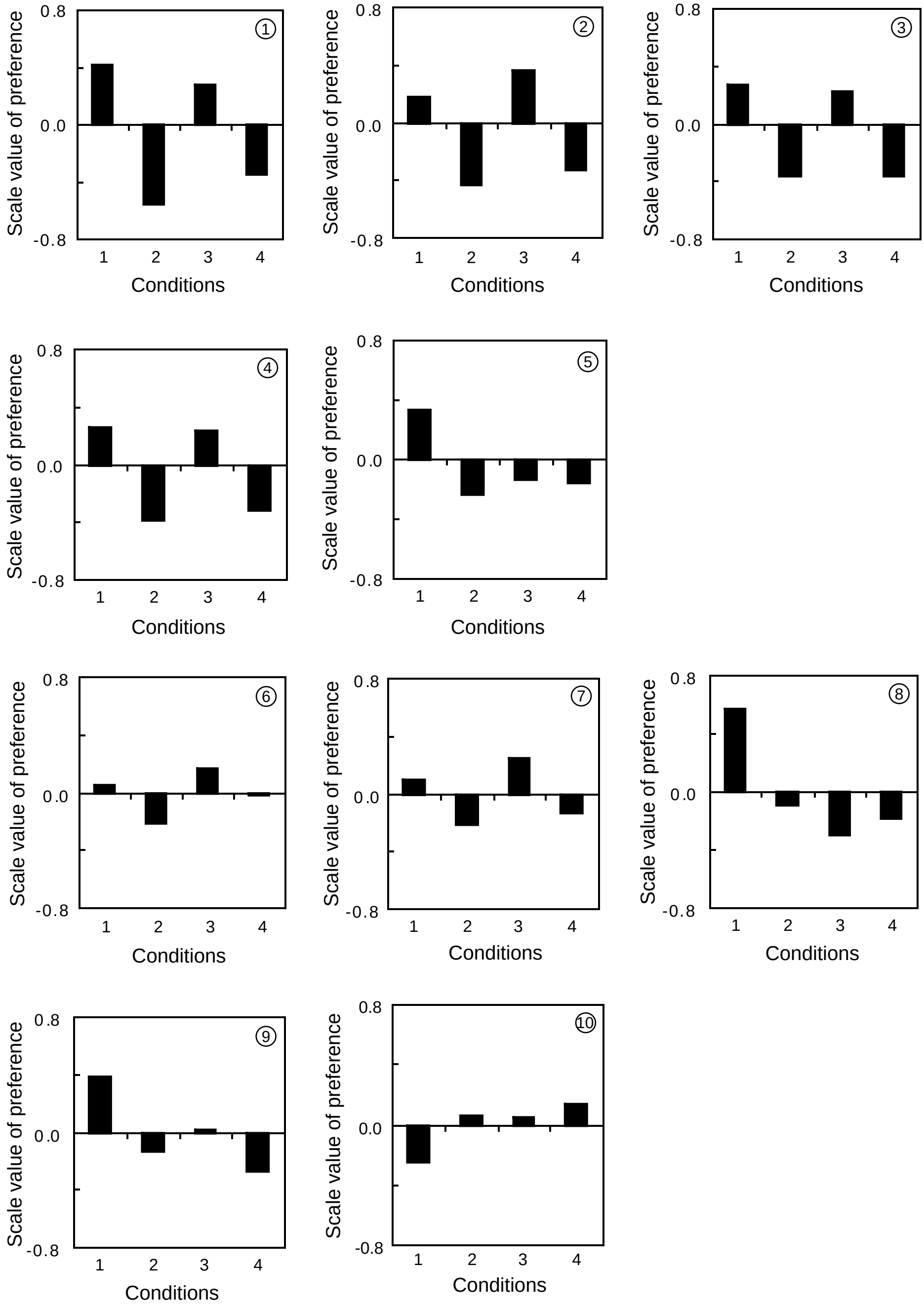

Figure 4. Results of paired-comparison tests at each listener's location. The numbers correspond to the listener locations shown in figure 2.

seat 1 in the stall). The τ_{IACC} values of almost all listening positions were less than 0·1 ms because the receiver faced the sources during the measurement. The W_{IACC} values were centred around 0·12 ms.

Values of Δt_1 in the stall for the source in the pit are almost constant, around 10 ms for the frontal source and 3 ms for the rear source. These reflections came from the rear wall of the pit. Values of Δt_1 in the stall for the source on the stage are greater than those for the source in the pit. The reflections may have come from the side wall of the audience area. In the boxes, the first reflections come from the walls inside the box. In the gallery, the first reflections for all four sources come from the ceiling. As for the values of T_{sub} in the stall for the source on the stage, the rear source gives a greater value than the frontal source. This greater T_{sub} was due to the large volume of the stagehouse.

4. MULTIPLE DIMENSIONAL ANALYSIS

4.1. CORRELATION BETWEEN PHYSICAL FACTORS

The relationship between scale values of subjective preference and physical factors obtained by acoustical measurements was examined by factor analysis described in Appendix B. Of the physical parameters, W_{IACC} is the significant factor in apparent source width (ASW) only when a source signal with a predominately low frequency is compared with a source signal with a predominately high frequency [13]; and thus it is not included in this analysis. Correlation coefficients between the physical factors are listed in Table 2. There is a certain degree of coherence between physical factors, for example, *LL* and Δt_1 of sound fields in existing concert halls. This coherence is a physical phenomenon that depends on the characteristics of the sound field.

The outside variable to be predicted was the scale values of preference obtained by subjective judgments, and the explanatory variables were: (1) *LL*, (2) *IACC*, (3) τ_{IACC}, and (4) Δt_1 for the pit source, (5) Δt_1 for the stage source, (6) T_{sub} of 1 kHz for the pit source, and (7) T_{sub} of 1 kHz for the stage source. Iterations for the possible subdivision of the subcategories for each factor were conducted.

4.2. RESULTS AND DISCUSSION

The scores which give the best correlation between the scale value of preference and the total score obtained from the factor analysis are shown in Figure 5. It is clear that the *LL* scores increased with an increase in *LL*. The scores decreased with an increase in *IACC*. The τ_{IACC} scores slightly decreased with an increase in τ_{IACC}. The effect of τ_{IACC} on the total scores was minor because all the loudspeakers were located on the centre axis of the hall and the listeners faced the centre of the stage. The scores for the three factors (*LL*, *IACC*, and τ_{IACC}) agree with those obtained for sound fields in a concert hall [7].

TABLE 2

Correlation coefficients among orthogonal physical factors obtained from the acoustical measurements in an opera house

	LL	*IACC*	τ_{IACC}	Δt_1 (pit)	Δt_1 (stage)	T_{sub} (1 kHz; pit)	T_{sub} (1 kHz; stage)
LL	—	0·12	−0·04	0·74**	0·56**	0·10	−0·17
IACC		—	0·23	0·22	0·25	0·23	0·61**
τ_{IACC}			—	0·02	−0·22	0·13	−0·02
Δt_1 (pit)				—	0·44	0·27	−0·12
Δt_1 (stage)					—	−0·08	−0·06**
T_{sub} (1 kHz; pit)						—	0·02
T_{sub} (1 kHz; stage)							—

$^{**}p<0{\cdot}01$.

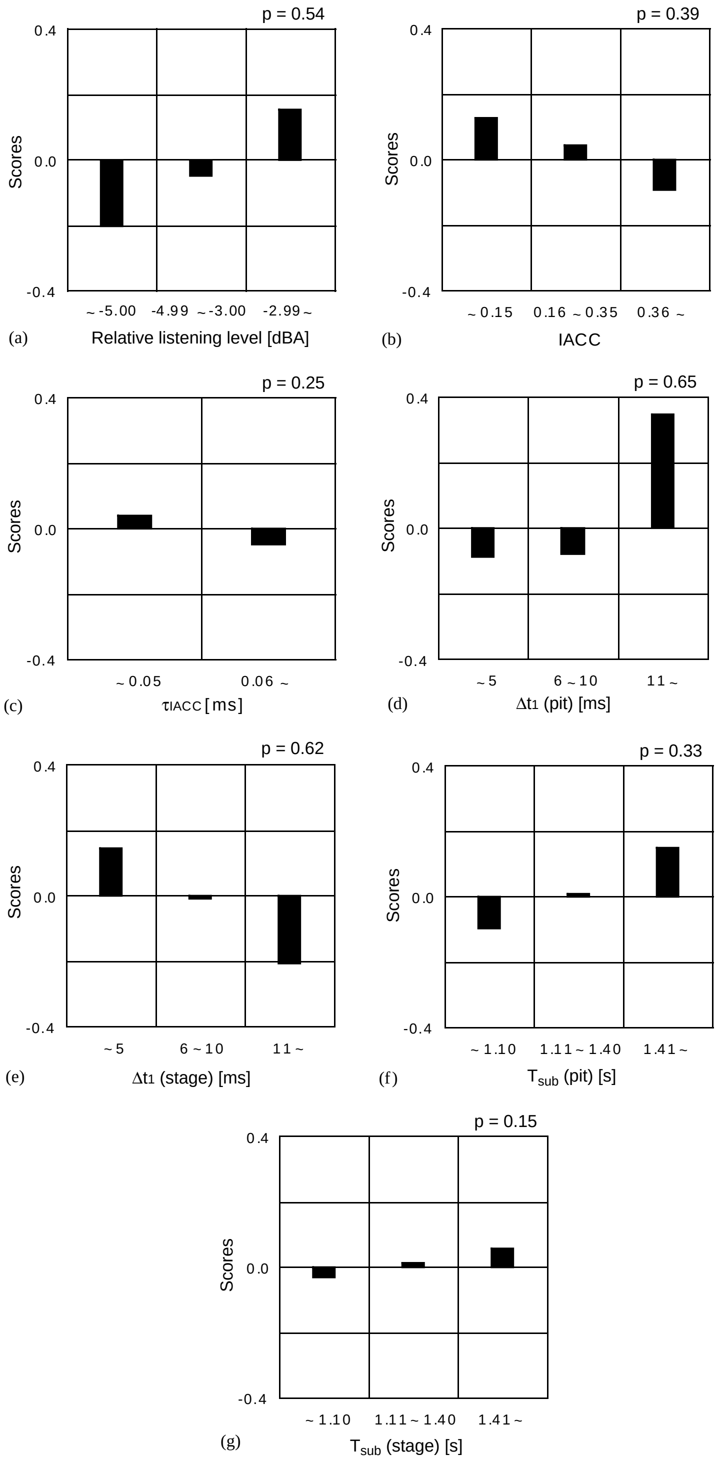

Figure 5. Scores for each category of physical factors obtained by factor analysis: (a) *LL*; (b) *IACC*; (c) τ_{IACC}; (d) Δt_1 for the pit source; (e) Δt_1 for the stage source; (f) T_{sub} of 1 kHz for the pit source; and (g) T_{sub} of 1 kHz for the stage source (p: partial correlation coefficient of each factor).

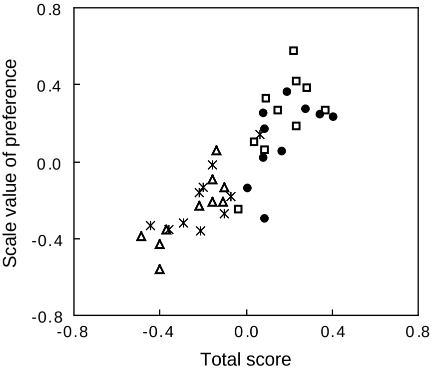

Figure 6. Relationship between scale values obtained by subjective judgments and total scores calculated by factor analysis using scores shown in Figure 5 (□, condition 1; △, condition 2; $*$, condition 3; ●, condition 4).

The partial correlation coefficients for Δt_1 are the largest among the physical factors. The Δt_1 scores for the pit increased with a decrease in Δt_1. On the other hand, the Δt_1 scores for the stage increased with an increase in Δt_1. Preferred Δt_1 of the source signal with longer τ_e is longer than that with shorter τ_e [4]. The piano signal reproduced from the loudspeakers in the pit has longer τ_e ($\approx 160\,\mathrm{ms}$) than the vocal signal ($\approx 8\,\mathrm{ms}$) from the loudspeakers on the stage. The scores of Δt_1 may be related to the τ_e values of the source signals. The T_{sub} scores for the stage and in the pit increased with an increase in T_{sub}. The effects of T_{sub} on the scores are rather minor in this investigation because of the limited range of T_{sub} in the opera house.

The relationship between the scale value obtained by preference judgments and the total score at each listening position are shown in Figure 6. The scale values of preference are calculated from the total score under each of the four conditions ($r = 0{\cdot}86$, $p < 0{\cdot}01$).

5. CONCLUDING REMARKS

To determine whether the subjective preference theory can be applied to the sound field of an opera house, factor analysis was used to examine the relationship between the scale values of preference and the orthogonal physical factors. The scale values of preference for different source locations on the stage and in the orchestra pit were obtained by using a paired-comparison method. The physical factors were obtained from the interaural cross-correlation functions and binaural impulse responses measured at each listening position. Results of the factor analysis show that the scale values of preference can be calculated from the total scores of orthogonal acoustical factors.

ACKNOWLEDGMENTS

The authors wish to thank the staff of Teatro Comunale in Ferrara for their cooperation during the experiments. We express our gratitude to Professor Roberto Pompoli of the University of Ferrara and Professor Yoichi Ando of Kobe University for their valuable comments and suggestions. The authors would also like to thank the students who participated in the experimental sessions and Takuya Hotehama for his help with the acoustical measurements. This work was supported in part by a grant-in-aid for scientific research from the Japan Society for the Promotion of Science.

REFERENCES

1. R. Pompoli and N. Prodi 2000 *Journal of Sound and Vibration* **232**, 281–301. Guidelines for acoustical measurements inside historical opera houses: procedures and validation.
2. T. Hidaka and L. L. Beranek 2000 *Journal of the Acoustical Society of America* **107**, 368–383. Objective and subjective evaluations of twenty-three opera houses in Europe, Japan, and the Americas.
3. L. Tronchin and A. Farina 1997 *Journal of Audio Engineering Society* **45**, 1051–1062. Acoustics of the former teatro "La Fenice" in Venice.
4. Yoichi Ando 1985 *Concert Hall Acoustics*. Heidelberg: Springer-Verlag.
5. Yoichi Ando 1998 *Architectural Acoustics—Blending Sound Sources, Sound Fields, and Listeners*. New York: AIP Press/Springer-Verlag.
6. A. Cocchi, A. Farina and L. Rocco 1990 *Applied Acoustics* **30**, 1–13. Reliability of scale-model researches: a concert hall case.
7. S. Sato, Y. Mori and Y. Ando 1997 in *Music and Concert Hall Acoustics* (Y. Ando and D. Noson, editors). London: Academic Press. On the subjective evaluation of source locations on the stage by listeners.
8. Y. Ando, T. Okano and Y. Takezoe 1989 *Journal of the Acoustical Society of America* **86**, 644–649. The running autocorrelation function of different music signals relating to preferred temporal parameters of sound field.
9. K. Mouri, K. Akiyama and Y. Ando 2000 *Journal of Sound and Vibration* **232**, 139–147. Relationship between subjective preference and the alpha-brain wave in relation to the initial time delay gap with vocal music.
10. T. Taguti and Y. Ando 1997 in *Music and Concert Hall Acoustics* (Y. Ando and D. Noson, editors). London: Academic Press; chapter 23. Characteristics of the short-term autocorrelation function of sound signals in piano performances.
11. L. L. Thurstone 1927 *Psychological Review* **34**, 273–289. A law of comparative judgment.
12. F. Mosteller 1951 *Psychometorica* **16**, 207–218. Remarks on the method of paired comparisons: III. A test of significance for paired comparisons when equal standard deviations and equal correlations are assumed.
13. Y. Ando, S. Sato and H. Sakai 1999 in *Computational Architectural Acoustics in Architecture* (J. J. Sendra, editor). Southampton: WIT Press; Chapter 4. Fundamental subjective attributes of sound fields based on the model of auditory–brain system.
14. C. Hayashi 1952 *Annals of the Institute of Statistical Mathematics* **III**, 69–98. On the prediction of phenomena from qualitative data and the quantification of qualitative data from the mathematico-statistical point of view.
15. C. Hayashi 1954 *Proceedings of Japan Academy* **30**, 61–65. Multidimensional quantification. I.
16. C. Hayashi 1954 *Proceedings of Japan Academy* **30**, 165–169. Multidimensional quantification. II.

APPENDIX A: DEFINITIONS AND PROCEDURES FOR CALCULATING ACOUSTICAL FACTORS

All the physical factors described in the paper were calculated from recorded signals p_{jl} and p_{jr}. Index j indicates the sampled elements of the signal with constant interval σ ($j = 0, 1, 2, \ldots, L-1$). Indices l and r represent the left and right ears respectively.

A.1. LISTENING LEVEL (LL)

Listening level LL (dB) is defined as sound pressure level at each ear of the receiver relative to SPL at the reference position. LL is given as a geometrical mean of the left and right LLs. The value of LL at each ear is calculated as the autocorrelation function $\Phi_{ll,rr}(\tau)$ at $\tau = 0$ of the left and right signals p_{jl} and p_{jr}

$$\Phi_{ll,rr}(0) = \sum_{j=0}^{L-1} p_{jl,r}^2. \tag{A1}$$

Relative LL is obtained by

$$LL = 10\log_{10}\frac{\sqrt{\Phi_{ll}(0)\Phi_{rr}(0)}}{\Phi^{(ref)}(0)} \quad \text{if } p_{jl,r} \neq 0, \tag{A2}$$

where

$$\Phi^{(ref)}(0) = \sqrt{\Phi_{ll}^{(ref)}(0)\Phi_{rr}^{(ref)}(0)}. \tag{A3}$$

Here, $\Phi^{(ref)}(0)$ is the geometrical mean of the autocorrelation functions of signals at $\tau = 0$ at the reference position.

A.2. INITIAL TIME DELAY GAP BETWEEN THE DIRECT SOUND AND THE FIRST REFLECTION (Δt_1)

Initial time delay gap between the direct sound and the first reflection Δt_1 is defined by the time difference between the arrival time of the direct sound and that of the first reflection arriving at the ears. Here, the shorter of the left and right Δt_1's obtained from binaural impulse responses was selected as Δt_1 because the sound paths for the first reflection are different for each ear.

A.3. SUBSEQUENT REVERBERATION TIME (T_{sub})

Subsequent reverberation time T_{sub} was defined as the time required for a sound to decrease 60 dB after the arrival of the first reflection for an integrated decay curve. The integrated decay curve as a function of time can be obtained by squaring and integrating the impulse response. Linear regression for the initial 10-dB attenuation is done by the logarithmic transformation of the integrated decay curve. The T_{sub} value for each position is given by an arithmetic mean of the left and right T_{sub} values.

A.4. FACTORS OF INTERAURAL CROSS-CORRELATION FUNCTION ($IACC, \tau_{IACC}$, AND W_{IACC})

The definitions of $IACC$, τ_{IACC} and W_{IACC} as representative factors of interaural cross-correlation function are shown in Figure A1. The normalized interaural cross-correlation function is given by

$$\phi_{lr}(j\sigma) = \frac{\Phi_{lr}(\mathrm{j}\sigma)}{\sqrt{\Phi_{ll}(0)\Phi_{rr}(0)}}, \tag{A4}$$

where $\Phi_{ll}(0)$ and $\Phi_{rr}(0)$ represent the autocorrelation function ($\tau = 0$) of the signal at each ear respectively. The denominator is the geometrical mean of the sound energies arriving at the two ears, and $\Phi_{lr}(\mathrm{j}\sigma)$ is the crosscorrelation of the signals at both ears. The magnitude of interaural cross-correlation function is defined by

$$IACC = |\phi_{l,r}(\mathrm{j}\sigma)|_{max}, \tag{A5}$$

where $|\tau| \leqslant 1$ (ms).

IACC is a significant factor in determining the degree of subjective diffuseness as well as subjective preference in the sound field [4, 5, 13]. It represents the degree of similarity in the incident sound waves arriving at the two ears.

The interaural time delay at which the $IACC$ is determined as shown in Figure A1 is denoted as τ_{IACC}. When τ_{IACC} is zero, the frontal-sound source image and a well-balanced sound field may usually by perceived.

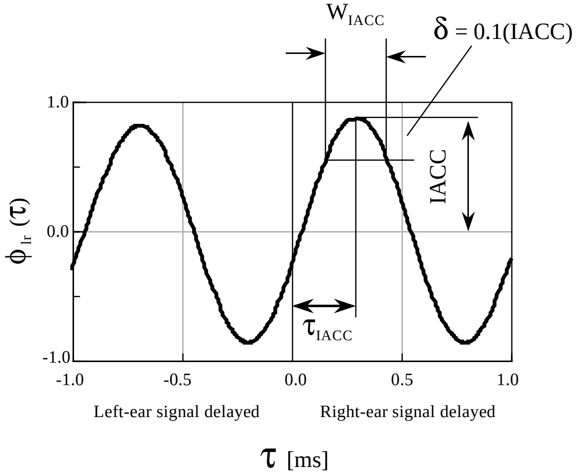

Figure A1. Definitions of the $IACC$, τ_{IACC}, and W_{IACC} for the interaural cross-correlation function.

The width of the interaural cross-correlation function, W_{IACC}, is defined as the interval of the delay time at 10% below the $IACC$. W_{IACC} is significant and is related to the ASW, which can be calculated by using $IACC$ and W_{IACC}.

APPENDIX B: FACTOR ANALYSIS [14-16]

The multiple-dimensional-factor analysis is briefly described here. The numeric values are given for each subcategory for each item and the responses synthesized.

In this analysis, all items do not need to be scalable. n cases are assumed. Let A be an outside variable and define s as 1, 2, ..., R (R is the number of items) and k as 1, 2, ..., K_s (K_s is the number of subcategories in the sth item). Since each case checks only one subcategory in each item, the behaviour pattern of the ith case is synthesized in the form

$$\alpha_i = \sum_{s=1}^{K_s} X_s(i) = \sum_{s=1}^{R} \left\{ \sum_{k=1}^{K_s} \delta_i(sk) X_{sk} \right\}, \tag{B1}$$

where

$$\sum_{k=1}^{K_s} \delta_i(sk) = 1 \tag{B2}$$

and $\delta_i(sk) = 1$ if the ith case comes under the kth subcategory in the sth item, 0 otherwise.

The total score of the ith case, α_i has a numerical value, since X_{sk} also has a numerical value.

The correlation coefficient ρ between A and a_i is written as

$$\rho(A, \alpha_i) = \frac{\frac{1}{n}\sum_{i=1}^{n}(A_i - \bar{A})(\alpha_i - \bar{\alpha})}{\sigma_A \sigma_\alpha}, \tag{B3}$$

where

$$\bar{A} = \frac{1}{n}\sum_{i=1}^{n} A_i, \qquad \sigma_A^2 = \frac{1}{n}\sum_{i=1}^{n}(A_i - \bar{A})^2 \tag{B4}$$

and

$$\bar{\alpha} = \frac{1}{n}\sum_{i=1}^{n}\alpha_i \qquad \sigma_\alpha^2 = \frac{1}{n}\sum_{i=1}^{n}(\alpha_i - \bar{\alpha})^2.$$

In order to obtain a maximum value, ρ, or to estimate the outside variable from the behaviour pattern, put $\bar{A} = 0$ and $\bar{\alpha} = 0$, because ρ is invariant under a shift of origin. Then the score of each subcategory can be determined by solving

$$\frac{\partial\rho}{\partial X_{sk}} = 0 \quad (s = 1, 2, \ldots, R;\; k = 1, 2, \ldots, K_s). \tag{B5}$$