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# Identification and updating of loading in frameworks using dynamic measurements

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**Abstract**

This paper is concerned with the effect of structural loading on dynamic performance. This topic is recognised as being of importance when validating finite element (FE) models with experimental data. A strategy for including axial load effects in a model updating procedure is developed. The method can be used to identify loading in structural frameworks using measured dynamic data.

The effectiveness of the new method is demonstrated by means of case studies involving both simulated and experimental data. The theoretical study allows aspects of the sensitivity of the method to realistic levels of experimental noise to be studied as well as the way in which dynamic load identification can be enhanced with static measurements. The experimental case study proves the practical success of the technique. Updated axial load parameters are compared with static measurements of the same quantities.
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## 1. Introduction

It is well known that membrane loads in slender structural elements result in a lack of stiffness in the transverse direction. In practical dynamic terms this stiffness alteration leads to a dependence of natural frequency upon structural loading. Rayleigh [1], for instance, set out the effect of axial load on natural frequency. The buckling capacity of a perfect column is found to reduce the first natural frequency to zero. The commonly used term stress stiffening will be used to describe the phenomenon here.

The relationship between natural frequency and axial load has proved to be of practical interest in determining the load upon structural members resulting from dynamic measurements. Stephens

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[2] attempted to use dynamic measurements to identify both level of loading and degree of fixity in as-built columns. Lurie [3], while deducing that Stephens method did not allow fixity to be uniquely identified, went on to suggest a practical method for determining buckling loads from dynamic measurements [4]. The present work extends the work of Stephens and Lurie, using the finite element method to enable identification of loads in multiple members of a structural framework.

The work described in this paper falls under the heading of finite element model updating. The background to this area is described in detail in several publications, for instance Refs. [5,6]. Model updating involves making modifications—often iteratively—to an a priori finite element model to reconcile the differences between dynamic measurements and predictions of dynamic behaviour by the FE model. The goal of updating is that an initial best estimate of the finite element model of a structure can be made a closer representation of a prototype version by calibration using dynamic testing data.

Model updating offers the possibility of enriching a priori FE models using experimental test data. Any updating process, however, is limited by the parameters which can be altered. This paper extends the existing arsenal of updating parameters to include those which account for the effects of loading. The reverse process implies that updating load-dependent parameters equates to *load identification*.

Stress stiffening potentially affects all structural elements which carry load. Significant variations to lateral stability and vibration characteristics are only noted for slender elements such as strings, beams, thin plates and shells. This paper concentrates on the example of the simple beam where *axial load* is the critical parameter. Consideration of beam-type elements allows load identification techniques to be developed for structural frameworks. The technique is conceptually applicable to plates although investigation of this area is beyond the scope of this paper.

The relationship between load in slender beams described in this paper holds when no static transverse deflection exists. Such deflections could arise as a side effect of the axial loading or an initial imperfection in the beam resulting, for example, from lack-of-fit. This change in geometry also influences dynamic behaviour and is additionally sensitive to axial load. The effect of initial deformity is particularly important in the case of thin plates. Massonet [7] shows that relatively small initial curvatures in circular plates leads to large excursions from an otherwise approximately linear relationship between the square of the circular natural frequency ($\omega^2$) and membrane load.

## 2. Stress stiffening as an updating parameter

### 2.1. Stress stiffening

The equation of motion for the free vibration of a simple beam with the usual assumptions of small deflection is given by

$$EI\frac{\partial^4 v}{\partial x^4} + \rho A\frac{\partial^2 v}{\partial t^2} = 0, \tag{1}$$

where $EI$ is the flexural stiffness of the beam, $v$ the transverse displacement, $x$ is the distance along the length of the beam, $\rho$ the density, $A$ the cross-sectional area and $t$ is time. A further term is required to account for a constant axial load $P$, where $P > 0$ implies that the beam is in tension. Thus Eq. (1) can be written as

$$EI\frac{\partial^4 v}{\partial x^4} + P\frac{\partial^2 v}{\partial x^2} + \rho A\frac{\partial^2 v}{\partial t^2} = 0. \tag{2}$$

For a pinned beam assuming a harmonic solution and applying the appropriate boundary conditions, a series of solutions for natural frequency which are dependent upon the axial load

$$\omega_n = \left(\frac{n\pi}{L}\right)^2 \left[\frac{EI}{\rho A}\left(1 - \frac{P}{P_{Euler}}\right)\right]^{1/2}, \quad n = 1, 2, 3..., \tag{3}$$

where $P_{Euler}$ is the $n$th critical buckling load given by

$$P_{Euler} = \frac{n^2\pi^2 EI}{L^2}. \tag{4}$$

Note that this applies for low-level vibrations where the amplitude is sufficiently small that the axial load in the beam can be considered as being constant. Lurie [4] showed that the relationship for other conditions of end fixity obeyed an approximately linear relationship. He found that the non-linearity arising from semi-fixed boundary conditions was indistinguishable from the error in experimental readings of the same phenomena. Lurie further noted that the approximate relationship between axial load and natural frequency enabled the Euler buckling load for an in situ column to be assessed given dynamic readings of the column under at least two *known* axial loads. The practical value of this application is limited since any column forming part of an existing structure would be subjected to initial loading which could only be estimated, the buckling load itself would not be attainable. However, given a reasonable estimation of material properties and geometry—as well as an estimate of the fixity of the column—an estimate of the current axial loading could be attained.

The finite element method devised in the 1950s offers far wider scope for establishing relations between membrane loading and both stability considerations and dynamic response. The effects of a time-invariant axial load can be included in the finite element analysis by an additional stiffness term which is a function of the geometry of the element. The inclusion of such a term does not per se imply non-linearity in the relationship between load and deflection although it is often linked with non-linear geometrical analyses. The relationship between stress stiffening and large deflection analyses is explored in Section 4 below. It is common in commercial finite element packages for the effects of an axial load in a single step linear static analysis to be accounted for in a subsequent dynamic analysis.

The additional stiffness term for a 2-D beam—$[\mathbf{k}_\sigma]$—(along with corresponding displacements) by

$$[\mathbf{k}_\sigma]\{\mathbf{v}\} = \frac{P}{L}\begin{bmatrix} \frac{6}{5} & \frac{L}{10} & -\frac{6}{5} & \frac{L}{10} \\ & \frac{2L^2}{15} & -\frac{L}{10} & -\frac{L^2}{30} \\ & & \frac{6}{5} & -\frac{L}{10} \\ sym & & & \frac{2L^2}{15} \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix}, \tag{5}$$

where $\theta_1$ and $\theta_2$ are the rotational end displacements and $v_1$ and $v_2$ are the corresponding transverse displacements. The length of the element is given by $L$.

### 2.2. Updating of stress stiffening parameter

A number of approaches to model updating have been introduced over the last 20 years or so [5,6]. Amongst the most enduring are the so-called sensitivity methods which determine the relationship of differences observed and predicted dynamic behaviour with certain *updating parameters*. The updating parameters share a functional relationship with certain of the design parameters of an a priori FE model. It is generally accepted that these parameters should be chosen such that the updated FE model is physically realistic. The simplest and most popular approach has been to consider a set of updating parameters, $p_j, j = 1, 2, ..., n_p$ as factors upon the elemental mass and stiffness matrices such that

$$[\mathbf{Z}_u] = \sum_j p_j[\mathbf{z}], \tag{6}$$

where the summation represents *matrix building* and the parameters $[\mathbf{Z}]$ and $[\mathbf{z}]$ represent, respectively, global and local representations of either mass or stiffness. Clearly for the updated model to remain the same as the initial model it is required that

$$p_j = 1, \quad j = 1, 2, ..., n_p, \tag{7}$$

where $n_p$ is the number of updating parameters.

The sensitivity of the $i$th eigenvalue to a change in an updating parameter was shown by Wittrick [8] to be

$$\frac{\partial \lambda_i}{\partial p_j} = \{\boldsymbol{\phi}\}_i^{\mathrm{T}} \left( \frac{\partial}{\partial p_j}[\mathbf{K}] - \lambda_i \frac{\partial}{\partial p_j}[\mathbf{M}] \right) \{\boldsymbol{\phi}\}_i, \tag{8}$$

where $\lambda_i$ is the $i$th eigenvector, $\{\boldsymbol{\phi}\}_i$ is the corresponding mass normalised eigenvector, $p_j$ is the $j$th updating parameter and $[\mathbf{K}]$, $[\mathbf{M}]$ are the global stiffness and mass matrices, respectively.

The next step is to use the premise that the correct (updated) structural stiffness matrix can be constructed in terms of the initial model by

$$[\mathbf{K}_u] = [\mathbf{K}_a] + [\Delta\mathbf{K}], \tag{9}$$

where the subscripts $u$ and $a$ represent the updated and a priori stiffness matrices, respectively. The assumption that the updated stiffness matrix is a function of a set of updating parameters $p_1, p_2, ..., p_{n_p}$ is then adopted. This can further be expressed as a Taylor expansion about the initial stiffness matrix in powers of $p$. The updated stiffness to the first order is given by

$$[\mathbf{K}_u] = [\mathbf{K}_a] + [\Delta\mathbf{K}] = [\mathbf{K}_a] + \sum_j \delta p_j \frac{\partial}{\partial p_j}[\mathbf{K}_a] + \mathrm{O}\left(p_j^2\right). \tag{10}$$

If the a priori stiffness matrix is considered as being built from a set of both unloaded and loaded component elemental matrices it can be expressed as

$$[\mathbf{K}_a] = \sum_{j=1}^{n_{elems}} \left([\mathbf{k}]_j + [\mathbf{k}_\sigma]_j\right). \tag{11}$$

This requires that the axial load present in an element—which is implicit in the stress stiffening term—is available. Recasting the original stiffness matrix in terms of a set of factors $p_1, p_2, ..., p_{n_{elems}}$ upon the stress stiffening terms results in a stiffness matrix given by

$$[\mathbf{K}_a] = \sum_{j=1}^{n_{elems}} \left([\mathbf{k}]_j + p_j[\mathbf{k}_\sigma]_j\right). \tag{12}$$

Differentiating with respect to $p_j$ gives

$$\frac{\partial[\mathbf{K}_a]}{\partial p_j} = [\mathbf{k}_\sigma]_j, \tag{13}$$

from which it can be seen that the second and higher derivatives are zero. The relationship between the updated structural matrix and the updating parameter becomes exact so that

$$[\Delta\mathbf{K}] = \sum_j \delta p_j[\mathbf{k}_\sigma]_j. \tag{14}$$

Noting that the stress stiffness matrix has axial load as a factor and defining $[\hat{\mathbf{k}}_\sigma]$ as the *normalised* stress stiffness such that

$$\left[\hat{\mathbf{k}}_\sigma\right] = \frac{[\mathbf{k}_\sigma]}{P}. \tag{15}$$

Substituting in Eq. (12) leads to the parameter $p_j$ representing the value of load, that is

$$[\mathbf{K}] = \sum_{j=1}^{n_{elems}} \left([\mathbf{k}_z]_j + p_j[\hat{\mathbf{k}}_\sigma]_j\right). \tag{16}$$

Substituting Eq. (13) into Eq. (8) and noting that the derivative of the mass matrix with respect to the $j$th stress-stiffening updating parameter is zero, for $p$ defined in Eq. (12) it is found that

$$\frac{\partial\lambda_i}{\partial p_j} = \{\boldsymbol{\phi}\}_i^{\mathrm{T}}[\mathbf{k}_\sigma]_j\{\boldsymbol{\phi}\}_i. \tag{17}$$

It is interesting to note that the elemental change while realistic, is not part of the family of generic elements suggested by Gladwell and Ahmadian [9].

### 2.3. Solution

Considering the rate of change of eigenvalues with respect to updating parameter as elements of a larger stiffness matrix $[\mathbf{S}]_{ij}$ such that

$$[\mathbf{S}]_{ij} = \frac{\partial\lambda_i}{\partial p_j}, \tag{18}$$

a sensitivity matrix can be constructed as

$$[\mathbf{S}]\{\delta\mathbf{p}\} = \{\delta\boldsymbol{\lambda}\}. \tag{19}$$

Solving for $\{\delta\mathbf{p}\}$ can be achieved in a least-squares sense using singular value decomposition [10].

$$\{\delta\mathbf{p}\}_{LS} = [\mathbf{S}]^+\{\delta\boldsymbol{\lambda}\}, \tag{20}$$

where $[\mathbf{S}]^+$ is the pseudo inverse of $[\mathbf{S}]$. Golub and Van Loan [11], for instance, show that this solution provides a unique set of values $\{\delta\mathbf{p}\}$ which minimise the square of the 2-norm of a residual term

$$\min(R_{LS}^2) = \|\{\delta\boldsymbol{\lambda}\} - [S]\{\delta\mathbf{p}\}_{LS}\|_2^2. \tag{21}$$

The method holds for all instances of Eq. (19), although for underdetermined cases the level of information required outweighs that which is available from dynamic data. It is common to seek to improve the conditioning of Eq. (19) by row and/or column weighting, a process known as regularisation. The requirement for regularisation with stress stiffening parameters is discussed with reference to a numerical study in the following section.

The solution proceeds iteratively with the running value of the residual term providing an indication of convergence upon a solution.

## 3. Numerical study

The characteristics of the method described in the previous section are most readily examined by means of a numerical study. A doubly braced rectangular frame will be considered. The framework was designed to allow the effects of loading on dynamic behaviour to be isolated. Some aspects of the dynamic behaviour of the framework are described in Ref. [12]. A case study using experimental data is presented in the following section. The frame has overall dimensions of $500 \times 300$ mm and comprises six spars of $6\,\text{mm} \times 15\,\text{mm}$ frame and is designed to vibrate predominantly out of its plane. An exploded view of the framework is shown in Fig. 1 which includes the numbering scheme for the six spars.

If $n_{modes}$ natural frequencies are identified and we wish to identify $n_{loads}$ elemental axial loads starting with an initial load distribution defined by $p_1, p_2, \ldots, p_{n_{loads}}$ the following equation is solved over a number of steps:

$$[\mathbf{S}]_{modes \times n_{loads}}\{\delta\mathbf{p}\}_{n_{loads}} = \{\delta\boldsymbol{\lambda}\}_{n_{modes}}. \tag{22}$$

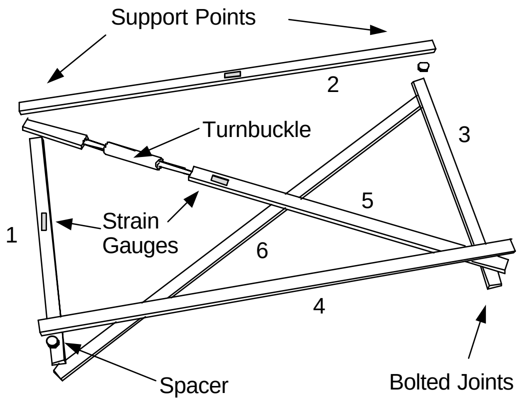


Fig. 1. General arrangement of experimental framework.

Based on the assumption that the load in certain sets of elements or substructures will be the same it is reasonable to decrease the number of $\delta p_i$. In this case Eq. (22) is reduced to

$$[\{\{\mathbf{S}\}_{1a} + \{\mathbf{S}\}_{1b} + \cdots\}\cdots]\left\{\begin{array}{c}\Delta p_{spar1}\\ \Delta p_{spar2}\\ \vdots\end{array}\right\} = \{\Delta\boldsymbol{\lambda}\}, \tag{23}$$

where the subscripts $1a, 1b, \ldots$ for instance refer to the elements which constitute spar 1. The same procedure is undertaken for all six spars of the framework leading to six independent loading-related parameters to be identified.

In the following discussion, the "experimental" eigenvalues are simulated using a finite element model developed in CALFEM [13] an elementary FE toolbox for MATLAB [14]. The finite element model includes the fact that the spars were not perfectly co-planar as was the case with the experimental arrangement. The model includes an optimised representation of the turnbuckle as a circular bar. The axial load in each spar is estimated by imposing an arbitrary load pattern approximating to the effect of introducing a tensile load of 1 kN in the diagonal spars. The assumed load in the compression members are –515 and −857 N. This loading pattern causes stresses of up to 5% of the yield stress and is therefore comfortably within serviceability levels.

The finite element implementation described above allowed the stress stiffening resulting from these loads to be accounted for in the finite element model independently of the structural deformity arising from loading. Fifteen non-rigid-body modes are found in the first 400 Hz.The perturbations to natural frequency predicted by the finite element model arising from loading up to 2 kN are shown in Fig. 2. The level of assumed loading in the axial spars of 1 kN is shown as a vertical grey line. It is clear that the stress stiffening has a dramatic effect on all of the modes. Modes 4 and 5 are seen to interchange at around 500 kN. The results of a full non-linear geometric analysis taking into account large deformations as well as stress stiffening are shown in Fig. 2 as dotted lines. These effects arise as a result of the out of plane bending of the spars. They will not be considered in this sections but their implications are noted in the following section. The *zero-load* natural frequencies are shown in along with the corresponding mode shapes Fig. 3

Taking the entirely unloaded structure as the original FE model, stress-stiffening parameters initially have no effect. The sensitivity matrix of the 15 natural frequencies shown above to the six updating parameters evaluated about the zero-load case is shown graphically in Fig. 4. The sensitivity of higher order modes to changes in load in the six spars are seen to be larger than their lower order counterparts, but by less than an order of magnitude. This suggests that the problem is well conditioned and requires no row or column weighting to regularise the matrix. A comparison of the sensitivity matrix and the predicted mode shapes (shown in Fig. 3) confirms that the distribution of strain energy in each mode shape relates clearly to the sensitivity of the associated resonance to the added stiffness in that area. For instance the flexure of spar five is characterised by mode 2. As one might expect, this manifests itself as a high sensitivity of mode 2 to changes in the stress stiffening of that particular spar.

The values of the six updating parameters after 10 iterations of the updating process are shown in Table 1. The tension loads are seen to have been exactly identified and the loads in the compression members are estimated to within approximately 10% of the target value. The convergence of identified loads upon target values is shown graphically in Fig. 5.

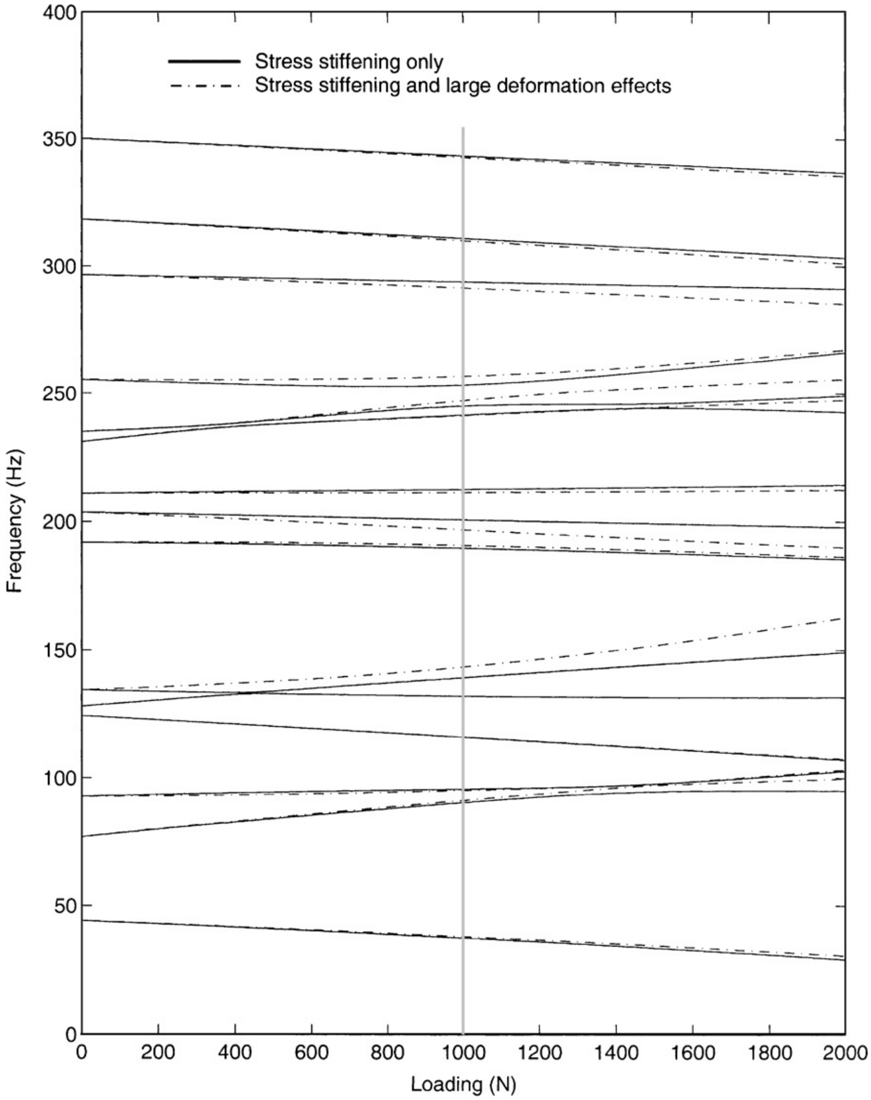


Fig. 2. FE predictions of natural frequency perturbations under loading.

Table 1
Identified load after 10 iterations, no initial knowledge of axial loading

| Spar | Theoretical load (N) | Identified load (N) |
|---|---|---|
| 1 | −515 | −475 |
| 2 | −857 | −902 |
| 3 | −515 | −579 |
| 4 | −857 | −800 |
| 5 | 1000 | 1000 |
| 6 | 1000 | 1000 |

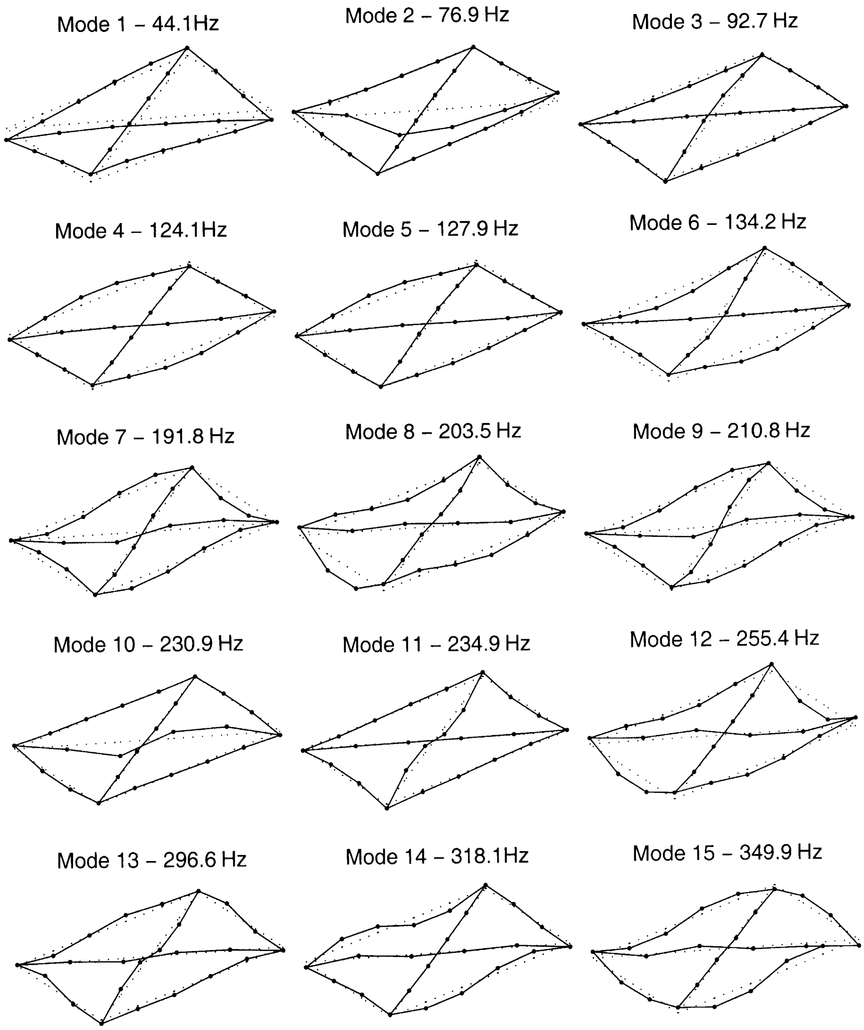


Fig. 3. Mode shapes corresponding to 15 FE modes.

The correct value of the loads in the diagonal are quickly converged upon in a single iteration of the updating procedure. This can be explained by the preponderance of modes which are sensitive to these particular parameters. The first step of the iteration of the updating process is seen to overestimate three of the spar loads substantially with subsequent steps producing convergence upon the correct values. That modes 4 and 5 can exchange places with loading level was beyond the sophistication of the updating algorithm and may have affected the stability of convergence of identification of the compressive forces.

To simulate the differences between experimental data and FE predictions deriving from inaccuracies other than those discussed in this paper "noise" is added to the simulated experimental data. An additive noise model using a random value between –10 and +10 Hz is added to each to the simulated experimental resonance frequency. Under these circumstances,

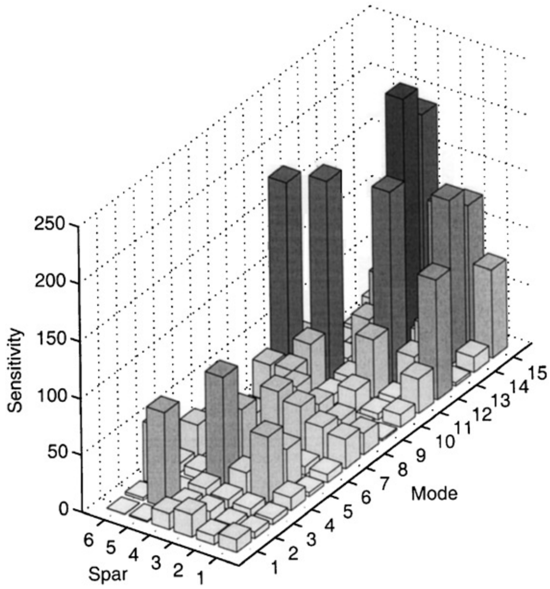


Fig. 4. Sensitivity matrix, $\{p\} = 0$.

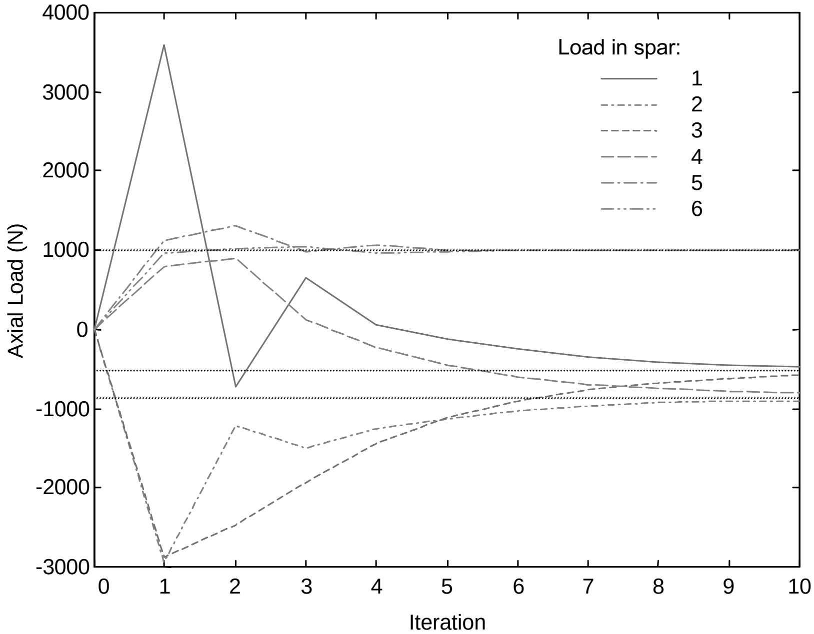


Fig. 5. Identified loads from no initial knowledge of loading: load in spar.

realistic estimation of the loading present in the structure is still found to be achievable. Twenty different target sets of data with different noise models were prepared as the experimental data and the updating procedure was invoked with no initial knowledge of load. Fifteen natural frequencies were used to characterise the loaded structure. Ten iterations of the updating process were performed and the 20 sets of identified loads are shown in Fig. 6. The load in the diagonal members are updated most robustly with useful estimates of the loading in the vertical and horizontal members also gained.

The preceding discussion has assumed that no initial knowledge of the loading is available, which is the harshest test that the updating process must face. In practical situations, while full instrumentation of a structure might prove impractical, some estimation of the loading might be possible. If the stress in one or more members of the framework is available then as well as being a reduction in the number of unknowns to identify, the initial FE model will be a closer representation of the loaded structure.

As one might expect from the previous results, the most valuable contribution to the conditioning of the updating problem is made by a knowledge of the loads upon which convergence is slowest. Fig. 7 shows the result of performing the load identification four times with known loading in, respectively, members 1, 2 and 5 compared to the zero initial load situation, the loads in members 3, 4 and 6 converge in the same manner as their counterparts, their omission from Fig. 7 is only to aid clarity. As anticipated, a knowledge of either of the compression loads dramatically improves the convergence upon the target loads.

The significance of this observation is that a formulation of the sensitivity of eigenvalues to elemental axial load gives insight into which member should be instrumented to enable the most successful identification of loading. A simulated study would allow this information to be gained.

An estimation of bounds on loads to be identified or the relationship between spar loads would further decrease the unknowns to be identified by the updating procedure and thereby increase its effectiveness.

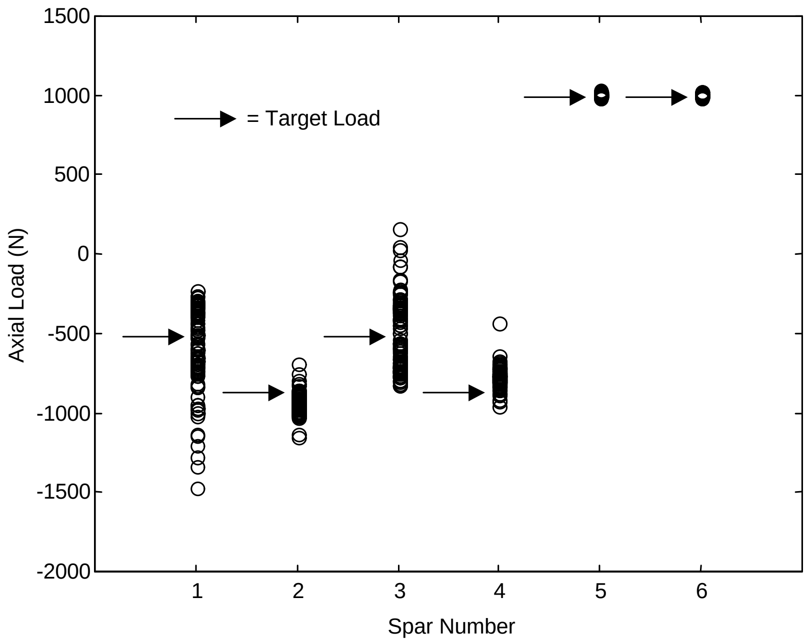


Fig. 6. Loads identified from noisy simulated experimental data.

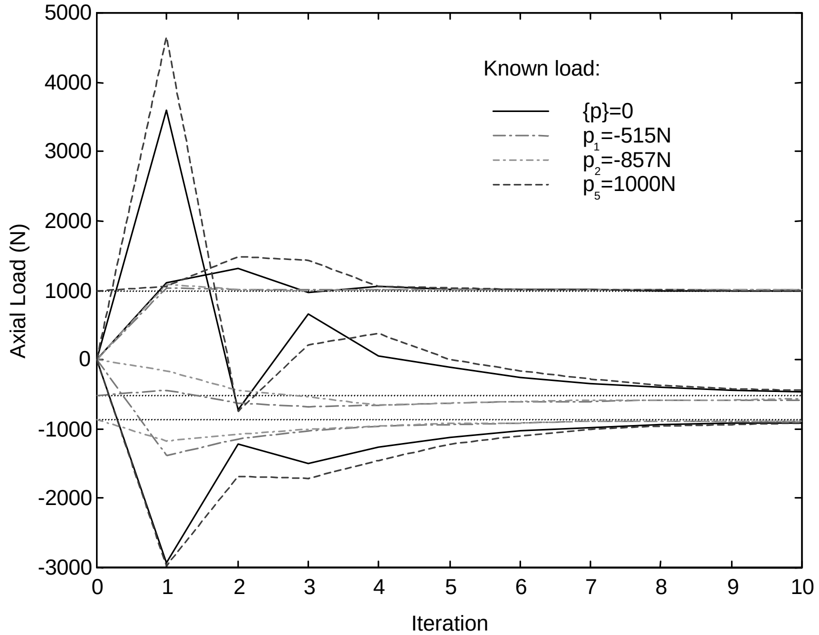


Fig. 7. Load identification with various initial known loads: known load.

## 4. Experimental case study

Three frameworks as described above were manufactured and tested. The magnitude of an arbitrary applied load in one of the frames was determined by strain gauges mounted on either side of and at the centre of each spar. The two remaining frames were constructed to allow some comparison between load-dependent effects and changes in behaviour resulting from test-to-test variability, a more detailed explaination of the testing of the framework by the authors [15] reveals that the effects of loading are clearly more significant than the effects of test-to-test variability.

Tension loading of up to approximately 2 kN in the diagonal members was found to be easily achievable. The application of loading resulted in noticeable curvature of the compression members but strain measurements indicated the structure to be comfortably within serviceability limits. Dynamic mobility measurements were taken at 26 points in the out of plane direction at an arbitrary level, approximately 50% of the 2 kN practical maximum load.

The symmetry of the structure along with the effect of loading on mode shape conspire to make the task of correlating modes between the FE prediction and the experimental readings a complex task [15]. Correlation of modes between the unloaded and loaded state also proves to be a thorny problem. Comparison of more than two sets of data using the MAC to compare mode shapes is a labour intensive process from which it was found that in the current case only seven modes could be reliably correlated. These correspond to modes 1–5, 14 and 15 of the unloaded finite element model. Referring back to Fig. 2 provides some explanation for the uncertain correlations; there appears to be a relationship between modes which are affected by geometry changes (shown by the dotted line) and those which cannot easily be paired. The difficulty in clearly correlating mode

Table 2
Initial and updated FE predictions and experimental resonance frequency measurements

| FE mode | Frequency (Hz) Experimental | Initial FE | Updated FE | Δ frequency (Hz) Initial–experimental | Initial–updated |
|---|---|---|---|---|---|
| 1 | 39.69 | 44.19 | 36.18 | 4.49 | −3.51 |
| 2 | 89.95 | 76.90 | 90.00 | −13.05 | 0.05 |
| 3 | 94.57 | 92.72 | 94.31 | −1.85 | −0.26 |
| 4 | 115.43 | 124.11 | 113.81 | 8.68 | −1.62 |
| 14 | 308.85 | 318.17 | 308.99 | 9.32 | 0.14 |
| 15 | 341.55 | 349.96 | 341.72 | 8.41 | 0.16 |

pairs is exacerbated by the similarity of some pairs of modes resulting from the near symmetry of the structure.

The updating procedure was applied using the axial load in each of the spars as updating parameters to minimise the differences in the resonant frequencies corresponding to the seven correlating modes. The updated natural frequencies are compared with initial and experimental values in Table 2. As one would expect from an updating procedure, the updated model produces a set of predicted natural frequencies which match the measured resonances.

Of more significance is Fig. 8 which shows the dynamically identified loads compared with those measured independently with strain gauges. The convergence of the loads upon their final updated values is also shown. A close match of the the compression loads is seen to be obtained. Tension loads are identified in the diagonal members at about 50% of the known level. As with any updating procedure the mismatch occurs since it has not been possible to access all of the incorrect parameters in the initial model. The comparitively crude model of the turnbuckle is the most likely candidate for causing the disparity. Validation of finite element model updating procedures using real experimental data is unusual in itself and the ability to independently confirm the veracity of updated parameters more so. To this extent the results provide some confidence that finite element model updating will become viable tool in practical situations.

Although the loads have not been identified exactly it could still be argued that the updated model is a better representation of the loaded structure than the initial model. A better match could be expected if some knowledge of static loads or their inter-relationships was assumed.

## 5. Conclusions

A method allowing stress stiffening parameters in a finite element model to be updated has been introduced. The technique has been shown to provide a means of identifying axial loads in beam elements from the dynamic measurements of the structure in a loaded state. The prediction of unloaded behaviour is generated by a finite element model and the identification of elemental loads is performed as part of a more general identification of mis-modelled structural properties.

The validity of the method has been demonstrated using simulated experimental data from a redundant two-dimensional framework. Loads in the structure have been correctly identified using modal data. The likelihood of identification of structural loading is shown to be resilient to the addition of realistic levels of noise.

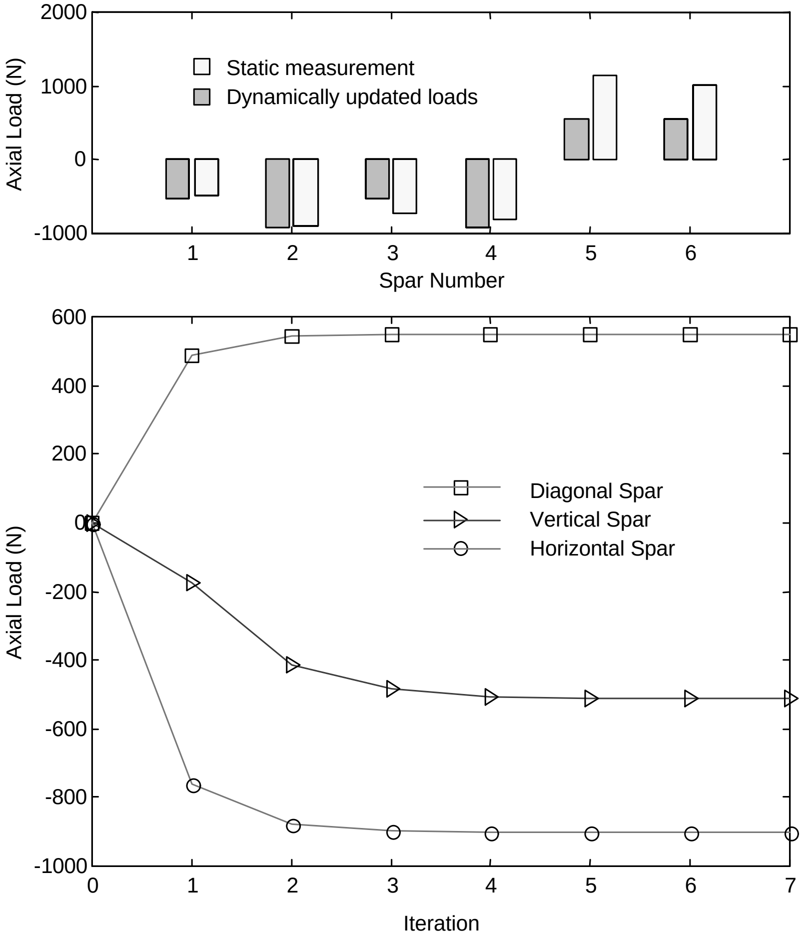


Fig. 8. Identified loads, convergence on updated values: diagonal spar; vertical spar; horizontal spar.

Some independent static measurement of the load within a structure is shown to improve the chance of successfully identifying elemental loading. The choice of location at which loading is specified is shown to be significant in determining the speed and accuracy of the dynamic load identification.

An experimental case study has provided further confidence in the method. A realistic estimation of loading in a simple framework is estimated from dynamic measurements.

The difficulty in correlating modes which have been affected by deformation is identified as an area for further investigation.

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