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ERRATA

THE BOUNDARY CONDITION AT AN IMPEDANCE WALL IN A NON-UNIFORM DUCT WITH POTENTIAL MEAN FLOW

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In reference [1] the Myers acoustic boundary condition at an impedance wall [2] is restructured for computational convenience. The first part of [1] recasts the boundary condition for application to Finite Element applications. Section 3 of [1] suggests a form of the Myers boundary condition which may be more convenient for other applications, and this is presented as equation (16). There is an error in procedure in deriving this equation and the result given is only valid for two-dimensional ducts. Here a formal procedure is adopted and a more general result is given.

An alternate approach to the simplified boundary condition is available which produces a boundary condition useful for numerical models which are not based on the weighted residuals formulation. The Myers boundary condition [2] is given by equation (4) of reference [1]:

$$\rho_r \mathbf{v}\cdot\mathbf{n} = \mathrm{i}\eta_r\rho_r\zeta + \rho_r\mathbf{V}_r\cdot\nabla\zeta - \rho_r\zeta\mathbf{n}\cdot(\mathbf{n}\cdot\nabla)\mathbf{V}_r. \tag{1}$$

With $W = 1$, equations (7) of reference [1] can be used to deduce the alternate forms

$$\rho_r \mathbf{v}\cdot\mathbf{n} = \mathrm{i}\eta_r\rho_r\zeta + \nabla\cdot\rho_r\zeta\mathbf{V}_r - \mathbf{n}\cdot(\mathbf{n}\cdot\nabla)\rho_r\zeta\mathbf{V}_r \tag{2}$$

and

$$\rho_r \mathbf{v}\cdot\mathbf{n} = \mathrm{i}\eta_r\rho_r\zeta + \mathbf{n}\cdot\nabla\times(\mathbf{n}\times\rho_r\zeta\mathbf{V}_r). \tag{3}$$

The mean flow velocity $\mathbf{V} = V_{r_\tau}\boldsymbol{\tau}$, with $\boldsymbol{\tau}$ the unit vector tangent to the mean flow streamline at the duct surface. Let $\mathbf{n}\times\boldsymbol{\tau} = \mathbf{k}$, where $\mathbf{n}$ is normal to the duct wall, and $\mathbf{k}$ completes the orthogonal, curvilinear co-ordinate system, based on the flow streamlines. Then

$$\rho_r \mathbf{v}\cdot\mathbf{n} = \mathrm{i}\eta_r\rho_r\zeta + \mathbf{n}\cdot\nabla\times(\rho_r\zeta V_{r_\tau}\mathbf{k}). \tag{4}$$

With the vector identity $\nabla\cdot(\mathbf{u}\times\mathbf{v}) = (\nabla\times\mathbf{u})\cdot\mathbf{v}$-$(\nabla\times\mathbf{v})\cdot\mathbf{u}$ and the relation $\mathbf{k}\times\mathbf{n} = \boldsymbol{\tau}$, it can be determined that equation (4) can be written as

$$\rho_r \mathbf{v}\cdot\mathbf{n} = \mathrm{i}\eta_r\rho_r\zeta + \boldsymbol{\tau}\cdot\nabla(\rho_r\zeta V_{r_\tau}) + \rho_r\zeta V_{r_\tau}(\nabla\times\mathbf{k})\cdot\mathbf{n}. \tag{5}$$

This result, obtained with standard vector operations should be compared to the original result, equation (16) of reference [1]. The final term accounts for the previously neglected contributions of the rotation of the normal and tangential unit vectors. These extra terms are now examined in the context of general orthogonal curvilinear co-ordinates.

Karmacheti [3] shows that in terms of the scale factors h_1, h_2, h_3, corresponding to co-ordinates q_1, q_2, q_3 and corresponding unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ the curl operation is given by

$$\nabla \times \mathbf{A} = \frac{1}{h_1 h_2 h_3} \det \begin{bmatrix} h_1 \mathbf{e}_1 & h_2 \mathbf{e}_2 & h_3 \mathbf{e}_3 \\ \dfrac{\partial}{\partial q_1} & \dfrac{\partial}{\partial q_2} & \dfrac{\partial}{\partial q_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{bmatrix}. \tag{6}$$

for $\mathbf{k}$, $A_1 = 0$, $A_2 = 0$, $A_3 = 1$. The operation $(\nabla \times \mathbf{k}) \cdot \mathbf{n}$ is

$$\nabla \times \mathbf{k} \cdot \mathbf{n} = \frac{1}{h_1 h_2 h_3} \det \begin{bmatrix} h_1 \mathbf{n} & h_2 \boldsymbol{\tau} & h_3 \mathbf{k} \\ \dfrac{\partial}{\partial q_1} & \dfrac{\partial}{\partial q_2} & \dfrac{\partial}{\partial q_3} \\ h_1 & 0 & h_3 \end{bmatrix} \cdot \mathbf{n} = \frac{1}{h_2 h_3} \frac{\partial h_3}{\partial q_2}. \tag{7}$$

Karamcheti's results also show that

$$\boldsymbol{\tau} \cdot \nabla (\rho_r \zeta V_{r_\tau}) = \frac{1}{h_2} \frac{\partial (\rho_r \zeta V_{r_\tau})}{\partial q_2}. \tag{8}$$

The Myers boundary condition is then

$$\rho_r \mathbf{v} \cdot \mathbf{n} = \mathrm{i} \eta_r \rho_r \zeta + \frac{1}{h_2 h_3} \frac{\partial (h_3 \rho_r \zeta V_{r_\tau})}{\partial q_2}. \tag{9}$$

In the case of a two-dimensional duct $h_2 = 1$, $h_3 = 1$, $\partial h_3 / \partial q_2 = 0$, and equation (5) is

$$\rho_r \mathbf{V} \cdot \mathbf{n} = \mathrm{i} \eta_r \rho_r \zeta + \frac{\partial}{\partial \tau} (\rho_r \zeta V_{r_\tau}). \tag{10}$$

This is equation (16) of reference [1]. In the more general case of axially symmetric flow in an axisymmetric duct, cylindrical co-ordinates are used. In this case $h_2 = 1$, $h_2 = r$. r is the local duct radius, and is a function of q_2, the coordinate τ. Equation (5) becomes

$$\rho_r \mathbf{V} \cdot \mathbf{n} = \mathrm{i} \eta_r \rho_r \zeta + \frac{1}{r} \frac{\partial}{\partial \tau} (r \rho_r \zeta V_{r_\tau}) \tag{11}$$

which is not contained in equation (16) of reference [1]. As noted in reference [1], the normal component of mean flow velocity in the flow field has no role.

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