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CRACK DETECTION THROUGH WAVELET TRANSFORM FOR A RUN-UP ROTOR

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1. INTRODUCTION

Propagating fatigue cracks can have detrimental effects on the reliability of the rotating machinery such as turbomachinery, process machinery, etc. Dynamic analysis of cracked rotors has been a subject of great interest for the last three decades and excellent reviews on this are available [1–4]. Most of the previous works focussed on detecting rotor cracks by analyzing the steady state vibrations of a rotor bearing system. But it is proved in earlier research [5–9] that it is easier to detect cracks during a startup or rundown process. Recent works include references [10–14] on vibration monitoring to detect crack using transient response during passage through a critical speed. However, the above works used speed-response, time-domain signals and/or the traditional signal processing technique such as FFT for the analyses to detect cracks.

The vibration signals during machine startup or rundown are non-stationary (frequency changes with time) in nature. Fourier transform gives the spectral content of the signal, but it gives no information regarding where in time those spectral components appear. Whereas, wavelets provide a time-scale information of a signal, enabling the extraction of features that vary in time. This property makes "wavelets" an ideal tool for analyzing signals of a transient or non-stationary nature.

The theory of the orthogonal wavelets and their application to signal analysis have been presented by Newland [15, 16]. An excellent recent review by Staszewski [17] gives various wavelet methodologies for damage detection with some application examples, such as gear faults [18] and beam cracks [19].

The present study is an extension of the previous work of the author [14], where for higher acceleration the detection through transient response fails. In the present study the continuous wavelet transforms (CWT) have been applied to detect traverse cracks in a rotor system passing through its critical speed. The present study aims at crack detection and monitoring i.e., the crack depth should be small. A simple hinge model has been proved in literature [3] to be a very good model for small cracks. Hence, Gasch [3] simple model has been considered for breathing action of crack. Time responses of a rotor system have been evaluated for different crack depths, various accelerations and the unbalance eccentricity with phase are investigated. The CWT has been used as a tool to detect the crack in a rotor system from the time-domain signals.

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2. EQUATION OF MOTION OF CRACKED ROTOR

A simple hinge model which represent the breathing crack as considered in reference [3] and applied by the author in reference [14] has been used in the present study also. A brief description of the model and equation of motion are given below.

The simple rotor system with cracked shaft both in inertial co-ordinates z-y and with rotating co-ordinates $\zeta - \eta$ is shown in Figure 1. The equation of motion of the cracked rotor system is given by

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{W}_z \\ \ddot{V}_y \end{Bmatrix} + \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} \begin{Bmatrix} \dot{W}_z \\ \dot{V}_y \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} W_z \\ V_y \end{Bmatrix}$$
$$= \begin{Bmatrix} mg \\ 0 \end{Bmatrix} + em\Omega^2 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

where $\theta = \Omega t + \beta$.

The above can be written as

$$M\ddot{u} + C\dot{u} + K(u, t)u = F_0 + F_u. \qquad (1)$$

As given in reference [3] the equations with the non-linear and time-variant stiffness matrix can be simplified to equations with linear and periodical time-variant stiffness terms by assuming weight dominance ($\Delta u(t) \ll u_0$), for the elastic deflection resulting in the following equation, where $\Delta K(t)\Delta u$ can be neglected if stability is guaranteed.

$$M\Delta\ddot{u} + C\Delta\dot{u} + [K_0 + \Delta K(t)]\Delta u = -\Delta K(t)u_0 + F_u, \qquad (2)$$

where $\Delta u(t)$ is the vector describing the vibration behavior and u_0 is the static deflection of uncracked shaft.

The static sag of the rotors is usually quite large, e.g., in turbo-generators often more than 1 mm. In such rotors Gasch [3] considered simple hinge model which is a very good model for small cracks. If a cracked shaft rotates slowly under the load of its own weight, then the crack will open and close per revolution, that is it breaths. The formulation of the flexibility matrix of a shaft with hinge in rotational co-ordinates [3, 14],

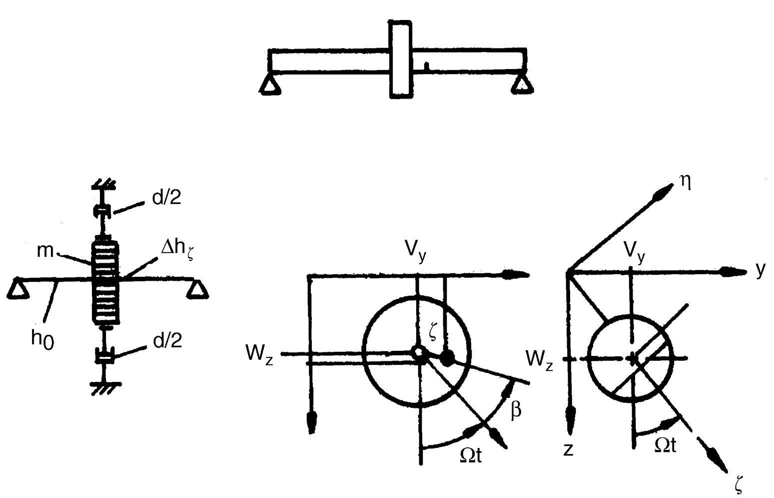

Figure 1. Details of cracked rotor model (from references [3,14]).

can be written as

$$\begin{Bmatrix} W_\zeta \\ V_\eta \end{Bmatrix} = \left(\begin{bmatrix} h_0 & 0 \\ 0 & h_0 \end{bmatrix} + f(t) \begin{bmatrix} \Delta h_\zeta, max & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{Bmatrix} f_\zeta \\ f_\eta \end{Bmatrix}, \tag{3}$$

where the additional flexibility $\Delta h_\zeta max$ is when there is fully open crack in addition to flexibility h_0 of the uncracked shaft. With f_ζ, f_η loadings, h_ζ is the main flexibility, as for small cracks the effect of cross-flexibility h_η can be neglected. The influence of the crack can be described by the stiffness parameter k_ζ/k_0. The stiffness ratio (k_ζ/k_0) variation with crack depth over shaft diameter can be obtained from reference [20]. The stiffness parameter k_ζ/k_0 reduces with increase in the crack depth. This will be used as a crack parameter in this paper.

The steering function, $f(t)$ for the hinge switches from 1 (open) to 0 (closed). In general, it is dependent on the position W_ζ: The rectangular function of the hinge model as shown in Figure 2 provides a better description of breathing for small cracks.

$$f(t) = \begin{Bmatrix} 0 & \text{for } W_\zeta \leqslant 0, \\ 1 & \text{for } W_\zeta > 0 \end{Bmatrix} \tag{4}$$

As shown in Figure 3 the change in sign of W_ζ depends on the angle of rotation i.e., at $\theta = 90^\circ$ and 270° and by assuming the dominance of weight the sign change in W_ζ can be determined by $W_{z,stat}$. The stiffness matrix for the cracked shaft can be derived [3,14] by inverting equation (3) and using the co-ordinating system as

$$K_0 + \Delta K(t) = \begin{bmatrix} k_0 & 0 \\ 0 & k_0 \end{bmatrix} - \frac{1}{2} f(t) \Delta K_\zeta \begin{bmatrix} 1 + \cos 2\theta & \sin 2\theta \\ \sin 2\theta & 1 - \cos 2\theta \end{bmatrix}. \tag{5}$$

3. TRANSIENT RESPONSE

The transient response as given in reference [14] is presented in the present study again as this will be used in the next sections on wavelets. The equation of motion of cracked rotor in transient or run-up case is given by

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \Delta & \ddot{W}_z \\ \Delta & \ddot{V}_y \end{Bmatrix} + \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} \begin{Bmatrix} \Delta & \dot{W}_z \\ \Delta & \dot{V}_y \end{Bmatrix} + \begin{bmatrix} k_0 & 0 \\ 0 & k_0 \end{bmatrix} \begin{Bmatrix} \Delta & W_z \\ \Delta & V_y \end{Bmatrix}$$
$$= \frac{1}{2} f(t) \Delta K_\zeta \begin{bmatrix} 1 + \cos 2\theta & \sin 2\theta \\ \sin 2\theta & 1 - \cos 2\theta \end{bmatrix} \begin{Bmatrix} W_{z,stat} \\ 0 \end{Bmatrix} + \begin{Bmatrix} F_z \\ F_y \end{Bmatrix}, \tag{6}$$

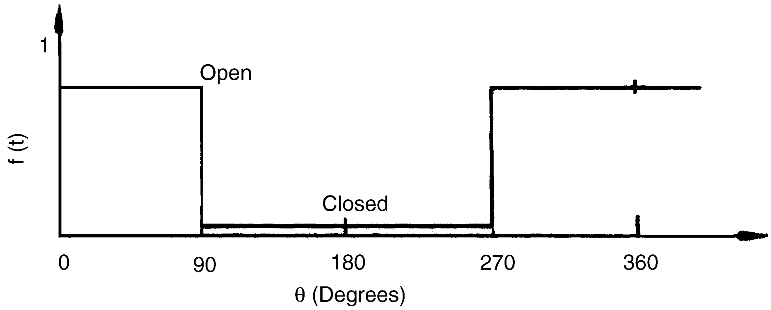

Figure 2. Breathing crack behaviour of hinge model (from references [3,14]).

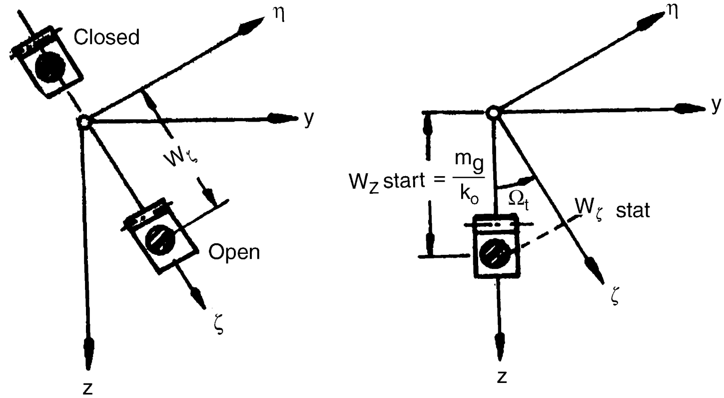

Figure 3. Hinge model showing the breathing crack (from references [3, 14]).

where for a rotation angle θ, the excitation forces F_z, F_y are given as

$$F_z = m\,e\left\{\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right\}, \quad (7)$$

$$F_y = m\,e\left\{-\ddot{\theta}\cos\theta + \dot{\theta}^2\sin\theta\right\}, \quad (8)$$

where $\theta = \beta + \Omega t$ (see Figure 1).

The steering function, $f(t)$ (see Figure 2) can be written as a Fourier series.

$$f(t) = \tfrac{1}{2} + (2/\pi)\cos\theta - (2/3\pi)\cos 3\theta + (2/5\pi)\cos 5\theta. \quad (9)$$

The stiffness of cracked shaft varies as a function of time. For steady state response, $\dot{\theta} = \Omega$ is a constant speed. The response can be obtained by assuming a harmonic solution. In the case of transient analysis, the dynamic response can be obtained using time-marching methods. When the rotating speed is changing $\theta(t) = \Omega_0 t + \frac{1}{2}at^2 + \beta$. where a is the rotor angular acceleration and Ω_0 the initial angular velocity.

4. CONTINUOUS WAVELET TRANSFORM

A great deal of interest has been emerged in recent times, in the application of wavelets, and they have been successfully implemented into many fields. Wavelet analysis is similar to Fourier analysis in the sense of breaking of the signal into its constituent parts for analysis. The Fourier transform breaks the signal into a series of sine waves of different frequencies, whereas the wavelet transform breaks the signal into its scaled shifted versions of the mother wavelet. Wavelets provide a time-scale information of a signal, enabling the extraction of features that vary in time. This property makes "wavelets" an ideal tool for analyzing signals of a transient or non-stationary nature.

Wavelet transforms can be divided into discrete or continuous. In discrete wavelet transform (DWT), the signal is broken into dyadic blocks (shifting and scaling is based on a power of 2). Whereas, in continuous wavelet transform (CWT) still uses discretely sampled data; however, the shifting is a smooth operation through out the sampled data, and the scaling can be defined from the minimum (original signal scale) to a maximum chosen by the user, thus giving a much finer resolution. In the present analysis CWT has been used with Morlet mother wavelet.

The CWT of f (t) is a time-scale method of signal processing that can be defined as the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet function $\Psi(t)$. Mathematically,

$$CWT(s,b) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} f(t) \Psi^* \left(\frac{t-b}{s} \right) \mathrm{d}t, \tag{10}$$

where $\Psi^*(t)$ denotes the complex conjugate of the mother wavelet. The parameter s represents the scale index which is reciprocal of frequency. The parameter b indicates the time shifting (or translation).

The CWT provides the time–frequency information of the signal. This means that any non-stationary events can be localized in time unlike Fourier analysis. Additionally, the frequency content of these events can be described for any position on the time axis. This property of CWT has been used in the present study to extract significant characteristics, which are embedded in time-domain signal of the cracked rotor passing through its critical speed.

5. RESULTS AND DISCUSSION

The cracked rotor considered in reference [14] is analyzed here again but with higher angular accelerations of rotor. Apart from time response, wavelet transforms analysis is also considered.

$$\text{Disc mass (m)}: \quad 51{\cdot}0\,\text{kg},\ e = 0{\cdot}01\,\text{mm},\ \Delta t = 0{\cdot}001\,\text{s},\ W_{stat} = 0{\cdot}5\,\text{mm},$$

$$\text{shaft stiffness}: \ 9{\cdot}99 \times 10^5\,\text{N/m},\ D = d/2m\omega_0 = 0{\cdot}1, \omega_0 = (g/W_{st})^{1/2},$$

$$\text{acceleration } (a) = \quad 30, 50, 75, 100\,\text{rad/s}^2$$

The Houbolt time-marching technique has been used to model the system in time domain with a time step of 0·001 s, due to better convergence of results [21], than for other schemes such as Newmark, Wilson θ, etc. methods. From Houbolt method the good convergence behavior is known, but on the other hand, the integrator adds a fictitious damping to the integrating system, which changes the system behavior especially in some cases of non-linear analysis. However, since the analysis considers the detection of small cracks, the linear analysis with Houbolt method yields good results. Though the stiffness varies with time when the shaft is cracked, for a small time step, for that time duration stiffness is assumed constant and same time-marching scheme is used.

The Morlet mother wavelet has chosen for all the CWTs. The CWT at a scale of 40, which is above critical speed has been chosen for the analysis. The analysis has been done for various rotor acceleration (a), for crack depth (stiffness parameter, k_ζ/k_0) and for crack orientation with unbalance (β).

Figure 4 shows the dynamic responses of the rotor without and with crack (K_ζ/K_0 = 0·95) passing through its critical speed in time domain, frequency domain (obtained by FFT of time duration of 2.5 s covering a frequency zone of 0–40 Hz) and CWT. The large increase in amplitudes of vibration is observed with crack in rotor compared to uncracked rotor in all the response plots, viz., time, FFT and CWT. In all the cases of the responses some change in pattern is observed. However, the characteristic sub critical response peaks at half the critical speed can be clearly observed only in the CWT of cracked rotor system. But these are not evident from the frequency and time responses of the cracked rotor. The presence of subharmonics in the response is due to breathing action of the crack and this can be used to detect a crack in a rotor system.

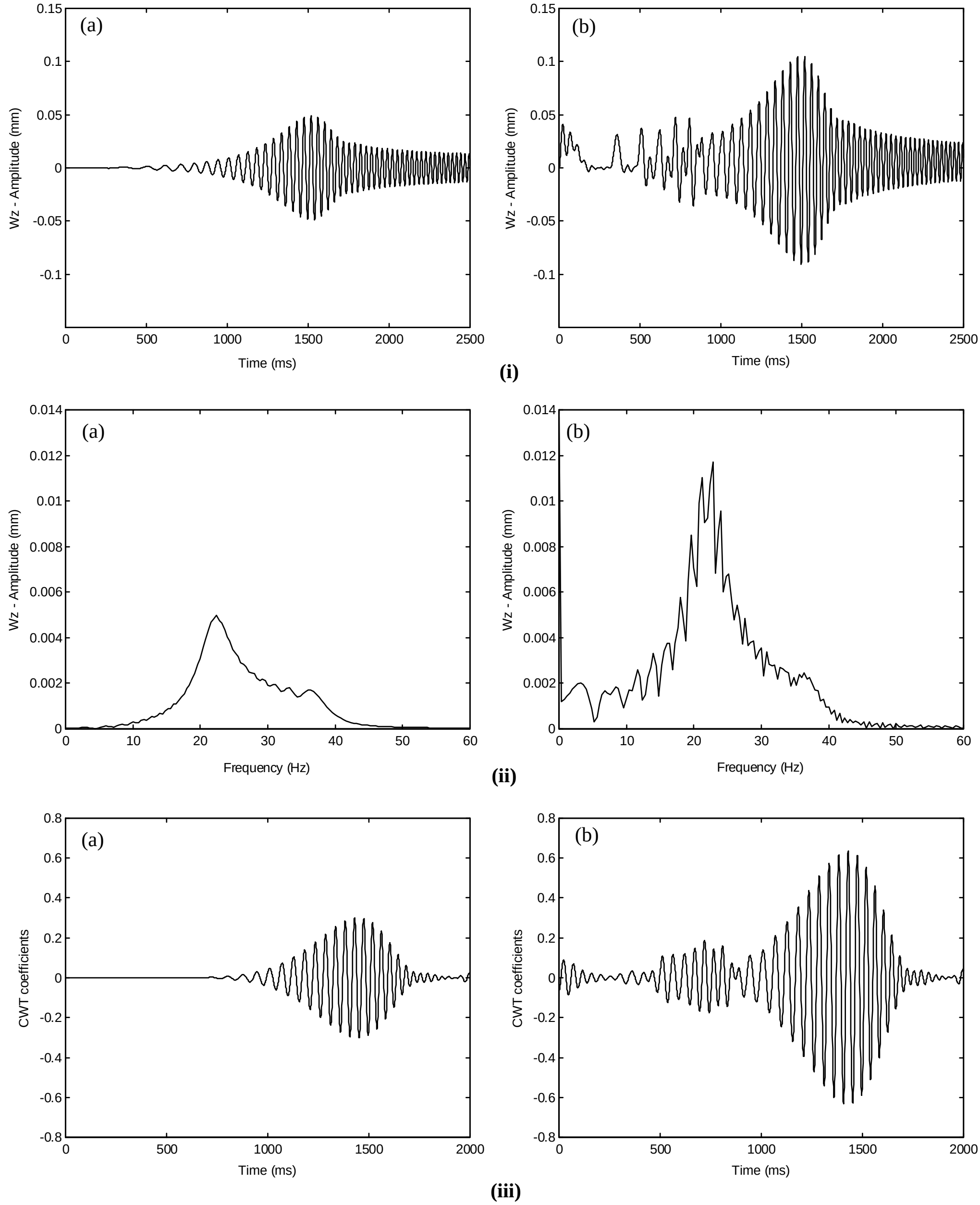

Figure 4. Comparison of (i) time response, (ii) FFT, (iii) CWT of cracked rotor $a = 100\,\mathrm{rad/s^2}$: (a) no crack; (b) with crack, $K_\xi/K_0 = 0{\cdot}95$.

CWT is also compared with time response for various rotor accelerations in the Figure 5. At low accelerations the subharmonic resonant peaks are clear from CWT plot and even in time response plot because the time taken to pass through the critical and subcritical speeds is more. However, as the acceleration increases, the subharmonic resonant peaks are embedded in time response and these can be extracted by using CWT. It is to be noted that CWT cycle-to-cycle fluctuations are much less and the plot is more

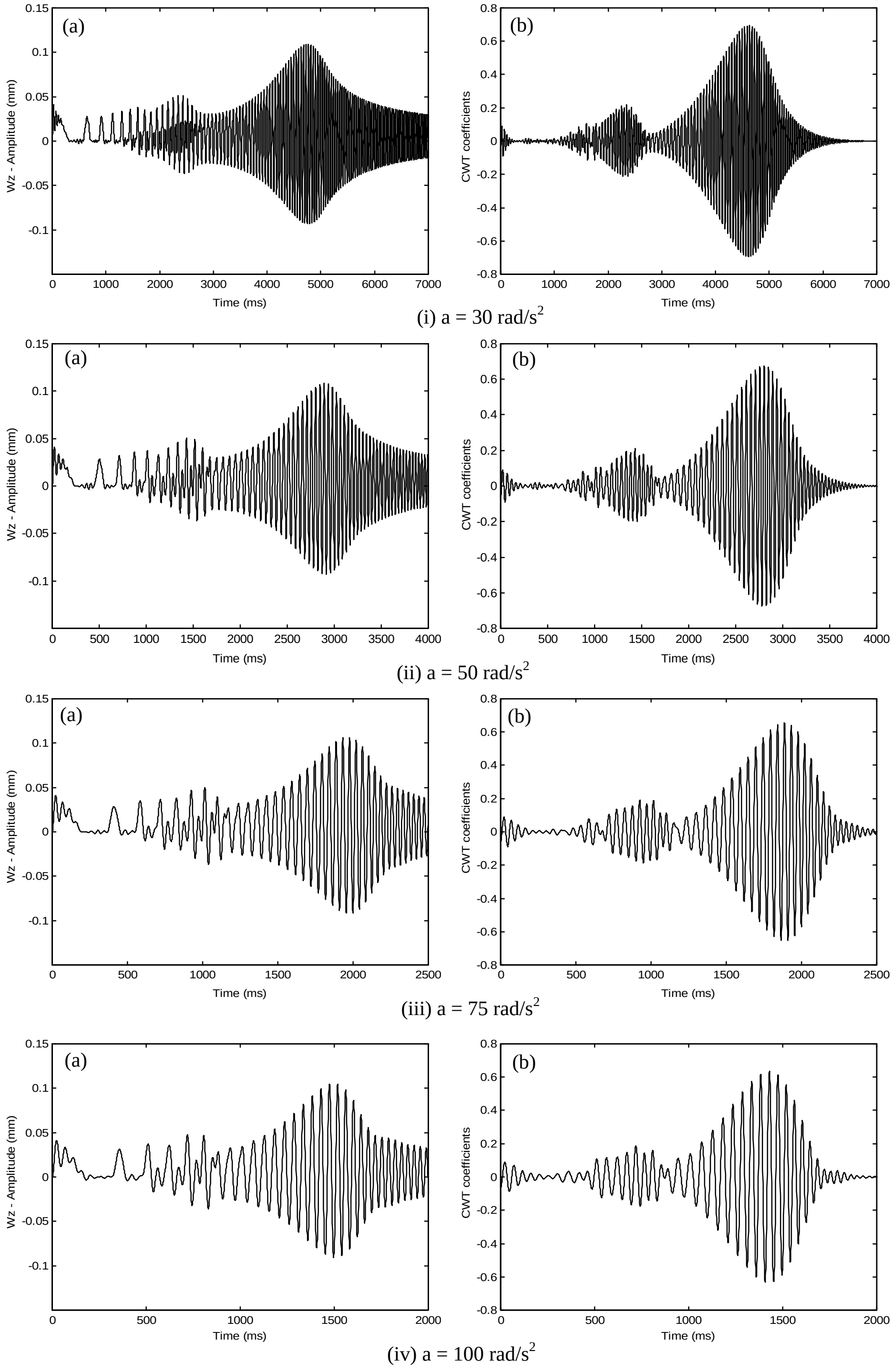

Figure 5. Comparison of CWT at a scale of 40 with time response for different accelerations, $K_\xi/K_0 = 0{\cdot}95$: (a) time; (b) CWT.

regular as compared to one in time domain. It is also observed from the figures that at high angular accelerations the vibration amplitudes are less. This can be expected since the increase in driving torque reduces the vibration due to the time needed to pass through critical and subcritical speed region is decreased.

Figure 6 shows the time responses for different crack depths (stiffness parameter, k_ζ/k_0) and the corresponding CWT at the scale of 40 and an acceleration of 100 rad/s^2. The rise of vibration amplitudes with crack depth can be seen from both the plots. However, the sub critical response peaks are clearly evident from the wavelet plots and these peaks are more apparent as the crack depth increases. This feature cannot be seen from the time-history plots.

The 1/3 critical is a better indicator of the presence of crack as compared to 1/2 critical speed resonance, which can also be caused by asymmetry of the rotor or supports. To show the importance of CWT, a stiff rotor with the following data is considered.

$$\text{Disc mass}(m): \quad 51{\cdot}0\,\text{kg}, \; e = 0{\cdot}01\,\text{mm}, \; \Delta t = 0{\cdot}001\,\text{s}, \; W_{stat} = 0{\cdot}245\,\text{mm}$$

$$\text{shaft stiffness}: \; 2{\cdot}04 \times 10^6\,\text{N/m}, \; D = d/2m\omega_0 = 0{\cdot}1, \; \omega_0 = (g/W_{st})^{1/2},$$

$$\text{acceleration } (a) = 30, 75, 100\,\text{rad/s}^2$$

Figure 7, shows the CWT comparison with time response for various rotor accelerations for the stiff rotor. Unlike the previous case, here the 1/3 critical is clearly seen for the CWT. At low accelerations the subharmonic resonant peaks are clear from the CWT plot and even in time-response plot because the time taken to pass through the critical and sub critical speeds is more. The reasons are explained before. But the importance of CWT is clearly seen for higher acceleration in this stiff rotor. Further the unbalance phase influence in the following paragraph explains the importance of CWT.

Figure 8 shows the influence of the phase between the unbalance eccentricity and crack. When the crack occurs in the direction to that of unbalance eccentricity the vibrations are severe compared to that of, when $\beta = \pi/2$. It becomes minimum when $\beta = \pi$. In some cases of β the sub criticals are clear in time response itself, but in all the cases CWT shows clear symptoms of crack 1/2 and 1/3 criticals.

It is clear that the CWT plots show subharmonic resonant peaks even for low crack depths, $k_\zeta/k_0 = 0{\cdot}98$ unlike time response. Thus, it is found that CWT is a powerful tool for detecting cracks particularly at high accelerations and low crack depths compared to time responses. From the discussion CWT is suggested for crack detection and monitoring in a rotor system.

6. CONCLUSIONS

The transient analysis of rotor system with transverse breathing crack has been studied for flexural vibrations. The CWT is found to be useful tool for extracting the silent features from time response of the cracked rotor passing through its critical speed.

The subharmonic resonant peaks are found by using CWT when the cracked rotor is passing through its critical speed. These peaks are not apparent in frequency spectrum as well as in time response. The CWT is more powerful for detecting cracks at high accelerations and low crack depths compared to time response.

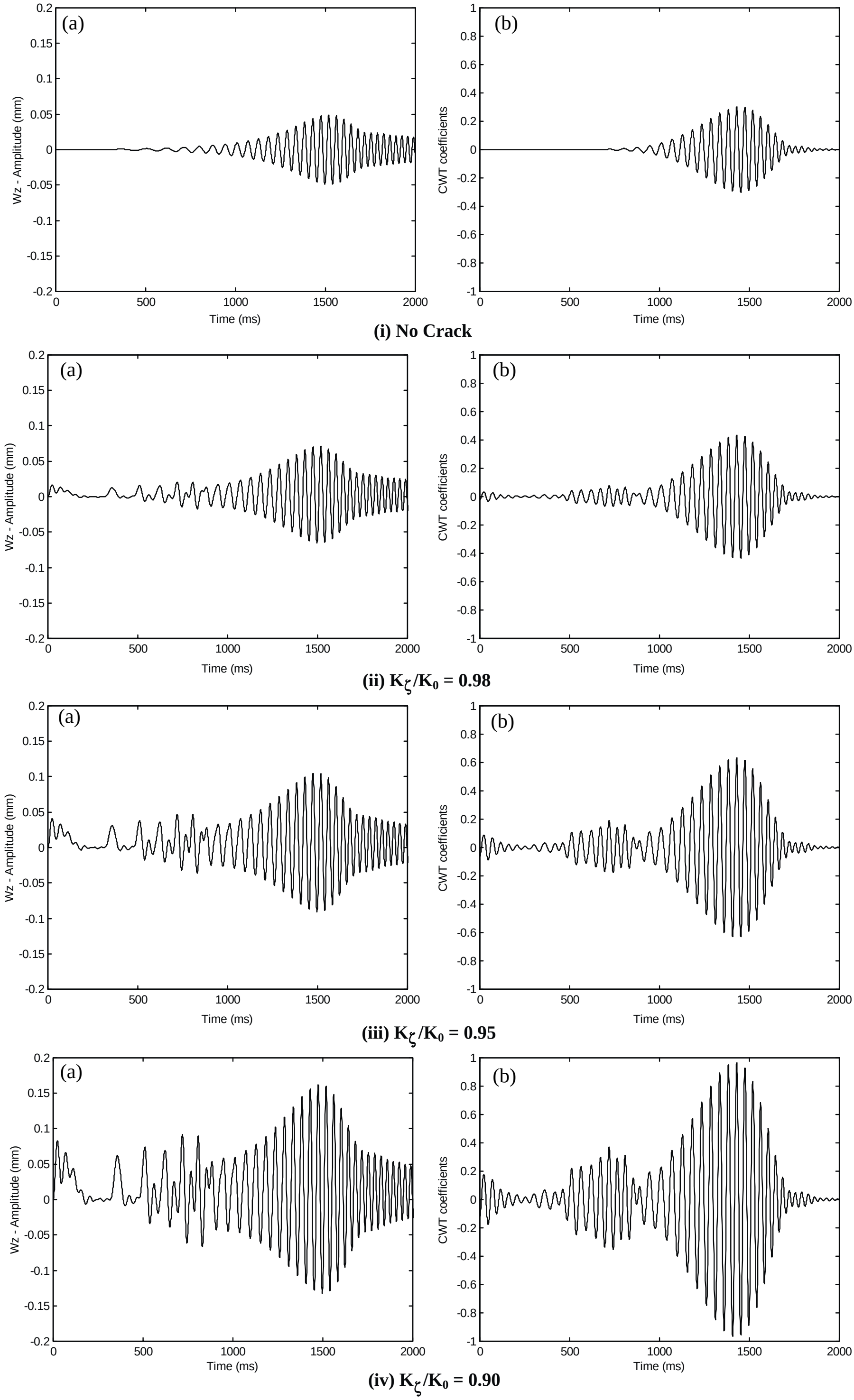

Figure 6. Time response and CWT (scale of 40) for different crack depths, acceleration $a = 100\,\text{rad/s}^2$: (a) time, (b) CWT

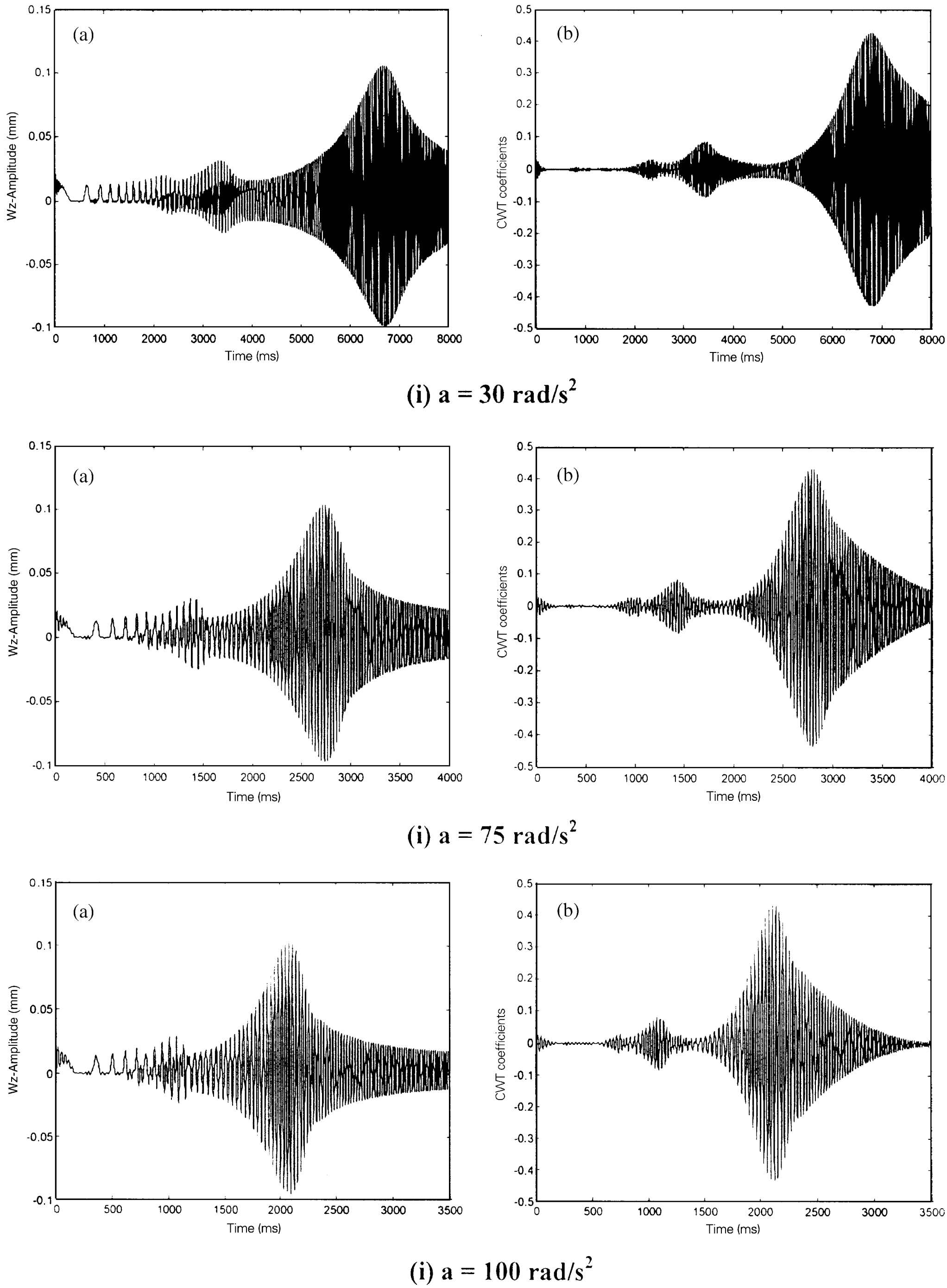

Figure 7. Comparison of CWT with time response for different accelerations of the stiff rotor for $K_\xi/K_0 = 0{\cdot}95$: (a) time; (b) CWT.

The phase between the unbalance eccentricity and crack is important. When the crack occurs in the direction of unbalance eccentricity the vibrations are severe compared to that of, when $\beta = \pi/2,\ \pi$.

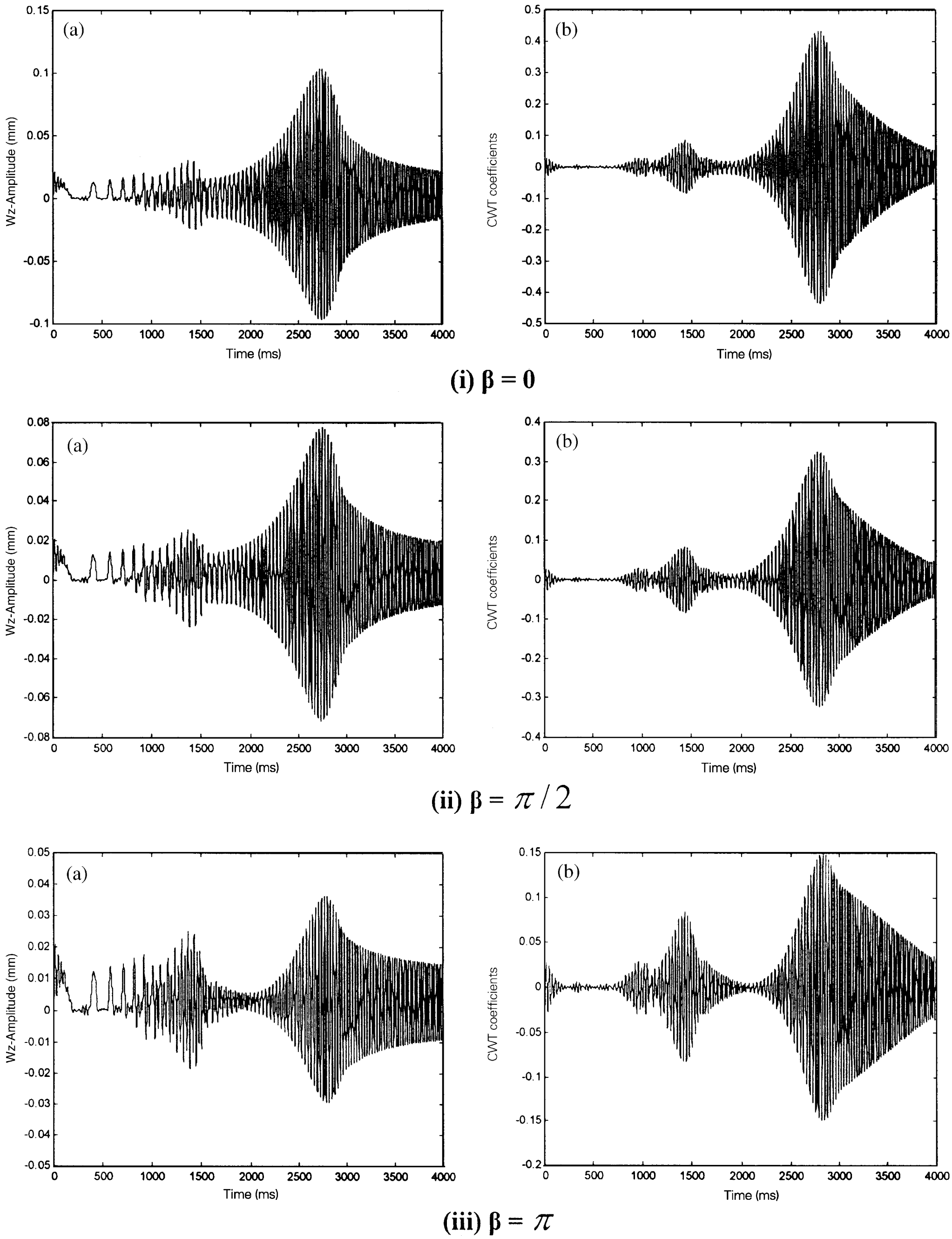

Figure 8. CWT compared with time and showing the influence of phase between eccentricity and crack, $a = 75\,\mathrm{rad/s^2}$, $K_\xi/K_0 = 0{\cdot}95$: (a) time; (b) CWT.

The results suggest that the analysis of transient response using CWT can be used for crack detection and monitoring when the rotor is passing through the critical speed, particularly for higher accelerations.

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