Journal of Sound and Vibration (2003) **259**(1), 177–192
doi:10.1006/jsvi.2002.5087, available online at http://www.idealibrary.com on IDEAL®

FREE-VIBRATION ANALYSIS OF LIQUID-FILLED TANK WITH BAFFLES

K. C. Biswal†

Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur 721302, *India*

S. K. Bhattacharyya

Department of Civil Engineering, Indian Institute of Technology, Kharagpur 721302, *India*

AND

P. K. Sinha

Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur 721302, *India.*
E-mail: pksinha@aero.iitkgp.ernet.in

(*Received* 25 *July* 2001, *and in final form* 2 *April* 2002)

The natural frequencies of liquid in a liquid-filled cylindrical rigid tank without and with baffles are evaluated. An annular plate is used as a baffle, which is fitted to the inner periphery of a cylindrical tank. Both rigid and flexible baffles are considered. Finite elements are used to discretize both the liquid and the structural domain. The slosh frequencies of liquid are computed for different dimensions, thicknesses and positions of baffles, both rigid and flexible considering the circumferential wave number as one. The axisymmetric and other asymmetric modes are not studied. The results obtained for rigid baffle case are comparable with the existing results. The coupled vibration frequencies of the tank–flexible-baffle system are computed considering the effect of sloshing of liquid.

1. INTRODUCTION

The motion of liquid with a free surface is of great concern in many engineering disciplines such as fuel sloshing of rocket propellant, oil oscillation in large storage tanks, water oscillation in a reservoir due to earthquake, sloshing of water in pressure-suppression pools of boiling water reactors and several others. The current practice of using lightweight structural materials has attracted the attention of many researchers to study the interaction of liquid with the elastic structural components. The knowledge of the natural frequencies of the liquid alone or the elastic structure alone is not adequate to understand the complex liquid–structure interaction problems. The study of coupled vibration frequencies arising out of interaction of liquid and structure is important.

Lindholm *et al.* [1] had determined experimentally the resonant breathing frequencies and mode shapes for a thin-walled, cylindrical shell containing an inviscid incompressible liquid. Arya *et al.* [2] had theoretically computed the dynamic properties of thin cylindrical containers. The isotropic plate submerged in an ideal incompressible fluid is analyzed under free vibration by Chowdhury [3]. Experimental investigations on the vibrational characteristics of fluid-coupled co-axial cylinders were carried out by Chu and Brown [4].

† On study leave from REC Rourkela 769 008, India.

0022-460X/02/$35.00

Haroun and Housner [5] had presented the results on the free lateral vibrations of ground-supported cylindrical liquid storage tanks. They had carried out both theoretical analysis and experimental investigation. Balendra *et al.* [6] had presented the finite element results on coupled system frequencies of liquid cylindrical tanks. The effect of sloshing of fluid was considered. A coupled fluid–structure finite element method which considers the sloshing effect was developed for the seismic analysis of fluid-filled system of various geometries by Liu *et al.* [7]. Olson and Bathe [8] had used the displacement-based fluid finite elements for calculating the frequencies of fluid and fluid–structure system. Experimental studies were carried out to investigate the dynamic stability of cantilever cylindrical shell partially filled with liquid under horizontal excitation by Chiba *et al.* [9]. Gupta and Hutchinson [10] carried out studies on ground-supported cylindrical storage tanks in an axisymmetric manner. A finite element method with a reduction technique had been used for solving the eigenproblem of liquid container coupling by Qinque and Lidu [11]. The vibration studies of base-isolated liquid storage tanks were carried out by Bo and Jia-Xiang [12]. The coupled hydroelastic frequencies of an incompressible and frictionless liquid in a circular container, of which the free fluid surface is covered by a flexible membrane or an elastic plate, had been determined analytically by Bauer [13]. Amabili [14] presented an analytical solution for the free vibration of simply supported, partially filled, horizontal circular shell, without considering the effect of sloshing. Goncalves and Ramos [15] presented an effective modal solution for evaluating the free vibration characteristics of vertical, thin, circular cylindrical shells, partially or completely filled with liquid and subjected to any variationally consistent set of boundary conditions on the lower and upper boundaries. Chiba [16] studied the free vibration characteristics of a partially liquid filled and partially submerged, clamped–free circular cylindrical shell. Bermudez *et al.* [17] carried out a finite element analysis for the solution of incompressible fluid–structure vibration problems. Analytical solution for the non-axisymmetric free vibrations of simply supported, fluid-filled orthotropic cylindrical shell was derived without considering the sloshing effect by Weiqiu *et al.* [18]. Amabili *et al.* [19] presented the dynamics of isotropic cylindrical shell tanks with a flexible bottom and ring stiffeners. Gedikli and Ergüven [20] had studied the effects of baffle on the natural frequencies of liquid in a cylindrical tank. However, the baffle was assumed to be rigid.

The rigidity of baffle may be achieved primarily by providing higher thickness of baffle, which as a result reduces the liquid storage capacity of the tank. The baffle preferably should be thin and lightweight for liquid-filled containers in general and liquid fuel-filled space vehicles in particular. Because of thinness and lightweight, the baffle remains no longer a rigid element. A dynamic interaction exists between the liquid and the flexible baffle. A coupled liquid–structure finite element method which considers the sloshing effect is developed in this study for calculating the natural frequencies of liquid and liquid–baffle system considering the circumferential wave number as one. The axisymmetric and other asymmetric modes are not considered in the present investigation.

2. GOVERNING EQUATIONS AND FINITE ELEMENT FORMULATION

A liquid-filled cylindrical rigid tank with a flexible baffle is shown in Figure 1. An annular plate is used as a baffle, which is fitted to the inner periphery of the tank. The base of the tank is also assumed as rigid. The liquid is assumed to be incompressible and inviscid resulting in an irrotational flow. The governing differential equation for the liquid in terms of pressure variable is

$$\nabla^2 P = 0 \tag{1}$$

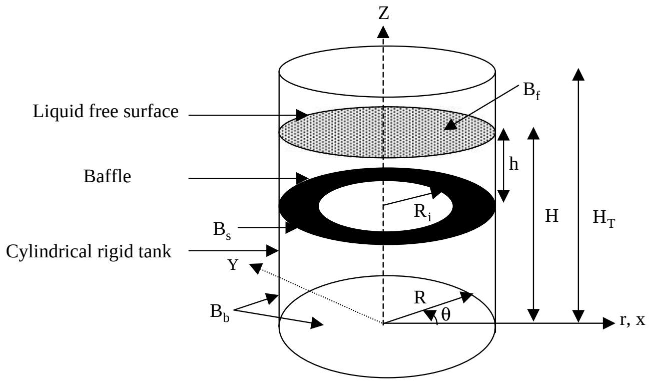

Figure 1. A rigid cylindrical tank with a baffle.

in which $P = P(x, y, z, t)$ is the liquid dynamic pressure and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$. Equation (1) is solved using finite element technique and with the appropriate time-dependent boundary conditions as specified below

(a) At liquid–baffle interface

$$\partial P/\partial n = -\rho_f \ddot{d}_n \quad \text{on } B_s. \tag{2}$$

where ρ_f is the density of the liquid, d_n is the displacement of the baffle and n is outwardly drawn normal to the baffle surface.

(b) At liquid-free surface

Considering the effect of small-amplitude waves on a liquid surface, the condition yields

$$\partial^2 P/\partial t^2 + g\partial P/\partial n = 0 \quad \text{on } B_f. \tag{3}$$

However, if sloshing is ignored, then $P = 0$.

(c) At the bottom of the tank and the tank wall

$$\partial P/\partial n = 0 \quad \text{on } B_b \tag{4}$$

2.1. IDEALIZATION OF BAFFLE

The baffle in this case is an annular plate which is fitted around the internal periphery of the rigid tank. The assumed displacement function for the transverse deflection w is expressed in the polar co-ordinates (r and θ) considering the circumferential wave number as one.

$$w = (\alpha_1 + \alpha_2 r + \alpha_3 r^2 + \alpha_4 r^3)\cos\theta \tag{5}$$

and the nodal displacements for each element are

$$\{d\} = \{w_1 \ \ \beta_1 \ \ w_2 \ \ \beta_2\}, \tag{6}$$

where w is the transverse displacement and β the rotation.

The element stiffness and mass matrices are given by

$$[K_s]^e = [C^{-1}]^{\mathrm{T}}[K][C^{-1}], \qquad [M_s]^e = [C^{-1}]^{\mathrm{T}}[M][C^{-1}], \tag{7, 8}$$

where

$$[C]=\begin{bmatrix}1 & R_1 & R_1^2 & R_1^3\\ 0 & 1 & 2R_1 & 3R_1^2\\ 1 & R_2 & R_2^2 & R_2^3\\ 0 & 1 & 2R_2 & 3R_2^2\end{bmatrix},\qquad [K]=\frac{\pi E t^3}{12(1-\upsilon^2)}\begin{bmatrix}k11 & 0 & k13 & k14\\ 0 & 0 & 0 & 0\\ k31 & 0 & k33 & k34\\ k41 & 0 & k43 & k44\end{bmatrix},$$

$$[M]=\pi\rho t\begin{bmatrix}m11 & m12 & m13 & m14\\ m21 & m22 & m23 & m24\\ m31 & m32 & m33 & m34\\ m41 & m42 & m43 & m44\end{bmatrix}$$

where R_1 and R_2 are inner and outer radii, respectively, for the annular ring element.
The elements of the matrices are:

$$k11=-(1{\cdot}5-\upsilon)(R_2^{-2}-R_1^{-2}),\qquad k31=k13=-3{\cdot}0\ln(R_2/R_1),$$

$$k33=(3{\cdot}5+\upsilon)(R_2^2-R_1^2),\qquad k41=k14=-(6+2\upsilon)(R_2-R_1),$$

$$k43=k34=(6+2\upsilon)(R_2^3-R_1^3),\qquad k44=(16+4\upsilon)(R_2^4-R_1^4),$$

$$m11=(R_2^2-R_1^2)/2,\qquad m12=m21=(R_2^3-R_1^3)/3,$$

$$m13=m31=(R_2^4-R_1^4)/4=m22,$$

$$m14=m41=(R_2^5-R_1^5)/5=m23=m32$$

$$m24=m42=(R_2^6-R_1^6)/6=m33,\qquad m34=m43=(R_2^7-R_1^7)/7,$$

$$m44=(R_2^8-R_1^8)/8.$$

2.2. IDEALIZATION OF LIQUID

The finite element formulation is based on Galerkin weighted residual method. A four-noded isoparametric quadrilateral axisymmetric element is employed to discretize the liquid domain. The liquid dynamic pressure ($\bar{P}$) is approximated as

$$\bar{P}(x,y,z,t)=\sum_{j=1}^{N}N_j(x,y,z)P_j(t) \tag{9}$$

in which N_j are the shape functions and $P_j(t)$ are the time-dependent nodal pressures. Applying divergence theorem to the residual form of governing differential equation for the liquid and minimizing the energy function , we get

$$\int_V\left(\frac{\partial N_i}{\partial x}\sum_1^N\frac{\partial N_j}{\partial x}P_j+\frac{\partial N_i}{\partial y}\sum_1^N\frac{\partial N_j}{\partial y}P_j+\frac{\partial N_i}{\partial z}\sum_1^N\frac{\partial N_j}{\partial z}P_j\right)\mathrm{d}V=\int_B N_i\frac{\partial\bar{P}}{\partial n}\,\mathrm{d}s \tag{10}$$

in which $B=B_s+B_f+B_b$ (B_s, B_f and B_b are defined in Figure 1).

Substituting equations (2), (3) and (4) into equation (10), we have

$$\int_V \left(\frac{\partial N_i}{\partial x} \sum_1^N \frac{\partial N_j}{\partial x} P_j + \frac{\partial N_i}{\partial y} \sum_1^N \frac{\partial N_j}{\partial y} P_j + \frac{\partial N_i}{\partial z} \sum_1^N \frac{\partial N_j}{\partial z} P_j \right) \mathrm{d}V$$

$$= - \int_{B_s} \rho_f N_i \ddot{d}_n ds - \frac{1}{g} \int_{B_f} N_i \sum_1^N N_j \ddot{P}_j \, \mathrm{d}s. \tag{11}$$

Equation (11) may be reduced to

$$[M_f]\{\ddot{P}\} + [K_f]\{P\} = \{F_p\} \tag{12}$$

in which the elements of matrices $[M_f]$, $[K_f]$, and $\{F_p\}$ are given by

$$M_{ij} = \frac{1}{g} \sum \int_{B_f} N_i N_j \, \mathrm{d}s \tag{13}$$

$$K_{ij} = \sum \int_V \left[\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right] \mathrm{d}V \tag{14}$$

$$F_i = - \sum \int_{B_s} \rho_f N_i \ddot{d}_n \, \mathrm{d}s. \tag{15}$$

Equation (12) may be rewritten as

$$[M_f]\{\ddot{P}\} + [K_f]\{P\} = -\rho_f [S]\{\ddot{d}\} \tag{16}$$

where

$$[S] = \int_{B_s} [N_f]^{\mathrm{T}} [N_s] \, \mathrm{d}s, \tag{17}$$

with $[N_f]$ the shape functions for the liquid element and $[N_s]$ the shape functions for the structural element.

The equations of motions of flexible baffle when subjected to nodal forces due to liquid dynamic pressure takes the following form:

$$[M_s]\{\ddot{d}\} + [K_s]\{d\} = [S]^{\mathrm{T}}\{P\}. \tag{18}$$

Equations (16) and (18) may be written in matrix form as

$$\begin{bmatrix} [M_s] & [0] \\ \rho_f[S] & [M_f] \end{bmatrix} \begin{Bmatrix} \ddot{d} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} [K_s] & -[S]^{\mathrm{T}} \\ [0] & [K_f] \end{bmatrix} \begin{Bmatrix} d \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \tag{19}$$

Equation (19) leads to a non-standard, unsymmetric eigenvalue problem. Thus, it is difficult to find the eigenvalues and corresponding eigenvectors for large-size matrices. To avoid the numerical difficulty, the sloshing effect was neglected by many researchers. The effect of sloshing is taken into account in the present coupled formulation. Equation (19) is rearranged in order to obtain the symmetric matrices for the coupled system, as given by

$$\begin{bmatrix} [K_s] & [0] \\ [0] & [M_f] \end{bmatrix} \begin{Bmatrix} \ddot{d} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} [K_s][M_s]^{-1}[K_s] & -[K_s][M_s]^{-1}[S]^{\mathrm{T}} \\ -\rho_f[S][M_s]^{-1}[K_s] & [K_f]+\rho_f[S][M_s]^{-1}[S]^{\mathrm{T}} \end{bmatrix} \begin{Bmatrix} d \\ P \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \tag{20}$$

The derivation of equation (20) is given in Appendix B.

Denoting ω_n the nth natural frequency of the coupled system and $\{\phi_n\}$ the corresponding mode shape vector, equation (20) becomes

$$[\hat{K}]\{\phi_n\} - \omega_n^2[\hat{M}]\{\phi_n\} = \{0\}, \tag{21}$$

where

$$[\hat{K}] = \begin{bmatrix} [K_s][M_s]^{-1}[K_s] & -[K_s][M_s]^{-1}[S]^{\mathrm{T}} \\ -\rho_f[S][M_s]^{-1}[K_s] & [K_f]+\rho_f[S][M_s]^{-1}[S]^{\mathrm{T}} \end{bmatrix}, \tag{22}$$

$$[\hat{M}] = \begin{bmatrix} [K_s] & [0] \\ [0] & [M_f] \end{bmatrix}. \tag{23}$$

In added mass formulation, the sloshing is neglected and the pressures at free surface nodes are considered as zero. The free vibration equation for coupled system is derived as

$$[K_s]\{d\} + [[M_s] + \rho_f[S]^{\mathrm{T}}[K_f]^{-1}[S]]\{\ddot{d}\} = \{0\}. \tag{24}$$

The free vibration equation for tank–rigid-baffle system is obtained from equation (16) by setting the acceleration vector $\{\ddot{d}\}$ equal to zero, as given below

$$[M_f] + \{\ddot{P}\} + [K_f]\{P\} = \{0\}. \tag{25}$$

The liquid mass matrix has contribution only from the free surface nodes. Guyan condensation technique is applied to equations (20) and (25) prior to the solution of eigenvalue problem.

3. NUMERICAL EXAMPLES, RESULTS AND DISCUSSION

3.1. EXAMPLE—1

A cylindrical rigid tank of radius (R) and liquid depth (H) is considered for the analysis of natural frequencies of the liquid. An annular plate is used as a baffle, which is fitted to the inner periphery of the tank. The thickness of the baffle is 0·004 m and assumed to be rigid. The density of the liquid (ρ_f) is 1000·0 kg/m^3. A single baffle is placed at a depth of h from the liquid-free surface. The natural frequencies expressed as non-dimensional parameters ($\bar{\omega}_n = \omega_n\,(R/g)^{1/2}$) of liquid in the cylindrical tank without baffle are computed for different H/R ratios and for two different radial modes ($n = 1$ and 2) and its squared values are presented in Figure 2 for comparison. It is observed that the computed results are quite comparable with the existing results [20]. The squared natural frequency parameters of

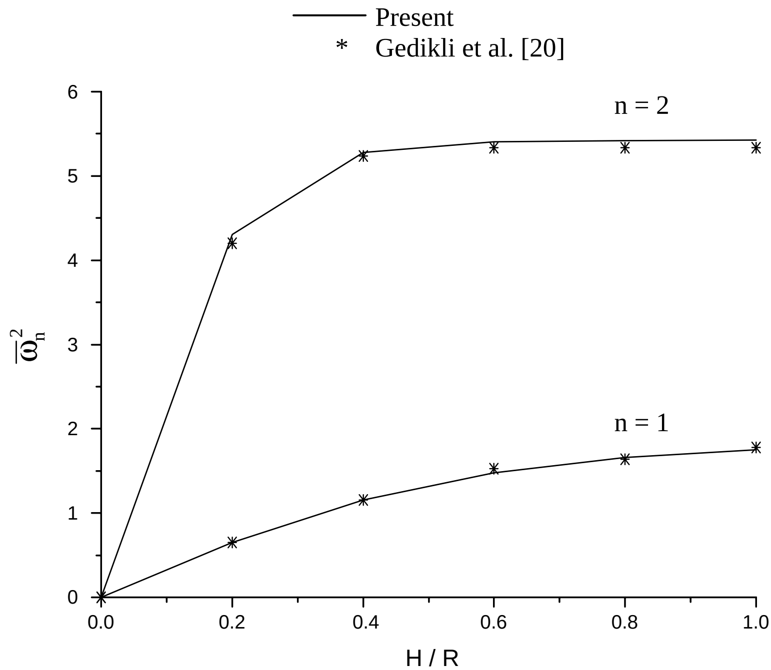

Figure 2. Variation of natural frequencies of liquid with H/R.

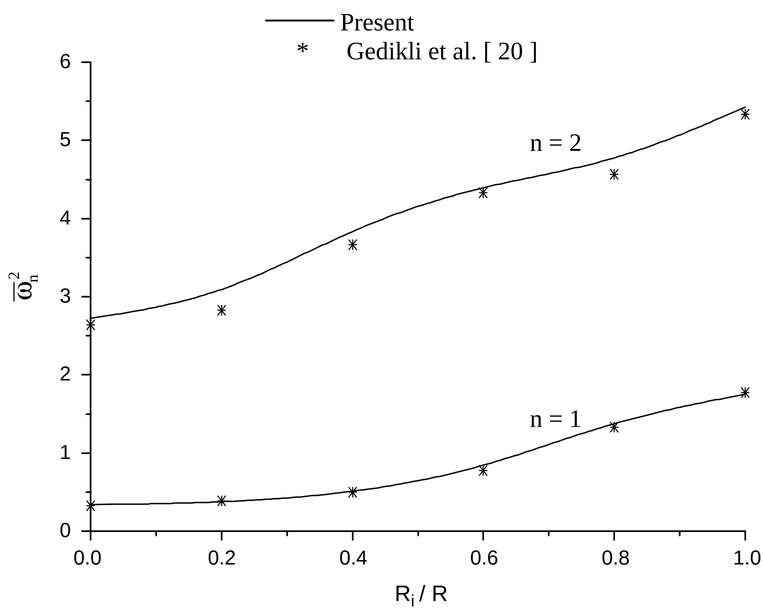

Figure 3. Variation of natural frequencies of liquid with $R_i/R(H/R = 1{\cdot}0$ and $h/H = 0{\cdot}1)$.

liquid in the presence of rigid baffle are evaluated and presented in Figures 3 and 4 for baffle positions $h/H = 0{\cdot}1$ and $0{\cdot}3$ respectively. The results are compared with the existing ones [20]. The present results are in good agreement with the available results.

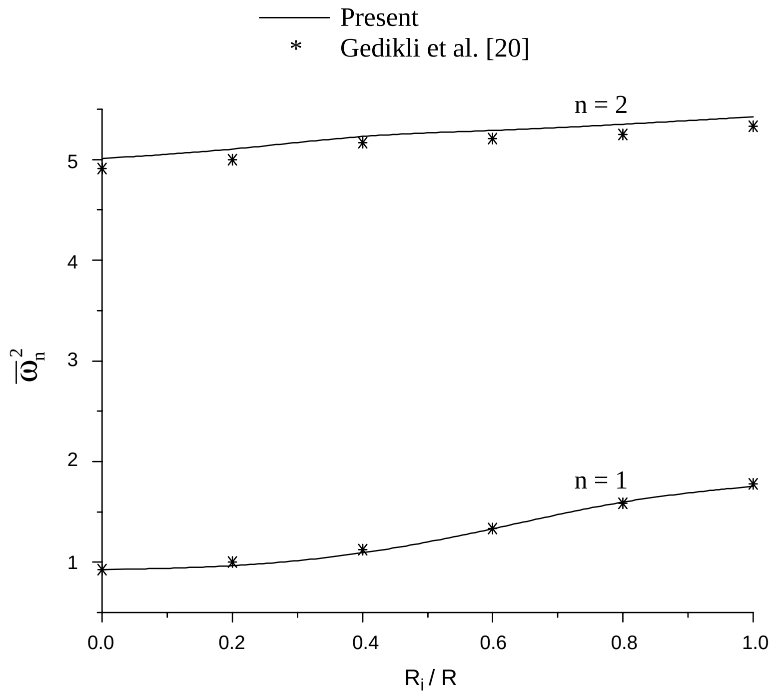

Figure 4. Variation of natural frequencies of liquid with $R_i/R(H/R = 1{\cdot}0$ and $h/H = 0{\cdot}3)$.

3.2. EXAMPLE—2

In this example, a rigid cylindrical tank–rigid-baffle system is considered for the analysis of slosh frequencies of the liquid. The radius (R) of the tank is 1·0 m and liquid depth (H) in the tank is 1·0 m. The density of the liquid (ρ_f) is 1000·0 kg/m^3. An annular plate of thickness 0·004 m is used as baffle and placed at different depths (h) from the liquid-free surface. The slosh frequency parameters ($\bar{\omega}_1 = \omega_1(R/g)^{1/2}$) of liquid are determined for different (h/H) ratios and for different ratios of inner and outer radii R_i/R of baffle and are presented in Table 1.

From Table 1, it is observed that the baffle has a greater influence on the slosh frequency parameters of liquid when placed close to the liquid-free surface. The effect of baffle on the slosh frequency parameter gradually diminished when the baffle is moved towards the bottom of the container and is almost negligible when it is placed very near to the bottom. The slosh frequency parameters of liquid increase with the increase in R_i/R ratio. This is due to the fact that the area of baffle decreases with the increase in R_i/R ratio. The slosh frequency parameter of liquid for $h/H = 0{\cdot}8$ and $R_i/R = 0{\cdot}8$ is nearly equal to the slosh frequency parameter of the tank without baffle. It is observed that when the baffle is placed at a depth of 0·8 H , the maximum variation of slosh frequency parameters of liquid in comparison with tank without baffle is only 2·5%.

3.3. EXAMPLE—3

A cylindrical rigid tank with a flexible baffle is considered. The dimension of the tank and liquid depth are assumed similar to that of example 2. The baffle is placed at a depth of

TABLE 1

Slosh frequency parameters $\bar{\omega}_1 = \omega_1 (R/g)^{1/2}$ of liquid in a tank with a rigid baffle for various positions of baffle from the liquid-free surface and for different R_i/R ratios

h/H ratio	R_i/R ratio 0·2	0·4	0·6	0·8
0·0	3·95036	2·63205	2·06027	1·66898
0·01	0·19814	0·24632	0·36033	0·70242
0·05	0·43293	0·51788	0·70854	1·10269
0·1	0·60075	0·69277	0·88887	1·19425
0·2	0·82220	0·90030	1·05742	1·24909
0·3	0·97426	1·03400	1·14969	1·27612
0·4	1·08397	1·12789	1·20985	1·29339
0·5	1·16349	1·19493	1·25116	1·30502
0·6	1·22075	1·24267	1·27977	1·31295
0·7	1·26161	1·27635	1·29938	1·31825
0·8	1·29054	1·29979	1·31249	1·32168

Note: The slosh frequency parameter of liquid in a tank without baffle is 1·32400.

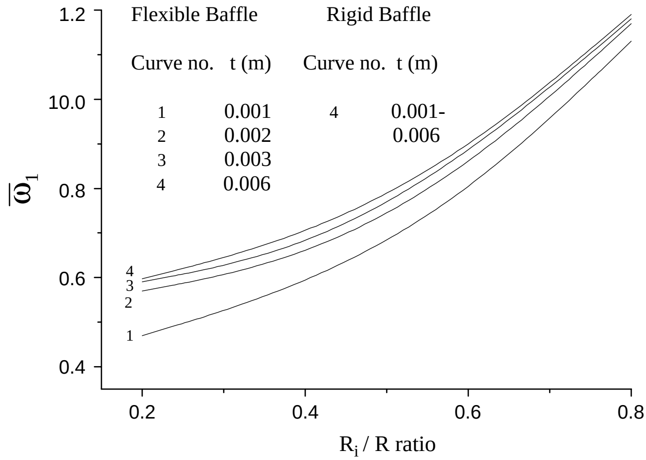

Figure 5. Effect of baffle thickness on slosh frequency parameter ($\bar{\omega}_1$) of liquid in a tank with a rigid/flexible baffle.

h from the liquid-free surface. The h/H ratio is taken as 0·1. The E and ρ_s of the flexible baffle material are $2{\cdot}0 \times 10^{11}$ N/m^2 and 7850 kg/m^3 respectively. The density of the liquid (ρ_f) is 1000·0 kg/m^3. The slosh frequency parameters ($\bar{\omega}_1 = \omega_1 (R/g)^{1/2}$) of the liquid are presented in Figure 5 for various thicknesses of flexible baffle and for different ratios of inner and outer radii R_i/R of baffle. The slosh frequency parameters of liquid for rigid baffle case are also presented in Figure 5.

The liquid slosh frequency parameters are computed for baffle thicknessess of 0·001, 0·002, 0·003, and 0·006 m. When the thickness of baffle is 0·001 m, there is a noticeable difference in values of slosh frequency parameters of liquid obtained for rigid and flexible baffles as shown in Figure 5. This illustrates the effect of flexibility of baffle on the slosh frequency parameters. The difference is reduced with the increase of baffle thickness. The

TABLE 2

Slosh frequency parameters $\bar{\omega}_1 = \omega_1\ (R/g)^{1/2}$ of liquid in a tank with two rigid/flexible baffles for various depths of the second baffle (h_2) from the liquid-free surface and for different R_i/R ratio

	R_i/R ratio			
h/H ratio	0·2	0·4	0·6	0·8
0·2	3·79248 (3·87686)	2·44074 (2·55091)	1·95892 (2·00353)	1·63857 (1·64276)
0·3	3·84165 (3·90976)	2·48720 (2·56590)	1·97246 (2·00466)	1·64108 (1·64447)
0·4	3·86432 (3·92719)	2·52102 (2·58399)	1·98961 (2·01541)	1·64710 (1·64990)
0·5	3·87584 (3·93664)	2·54467 (2·59936)	2·00000 (2·02787)	1·65344 (1·65586)
0·6	3·88226 (3·94210)	2·56110 (2·61117)	2·03615 (2·05651)	1·65893 (1·66106)
0·7	3·89004 (3·94960)	2·57260 (2·61994)	2·03968 (2·05868)	1·66345 (1·66541)
0·8	3·90928 (3·96946)	2·58443 (2·63101)	2·04181 (2·05882)	1·66611 (1·66796)

Bracketed values are for rigid baffle case.

Note:

(1) The slosh frequency parameters of liquid in a tank with a rigid baffle at free surface are 3·97825, 2·63783, 2·06263 and 1·66997 for $R_i/R = 0{\cdot}2$, 0·4, 0·6 and 0·8 respectively.

(2) The slosh frequency parameters of liquid in a tank with a flexible baffle at free surface are 3·91752, 2·59430, 2·04955 and 1·66878 for $R_i/R = 0{\cdot}2$, 0·4, 0·6 and 0·8 respectively.

liquid slosh frequency parameters for rigid baffle case are obtained for the above-mentioned thicknesses. It is seen from Figure 5 that the variation of thickness of rigid baffle has negligible effect on the liquid slosh frequency parameters. At higher thickness of baffle, i.e., $t = 0{\cdot}006$ m the slosh frequency parameters are determined using the tank–flexible-baffle formulation (equation (20)) and compared with slosh frequency parameters of liquid obtained from tank–rigid baffle formulation (equation (25)). Both formulations provide nearly same results for liquid slosh frequency parameters at a higher thickness of baffle. This comparison validates the accuracy of the present formulation for the coupled system.

3.4. EXAMPLE—4

A cylindrical tank with two baffles is considered. The dimension of the tank and the liquid depth are assumed to be same as in example 2. Both the baffles are either rigid or flexible. The first baffle is placed at the liquid-free surface and the second baffle of same geometry and dimension is placed towards the bottom of the container. The thickness of each baffle is taken as 0·002 m. The E and ρ_s of the flexible baffle material are $2{\cdot}0 \times 10^{11}$ N/m^2 and 7850 kg/m^3 respectively. The density of the liquid (ρ_f) is 1000·0 kg/m^3. The slosh frequency parameters ($\bar{\omega}_1 = \omega_1\ (R/g)^{1/2}$) of liquid for different ratios of inner and outer radii R_i/R of baffle are presented in Table 2.

It is observed from the results presented in Table 2 that the second rigid baffle irrespective of its position has negligible effect on the slosh frequency parameters of liquid. The reason is

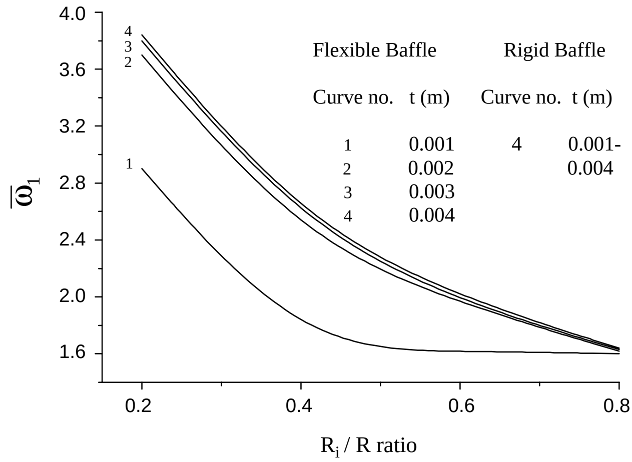

Figure 6. Effect of baffle thickness on slosh frequency parameter ($\bar{\omega}_1$) of liquid in a tank with two rigid/flexible baffles.

that the liquid below the first rigid baffle behaves more like a rigid mass. No additional effect is observed from the use of the second rigid baffle. However, the second flexible baffle placed below the first flexible baffle has a little effect on the slosh frequency parameters of liquid. The use of two or more baffles either rigid or flexible is of greater importance in a system, where the liquid level in the tank varies.

3.5. EXAMPLE—5

Example 4 is considered here again. But the second baffle is placed at $h_2/H = 0{\cdot}2$ from the liquid-free surface. The thickness of both first and second baffles is varied equally to study the effect of flexibility of baffles on the liquid slosh frequency parameters. The liquid slosh frequency parameters are computed for various thicknesses of baffles and different ratios of inner and outer radii, R_i/R of baffle and are shown in Figure 6.

It is observed from Figure 6 that there is no remarkable difference in slosh frequency parameters of liquid obtained from rigid and flexible baffle formulations for a higher thickness of baffle, i.e., $t = 0{\cdot}004$ m for all R_i/R ratios. This indicates that the flexible baffle behaves rigidly at higher thickness.

3.6. EXAMPLE—6

The cylindrical rigid container with a flexible baffle is considered. The dimensions of tank and liquid depth are assumed to be same as in example 2. The ratio of inner and outer radii, R_i/R of baffle is assumed to be 0·8. The E and ρ_s of the baffle material are $2{\cdot}0 \times 10^{11}$ N/m^2 and 7850 kg/m^3, respectively. The density of the liquid (ρ_f) is 1000·0 kg/m^3. The slosh frequency parameters of liquid and coupled vibration frequency parameters of the coupled system for baffle thickness, $t = 0{\cdot}0009$ and 0·001 m are presented in Tables 3 and 4 respectively . The baffle is positioned at 0·1 m from the liquid free surface, i.e., $h/H = 0{\cdot}1$.

It is observed from Table 3 that when the thickness of baffle is equal to 0·0009 m, there is no clear separation between the slosh frequency parameters of liquid and coupled vibration

TABLE 3

Natural frequency parameters $\bar{\omega}_n = \omega_n(R/g)^{1/2}$ *of the tank–flexible baffle system for a baffle thickness of* $t = 0{\cdot}0009\,\mathrm{m}$

Type of Frequency Parameters	Mode no. n	Tank–flexible-baffle system (sloshing considered)	Tank–flexible-baffle system (sloshing neglected)	Tank–rigid-baffle system (sloshing considered)
Sloshing frequency parameters	1	1·03085	—	1·19900
	2	2·16731	—	2·16942
	3	2·91188	—	2·91433
	4	3·56137	—	3·56698
	5	4·18144	—	4·18571
	6	4·80711	—	4·80826
	7	5·45028	—	5·45028
	8	6·08825	—	6·08835
	9	6·64022	—	6·64038
	10	6·95774	—	6·95922
	11	9·72226	—	9·72261
Coupled vibration frequency parameters	12	8·59862	8·64674	—
	13	155·46206	155·45819	—
	14	6534·20591	6534·20587	—
	15	34352·10610	34352·10606	—

TABLE 4

Natural frequency parameters $\bar{\omega}_n = \omega_n(R/g)^{1/2}$ *of the tank–flexible-baffle system for a baffle thickness of* $t = 0{\cdot}001\,\mathrm{m}$

Type of frequency Parameters	Mode no. n	Tank–flexible-baffle system (sloshing considered)	Tank–flexible-baffle system (sloshing neglected)	Tank–rigid-baffle system (sloshing considered)
Sloshing frequency parameters	1	1·13292	—	1·19884
	2	2·16821	—	2·16932
	3	2·91294	—	2·91431
	4	3·56375	—	3·56696
	5	4·18321	—	4·18567
	6	4·80759	—	4·80824
	7	5·45028	—	5·45028
	8	6·08829	—	6·08835
	9	6·64026	—	6·64034
	10	6·95848	—	6·95914
	11	9·72117	—	9·72117
Coupled vibration frequency parameters	12	10·19182	10·10441	—
	13	180·40726	180·40425	—
	14	7441·73804	7441·73802	—
	15	38591·17357	38591·17349	—

frequency parameters of tank–flexible baffle system. The liquid frequency parameters and coupled vibration frequency parameters are well separated at a thickness of 0·001 m as shown in Table 4, which is valid for higher thickness as well. Thus, beyond the thickness cited, though the baffle is still flexible, the baffle has negligible effect on the slosh frequency

TABLE 5

Thickness of baffle for well-separated slosh frequency parameters and coupled vibration frequency parameters in a tank–flexible-baffle system

R_i/R ratio	Thickness of baffle (t) (m)	$t/(R - R_i)$
0·2	0·0039	1/205
0·4	0·0037	1/162
0·6	0·0025	1/160
0·8	0·0010	1/200

parameters of the liquid and sloshing of liquid has no effect on the coupled vibration frequency parameters of the coupled system.

The thickness of baffle for obtaining a distinct spectrum of slosh frequency parameters and coupled vibration frequency parameters are presented in Table 5. It is observed that the thickness of baffle decreases with the increase in R_i/R ratios for obtaining a well-separated frequency spectrum.

3.7. EXAMPLE—7

The cylindrical container with two flexible baffles are now considered. The dimensions of tank and liquid depth are assumed to be same as in example 2. The first baffle is placed at liquid-free surface and the second baffle of same geometry and dimension is kept at 0·1 m from the liquid-free surface. The ratio of inner and outer radius of baffle, i.e., $R_i/R = 0{\cdot}8$. The E and ρ_s of the baffle material are $2{\cdot}0 \times 10^{11}$ N/m^2 and 7850 kg/m^3 respectively. The density of the liquid (ρ_f) is 1000·0 kg/m^3. The slosh frequency parameters of liquid and coupled vibration frequency parameters of the coupled system for the thickness of baffle, $t = 0{\cdot}0011$ m are presented in Table 6.

The following observations are made from the results listed in Table 6

(1) When thickness is 0·0011 m, a well-defined demarcation line exists for separation between the liquid frequency parameters and the coupled vibration frequency parameters. The flexibility of baffle does have negligible effects on the slosh frequency parameters of the liquid.
(2) The sloshing of liquid has no effect on the coupled vibration frequency parameters of the coupled system.

The thicknesses of baffle for obtaining well-separated slosh frequencies and coupled vibration frequencies are listed in Table 7.

4. CONCLUSIONS

The use of rigid baffle in a liquid-filled cylindrical rigid tank is examined. The slosh frequency parameters of liquid are computed for various locations of baffle in the tank. The baffle has appreciable effect on the slosh frequency parameters of liquid when placed very close to liquid-free surface for all R_i/R ratios. The flexibility of baffle is also taken into account. The flexibility of baffle does have an effect on the liquid slosh frequency parameters

TABLE 6

Natural frequency parameters $\bar{\omega}_n = \omega_n (R/g)^{1/2}$ of the tank–two-flexible-baffles system

Type of frequency parameters	Mode no. n	Tank–flexible-baffle system (sloshing considered)	Tank–rigid-baffle system (sloshing considered)
Sloshing frequency parameters	1	1·61821	1·64371
	2	2·68074	2·69785
	3	3·46525	3·48281
	4	4·20936	4·23083
	5	4·97334	5·00258
	6	5·76248	5·80411
	7	6·50089	6·55367
	8	7·00831	7·25295
	9	9·38139	9·38139
	10	10·10243	—
	11	35·91356	—
Coupled vibration frequency parameters	12	150·43592	—
	13	221·55265	—
	14	10598·60098	—
	15	11673·63356	—
	16	25542·33971	—
	17	35067·88949	—

TABLE 7

Thickness of baffle for well-separated slosh frequency parameters and coupled vibration frequency parameters in a tank–two-flexible-baffles system

R_i/R ratio	Thickness of baffle (t) (m)	$t/(R - R_i)$
0·2	0·0021	1/380
0·4	0·0026	1/230
0·6	0·0022	1/181
0·8	0·0011	1/181

up to certain thickness of baffle. Further increase in thickness of baffle does not have noticeable effect on the liquid slosh frequency parameters.

A liquid-filled rigid tank with two baffles is studied under free vibration analysis. Both baffles are either rigid or flexible. The first baffle is kept at liquid-free surface and the second baffle of same geometry and dimension is placed below it. The second baffle may not be appropriate for use in a rigid tank containing constant depth of liquid. However, for a varying liquid depth situation, the second baffle may be effective.

The validity and accuracy of the formulation developed in this investigation for the coupled system is checked by comparing the liquid slosh frequency parameters obtained through the rigid baffle formulation for a higher thickness. The present coupled formulation for the free vibration analysis of liquid-filled cylindrical tank–flexible-baffle system can be successfully used to compute the low frequencies associated with liquid sloshing modes and high frequencies associated with the coupled vibration modes.

REFERENCES

1. U. S. LINDHOLM, D. D. KANA and N. H. ABRAMSON 1962 *Journal of Aerospace Sciences* **29**, 1052–1059. Breathing vibrations of a circular cylindrical shell with an internal liquid.
2. A. S. ARYA, S. K. THAKKAR and A. GOYAL 1971 *Journal of Engineering Mechanics*, *American Society of Civil Engineers*, **EM2**, 317–331. Vibration analysis of thin cylindrical containers.
3. P. C. CHOWDHURY 1972 *International Journal of Ship Building Progress* **19**, 302–309. Fluid finite elements for added mass.
4. M. CHU and S. BROWN 1981 *Journal of Experimental Mechanics* **21**, 129–137. Experiments on the dynamics behaviour of fluid-coupled concentric cylinders.
5. M. A. HAROUN and G. W. HOUSNER 1981 *Journal of Applied Mechanics*, *American Society of Mechanical Engineers* **48**, 411–418. Earthquake response of deformable liquid storage tanks.
6. T. BALENDRA, K. K. ANG, P. PARAMASIVAM and S. L. LEE 1982, *International Journal of Mechanical Science* **24**, 47–59. Free vibration analysis of cylindrical liquid storage tanks.
7. W. K. LIU and D. C. MA 1982 *Nuclear Engineering and Design* **72**, 345–357. Coupling effect between liquid sloshing and flexible fluid-filled systems.
8. L. G. OLSON and K. J. BATHE 1983 *Nuclear Engineering and Design* **76**, 137–151. A study of displacement-based fluid finite elements for frequencies of fluid and fluid–structure system.
9. M. CHIBA, J. TANI, H. HASHIMOTO and S. SUDO 1986 *Journal of Sound and Vibration* **104**, 301–319. Dynamic stability of liquid-filled cylindrical shells under horizontal excitation. Part 1: experiment.
10. R. K. GUPTA and G. L. HUTCHINSON 1989 *Journal of Sound and Vibration* **135**, 357–374. Solid–water interaction in liquid storage tanks.
11. W. QINQUE and H. LIDU 1992 *Computers and Structures* **44**, 353–355. Eigen-problem of liquid-container coupling.
12. L. BO and T. JIA-XIANG 1994 *Computers and Structures* **52**, 1051–1059. Vibration studies of base-isolated liquid storage tanks.
13. H. F. BAUER 1995 *Journal of Sound and Vibration* **180**, 689–704. Coupled frequencies of a liquid in a circular cylindrical container with elastic liquid surface cover.
14. M. AMABILI 1996 *Journal of Sound and Vibration* **191**, 757–780. Free vibration of partially filled horizontal cylindrical shells.
15. P. B. GONCALVES and N. R. S. S. RAMOS 1996 *Journal of Sound and Vibration* **195**, 429–444. Free vibration analysis of cylindrical tanks partially filled with liquid.
16. M. CHIBA 1996 *Journal of the Acoustical Society of America* **1000**, 2170–2180. Free vibration characteristics of a partially liquid filled and partially submerged, clamped–free circular cylindrical shell.
17. A. BERMUDEZ, R. DURAN and R. RODRIGUEZ 1997 *International Journal of Numerical Methods in Engineering* **40**, 1435–1448. Finite element solution of incompressible fluid–structure vibration problems.
18. C. WEIQIU, H. J. DING, Y. M. GUO and Q. D. YANG 1997 *Journal of Engineering Mechanics*, *American Society of Civil Engineers* **123**, 1130–1133. Free vibrations of fluid-filled orthotropic cylindrical shells.
19. M. AMABILI, M. P. PAIDOUSSIS and A. A. LAKIS 1998 *Journal of Sound and Vibration* **213**, 259–299. Vibrations of partially filled cylindrical tanks with ring-stiffeners and flexible bottom.
20. A. GEDIKLI and M. E. ERGÜVEN 1999 *Journal of Sound and Vibration* **223**, 141–155. Seismic analysis of a liquid storage tank with a baffle.

APPENDIX A: DERIVATION OF EQUATION (20)

Equation (20) is derived as follows.

The first row of equation (19) is expressed as

$$[M_s] + \{\ddot{d}\} + [K_s]\{d\} - [S]^{\mathrm{T}}\{P\} = \{0\}, \tag{A1}$$

$$\{\ddot{d}\} = -[M_s]^{-1}[K_s]\{d\} + [M_s]^{-1}[S]^{\mathrm{T}}\{P\}. \tag{A2}$$

On substitution of equation (A2), the second row of equation (19) yields

$$\rho_f[S][-[M_s]^{-1}[K_s]\{d\} + [M_s]^{-1}[S]^{\mathrm{T}}\{P\}] + [M_f]\{\ddot{P}\} + [K_f]\{P\} = \{0\}$$

or

$$-\rho_f[S][M_s]^{-1}[K_s]\{d\} + [[K_f] + \rho_f[S][M_s]^{-1}[S]^{\mathrm{T}}]\{P\} + [M_f]\{\ddot{P}\} = \{0\}. \quad \text{(A3)}$$

Equation (A1) reduces to the following form on premultiplication with $[K_s][M_s]^{-1}$.

$$[K_s][M_s]^{-1}[M_s]\{\ddot{d}\} + [K_s][M_s]^{-1}[K_s]\{d\} - [K_s][M_s]^{-1}[S]^{\mathrm{T}}\{P\} = \{0\}. \quad \text{(A4)}$$

Equations (A3) and (A4) may be coupled to be expressed in the form of equation (20) as follows:

$$\begin{bmatrix} [K_s] & [0] \\ [0] & [M_f] \end{bmatrix} \begin{Bmatrix} \ddot{d} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} [K_s][M_s]^{-1}[K_s] & -[K_s][M_s]^{-1}[S]^{\mathrm{T}} \\ -\rho_f[S][M_s]^{-1}[K_s] & [K_f] + \rho_f[S][M_s]^{-1}[S]^T \end{bmatrix} \begin{Bmatrix} d \\ P \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$

APPENDIX B: NOMENCLATURE

R	radius of tank
H	depth of liquid in the tank
H_T	height of the tank
E	Young's modulus of baffle material
v	Poisson's ratio
ρ_f	mass density of liquid
ρ_s	mass density of baffle material
t	thickness of baffle
R_1	inner radius of annular ring element of baffle
R_2	outer radius of annular ring element of baffle
h	depth of baffle from liquid-free surface
h_2	depth of second baffle from liquid-free surface in case of two baffles system
$[K_f]$	global stiffness matrix of the liquid
$[M_f]$	global mass matrix of the liquid
$[K_s]$	global stiffness matrix of the flexible baffle
$[M_s]$	global mass matrix of the flexible baffle
$[S]$	coupling matrix
B_f	liquid-free surface boundary
B_s	baffle structure boundary
B_b	tank wall and bottom boundaries