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On one-port characterization of noise sources in ducts by using external loads

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Abstract

Equivalent acoustic source characterization of duct-borne fluid machinery noise is often undertaken by interpolating the results of two-microphone pressure measurements with different external acoustic loads over a linear one-port source model. If the source is time-invariant, the one-port source characteristics can be determined by using only two external loads. This is well known as the two-load method. An extension of the two-load method for time-variant sources is also available and known as the multiple-load method. In these methods the source is treated as a 'black-box'. This paper addresses the problem of one-port source characterization when the linear operations inherent in the 'black-box' are known explicitly. The equations governing the explicit one-port source models are derived and the source characteristics are shown to be measurable using only few acoustic loads. It is not the purpose of this paper to discuss the application of these models to any specific fluid machinery; however, of particular interest are the explicit source models that require only two loads. Numerical results are presented to show some features of such time-invariant and time-variant explicit one-port source models.

1. Introduction

Equivalent acoustic source characterization of duct-borne fluid machinery noise is often undertaken by using linear one-port models. The previous work and the measurement methods available for estimation of the characteristics of a linear one-port source are encompassed in the review article by Bodén and Åbom [1]. They consider the existing measurement methods in two groups. The methods in the first group require the use of an external sound source, whereas those in the second group require the use of external loads. The present paper will contribute to the second group of methods when complex pressure measurements can be made at the source plane.

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In this case, if the equivalent source is time-invariant, the one-port source characteristics can be determined by using only two external loads. This is known as the two-load method. Although the two-load method is strictly valid for linear time-invariant equivalent source characterization, several authors have reported that it may give useful results also in situations that are not exactly time-invariant or linear, if applied by using a number of extra loads to average out the modelling errors in the least-squares sense. An extension of the two-load method to characterization of linear time-variant periodic sources has been presented by Bodén [2]. This method, which is known as the multiple-load method, requires $1 + 2\,M$ loads, where M denotes the number of harmonics to be included in the source spectrum

The two and multiple-load methods are 'black-box' formulations, that is, no information is required for the formulation of the equivalent source system other than its linearity. In some cases, however, further information about the structure of the equivalent source system may be available or can be stipulated. The present paper addresses the problem of equivalent one-port source characterization, by using external loads, when the linear operations inherent in the 'black-box' are known explicitly. It will be shown that such explicit one-port source models can be measured by using only few loads. It is not the purpose of this paper to discuss the application of explicit one-port source models to any specific fluid machinery, however, of particular interest are the explicit source models that require only two loads and some numerical results obtained using these are included to show the features additionally available when using an explicit source model.

The present analysis also provides a unified formulation of equivalent linear one-port sources. The 'black-box' treatment is still possible, in which case one re-discovers the two-load method for time-invariant sources, and a generalization of the multiple-load method for time-variant periodic sources.

2. One-port equivalent source formulation

2.1. Assumptions and definitions

The Thévenin theorem on equivalent linear networks, which is carried over to duct acoustics by assuming fundamental mode propagation, forms the basis of measurements for one-port source characterization of duct-borne fluid machinery noise. This approach presumes that the fluid machinery noise source and the duct system that transmits it can be modelled as a two-terminal network of passive linear elements and independent pure sources. The section of the duct at which the Thévenin theorem is invoked for equivalent source characterization is referred to as the source plane. It is assumed that plane wave decomposition by complex pressure measurement can be made at the source plane using the two-microphone method. Then, the travelling pressure wave components at the source plane are considered as quantities known by measurement, and the measured acoustic pressure, $p'(t)$, at the source plane is given by the sum of the pressure wave components as

$$p'(t) = p^{+}(t) + p^{-}(t), \tag{1}$$

where t denotes the time and the superscripts '+' and '−' are used as usual to refer to the forward and backward travelling pressure wave components, respectively. Upon neglecting viscothermal effects, the pressure wave components are related to the particle velocity, $v'(t)$, by the relationship

$$\rho c v'(t) = p^+(t) - p^-(t) \tag{2}$$

Here, ρ denotes the ambient density and c denotes the speed of sound at the source plane, which is assumed to be constant temporally. Hence, the particle velocity at the source plane also constitutes a variable known by measurement. It should be noted that, when mean flow is present in a duct, acoustic pressure and particle velocity are not power variables as their electrical analogues, voltage and current. For a uniform duct carrying a uniform subsonic mean flow, $p'(t)$ and $v'(t)$ can be defined as acoustic power variables if Eqs. (1) and (2) are written with $p^+(t)$ replaced by $(1 + M_o)p^+(t)$ and $p^-(t)$ by $(1 - M_o)p^-(t)$, where M_o denotes the Mach number of the mean flow velocity, as the product of thus defined $p'(t)$ and $v'(t)$ gives the instantaneous axial acoustic intensity in the duct . Therefore, in what follows, $p'(t)$ and $v'(t)$ can be interpreted either as the physical acoustic pressure and particle velocity or, if the above-described modification is supposed to have been made, as acoustic power variables.

For fluid machinery running under steady conditions, invoking of the Thévenin theorem at the source plane of the flow-duct system gives, in the time domain,

$$p_O(t) = \left[\Im\{v'(t)\}\right]' + p'(t), \tag{3}$$

where a prime ($'$) denotes the fluctuating part of a quantity, $\Im$ denotes a linear integro-differential operator and $p_O(t)$ denotes the fluctuating part of the equivalent ideal pressure source, and all time-dependent quantities are periodic functions of time of period, say T. It is assumed that the operator $\Im$ is of the form

$$\Im = aD^\alpha + bD^\beta + \ldots, \tag{4}$$

where D^λ denotes λ-fold differentiation with respect to time for positive λ, or integration if λ is negative, $\alpha, \beta, \ldots$ denote non-equal integers and the coefficients $a, b, \ldots$ denote real constants or periodic function of time of period T. A typical form of Eq. (4) is $\Im = a + bD$, that is, $\alpha = 0$ and $\beta = 1$, which represents a predominantly resistive and inductive source.

The objective of the source measurements may now be posed as the determination of $p_O(t)$ and the source coefficients $a, b, \ldots$. The next section derives the equations that are needed for the determination of these parameters.

2.2. Derivation of complex source equations

It is convenient to represent the coefficients $a, b, \ldots$ in Eq. (4) generally by Fourier series expansions

$$a(t) = \sum_{k=-\infty}^{\infty} A_k \exp(\mathrm{i}k\omega t), \quad b(t) = \sum_{k=-\infty}^{\infty} B_k \exp(\mathrm{i}k\omega t), \quad \ldots \tag{5}$$

Here, A_k and B_k denote the complex Fourier coefficients, $\omega(= 2\pi/T)$ denotes the fundamental radian frequency and i denotes the unit imaginary number. Not all of these coefficients have to be periodic functions, some may be constant, in which case the corresponding Fourier series reduces to a real constant. For simplicity of the presentation, however, the following analysis assumes them to be all constant or periodic in time.

The periodic time dependence of the coefficients in Eq. (4), if any, is assumed to be due to periodic fluid machinery action that also determines the time dependence of the transmitted noise. Then, $p'(t)$ and $v'(t)$ may be assumed to be periodic of period T and have Fourier series representation as

$$v'(t) = \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} V'_k \exp(\mathrm{i}k\omega t), \quad p'(t) = \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} P'_k \exp(\mathrm{i}k\omega t), \tag{6}$$

where P'_k and V'_k denote the complex Fourier coefficients. Upon substituting the foregoing expansions in Eq. (3), it follows that

$$p_O(t) = \left[\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \left[\sum_{\substack{m=-\infty \\ m \neq k}}^{\infty} Z_m^{(k-m)} V'_{k-m} \right] \exp(\mathrm{i}k\omega t) \right] + p'(t). \tag{7}$$

Here,

$$Z_m^{(k)} = (\mathrm{i}k\omega)^{\alpha} A_m + (\mathrm{i}k\omega)^{\beta} B_m + \cdots . \tag{8}$$

Upon separating the convolution summation into parts corresponding to positive and negative values of m and noting the identity

$$Z_{-m}^{(k)} = \breve{Z}_m^{(-k)}, \tag{9}$$

Eq. (7) can be expressed as

$$p_O(t) = \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \left[\sum_{\substack{m=1 \\ m \neq k}}^{\infty} Z_m^{(k-m)} V'_{k-m} + \sum_{\substack{m=1 \\ m \neq -k}}^{\infty} \breve{Z}_m^{(-k-m)} V'_{k+m} + Z_0^{(k)} V'_k \right] \exp(\mathrm{i}k\omega t) + p'(t), \tag{10}$$

where an inverted over-arc ($\breve{A}$) denotes the complex conjugate. Thus, it follows from Eq. (10) and the second of Eq. (6) that, the Fourier series for the source pressure is

$$p_O(t) = \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} S_k \exp(\mathrm{i}k\omega t), \tag{11}$$

where

$$S_k = \sum_{\substack{m=1 \\ m \neq k}}^{\infty} Z_m^{(k-m)} V'_{k-m} + \sum_{\substack{m=1 \\ m \neq -k}}^{\infty} \breve{Z}_m^{(-k-m)} V'_{k+m} + Z_0^{(k)} V'_k + P'_k. \tag{12}$$

This infinite set of complex equations can be used for the determination of the equivalent source characteristics if the convolution summations are suitably truncated. The number of terms in these summations determines the number of harmonics in the spectrum of the integrodifferential operator $\Im$, whereas the number of complex equations determines the number of harmonics in the spectrum of the source pressure, $p_O(t)$. It is assumed that, using the first M harmonics can represent $\Im$ with sufficient accuracy. This would induce a finite number of unknowns in Eq. (12).

However, Eq. (12) are not independent for all $k = -\infty, \ldots, -1, 1, \ldots, \infty$. Indeed, a straightforward analysis shows that, $\breve{S}_{-k} = S_k$, that is, Eq. (12) are independent only for $k = 1, 2, \ldots, \infty$. Therefore,by truncating the number of harmonics in the convolution summations at M and the number of complex equations at K, the equations that can be used for source characterization can be expressed as

$$S_k = \sum_{\substack{m=1 \\ m \neq k}}^{M} Z_m^{(k-m)} V'_{k-m} + \sum_{m=1}^{M} \breve{Z}_m^{(-k-m)} V'_{k+m} + Z_0^{(k)} V_k + P'_k, \quad k = 1, 2, \ldots, K. \tag{13}$$

Eq. (13) will be referred to as the complex source equations.

In general, the total number of source equations that can be written for one set of measurement data turns out to be less than the total number of unknowns in these equations. The deficiency in the number of equations must, therefore, be met by using independent sets of measurement data that are derived by applying a number of different loads to the duct system. Obviously, this approach is strictly feasible if the equivalent source is independent of the loads. The next section presents an analysis of the minimum number of loads that will be needed in particular cases.

2.3. The number of measurement loads

The number of loads for which the number of source equations equals the number unknowns in Eq. (13) depends on whether the integrodifferential operator $\Im$ is considered implicitly, as a 'black-box', or the operations $\alpha, \beta, \ldots$ are assumed to be known explicitly. These are considered separately in the following.

2.3.1. Explicit time-variant source model

First consider the explicit treatment of operator $\Im$. In this case, the number of real unknowns which the complex Fourier coefficients $A_m, \breve{A}_m, B_m, \breve{B}_m, \ldots$, $m = 1, 2, \ldots, M$, and $A_0, B_0, \ldots$ present will be equal to $j(1 + 2M)$, where j denotes the number of the source coefficients $a, b, \ldots$. K complex equations are required to solve the unknowns, but every complex equation will introduce a new complex unknown, S_k, $k = 1, 2, \ldots, K$. Then,

$$N = 1 + \mu, \quad \mu = \frac{j(1 + 2M)}{2K}, \tag{14}$$

where N is the ratio of the total number of real unknowns to the total number of scalar equations. Obviously, if N is an integer, it denotes the number loads that will be needed for the determination of the source characteristics. Solutions of the first of Eq. (14) for an integer N can be expressed as

$$K|(1 + 2M) = 0, \quad \left(\frac{K}{j/2}\right)|(1 + 2M) = 0, \quad j = 2, 4, 6, \ldots, \tag{15}$$

where $A|B$ means the remainder of B divided by A. Some combinations of j, M and K, that satisfy Eq. (15) with two and three loads are given in Table 1. With such combinations, the operator $\Im$ is determined to the accuracy of M harmonics in j operators, whilst the source pressure is determined to the accuracy of K harmonics. The evaluation of the convolution summations requires, however, that the number of harmonics in the spectrum of the acoustic measurements is equal to $K + M$, at least.

Table 1
Integer combinations that solve Eq. (15); the pattern of this table can be continued in vertical and horizontal directions

j	K	M	N	j	K	M	N	j	K	M	N	j	K	M	N
2	7	3	2	2	9	4	2	2	11	5	2	2	13	6	2
4	14	3	2	4	18	4	2	4	22	5	2	4	26	6	2
6	21	3	2	6	27	4	2	6	33	5	2	6	39	6	2
4	7	3	3	4	9	4	3	4	11	5	3	4	13	6	3
8	14	3	3	8	18	4	3	8	22	5	3	8	26	6	3
12	21	3	3	12	27	4	3	12	33	5	3	12	39	6	3

2.3.2. Time-variant black-box model

The integrodifferential operator $\Im$ can be treated also as a 'black-box' by considering $Z_m^{(k-m)}$, $\breve{Z}_m^{(-k-m)}$ and $Z_0^{(k)}$in Eq. (13) as complex unknowns without explicit reference to the form of the integrodifferential operations in Eq. (4). Upon assuming for a moment that $M<K$, the total number of these terms will be $2\,K(M+1)-M$. Then, with K complex equations, the ratio of the number of complex unknowns to the number of complex equations is given by $N=2(1+M)-M/K$, which cannot be an integer; that is, the determination of the source characteristics is not feasible for $M<K$. For $M\geqslant K$, on the other hand, the total number of complex unknowns is $K(1+2\,M)$ and, therefore, the source characteristics can be determined by using

$$N = 1 + 2M \tag{16}$$

loads. M and K can be selected arbitrarily provided that the condition $M\geqslant K$ is satisfied and that the acoustic measurement spectrum covers the first $M+K$ harmonics. Bodén's multiple-load method [2] corresponds to the $M=K$ case of this black-box formulation.

2.3.3. Time-invariant black-box model

The foregoing analysis takes into account the possibility that the linear operator $\Im$ may be time-variant. If $\Im$ can be postulated to be time-invariant a priori, the coefficients $a, b, \ldots$ can be set to constant values. Then $A_m=0$, $B_m=0, \ldots$ for $m\neq 0$ and Eq. (13) reduces to

$$S_k = Z_0^{(k)} V_k' + P_k', \quad k = 1, 2, \ldots, K. \tag{17}$$

As these equations are uncoupled and every equation contains two complex unknowns, the source characteristics can be determined for all harmonics in the spectrum of the acoustic measurements by using only two loads. This is the popular two-load method, and $Z_0^{(k)}$ is known as the source impedance. In this method, it is clear that, the operator $\Im$ is treated as a 'black-box'.

2.3.4. Time-invariant explicit source model

An explicit treatment of Eq. (17) is also possible. If $Z_0^{(k)}$is treated explicitly, the total number of real unknowns, the coefficients, $a, b, \ldots$ and the real and imaginary parts of S_k, in Eq. (17) will be equal to $j+2\,K$, where K is the number of harmonics required for the representation of the source strength, which also determines the number of complex equations that are to be written. Then, from Eq. (14) for $M=0$, taking $j=2\mu K$, where μ denotes any integer, will determine the source characteristics by using $N = 1 + \mu$ loads. The best choice for μ is $\mu = 1$, as this gives the source pressure strength with the largest number of harmonics for a given j (see Section 3.3).

3. Solution of source equations

This section presents numerical algorithms for the determination of the source characteristics with explicit and 'black-box' treatment of the governing integrodifferential operator. An application is presented in the next section.

3.1. The explicit source formulation

With Eq. (8) applied explicitly, Eq. (13) can be expressed in matrix notation by using the following vector definitions:

$$\widehat{\mathbf{S}} = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_K \end{bmatrix}, \quad \widehat{\mathbf{P}} = \begin{bmatrix} P'_1 \\ P'_2 \\ \vdots \\ P'_K \end{bmatrix}, \quad \widehat{\mathbf{A}} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_M \end{bmatrix}, \quad \widehat{\mathbf{B}} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_M \end{bmatrix} \ldots, \quad \mathbf{X}_0 = \begin{bmatrix} A_0 \\ B_0 \\ \vdots \end{bmatrix}, \tag{18}$$

where an over-arc ($\widehat{A}$) is used to denote a vector or matrix with complex elements. Then, in matrix form, Eq. (13) is

$$\widehat{\mathbf{S}} = \widehat{\mathbf{V}}_\alpha^- \widehat{\mathbf{A}} + \widehat{\mathbf{V}}_\alpha^+ \breve{\mathbf{A}} + \widehat{\mathbf{V}}_\beta^- \widehat{\mathbf{B}} + \widehat{\mathbf{V}}_\beta^+ \breve{\mathbf{B}} + \cdots + \widehat{\mathbf{V}}_0 \mathbf{X}_0 + \widehat{\mathbf{P}}, \tag{19}$$

where

$$\widehat{\mathbf{V}}_\lambda^\pm = (\mathrm{i}\omega)^\lambda \begin{bmatrix} (1\pm 1)^\lambda V'_{1\pm 1} & (1\pm 2)^\lambda V'_{1\pm 2} & \cdots & (1\pm M)^\lambda V'_{1\pm M} \\ (2\pm 1)^\lambda V'_{2\pm 1} & (2\pm 2)^\lambda V'_{2\pm 2} & \cdots & (2\pm M)^\lambda V'_{2\pm M} \\ \vdots & \vdots & \vdots & \vdots \\ (K\pm 1)^\lambda V'_{K\pm 1} & (K\pm 2)^\lambda V'_{K\pm 2} & \cdots & (K\pm M)^\lambda V'_{K\pm M} \end{bmatrix}, \quad \lambda = \alpha, \beta, \ldots, \tag{20}$$

$$\widehat{\mathbf{V}}_0 = \begin{bmatrix} (\mathrm{i}\omega)^\alpha V'_1 & (\mathrm{i}\omega)^\beta V'_1 & \cdots \\ (\mathrm{i}2\omega)^\alpha V'_2 & (\mathrm{i}2\omega)^\beta V'_2 & \cdots \\ \vdots & \vdots & \vdots \\ (\mathrm{i}K\omega)^\alpha V'_K & (\mathrm{i}K\omega)^\beta V'_K & \cdots \end{bmatrix}. \tag{21}$$

Separating the real and imaginary parts of Eq. (19), one obtains

$$\begin{bmatrix} \widehat{\mathbf{S}}_R \\ \widehat{\mathbf{S}}_I \end{bmatrix} = \begin{bmatrix} \widehat{\mathbf{V}}_{0R} & \widehat{\mathbf{V}}^-_{\alpha R} + \widehat{\mathbf{V}}^+_{\alpha R} & -\widehat{\mathbf{V}}^-_{\alpha I} + \widehat{\mathbf{V}}^+_{\alpha I} & \widehat{\mathbf{V}}^-_{\beta R} + \widehat{\mathbf{V}}^+_{\beta R} & -\widehat{\mathbf{V}}^-_{\beta I} + \widehat{\mathbf{V}}^+_{\beta I} & \cdots \\ \widehat{\mathbf{V}}_{0I} & \widehat{\mathbf{V}}^-_{\alpha I} + \widehat{\mathbf{V}}^+_{\alpha I} & \widehat{\mathbf{V}}^-_{\alpha R} - \widehat{\mathbf{V}}^+_{\alpha R} & \widehat{\mathbf{V}}^-_{\beta I} + \widehat{\mathbf{V}}^+_{\beta I} & \widehat{\mathbf{V}}^-_{\beta R} - \widehat{\mathbf{V}}^+_{\beta R} & \cdots \end{bmatrix} \begin{bmatrix} \mathbf{X}_0 \\ \widehat{\mathbf{A}}_R \\ \widehat{\mathbf{A}}_I \\ \widehat{\mathbf{B}}_R \\ \widehat{\mathbf{B}}_I \\ \vdots \end{bmatrix} + \begin{bmatrix} \widehat{\mathbf{P}}_R \\ \widehat{\mathbf{P}}_I \end{bmatrix}, \tag{22}$$

or, briefly

$$\mathbf{S} = \mathbf{V}\mathbf{X} + \mathbf{P}. \tag{23}$$

In Eq. (22), the attached subscripts 'R' and 'I' denote, respectively, the real and imaginary parts of a complex quantity, and the meanings of the symbols in Eq. (23) are obvious from Eq. (22).

Eq. (23) constitutes $2K$ scalar equations for $j(1 + 2M) + 2K$ real unknowns. The number of loads required to determine the source characteristics are determined from Eq. (14). Thus, assuming N loads solve the source characteristics, Eq. (23) will apply for every load separately:

$$\mathbf{S} = \mathbf{V}^{(1)}\mathbf{X} + \mathbf{P}^{(1)},\ \ \mathbf{S} = \mathbf{V}^{(2)}\mathbf{X} + \mathbf{P}^{(2)}, \ldots,\ \ \mathbf{S} = \mathbf{V}^{(N)}\mathbf{X} + \mathbf{P}^{(N)}, \tag{24}$$

where the superscripts refer to the loads. Subtracting the first of Eq. (24) from the remaining equations in turn gives

$$\begin{bmatrix} \mathbf{V}^{(2)} - \mathbf{V}^{(1)} \\ \mathbf{V}^{(3)} - \mathbf{V}^{(1)} \\ \vdots \\ \mathbf{V}^{(N)} - \mathbf{V}^{(1)} \end{bmatrix} \mathbf{X} = - \begin{bmatrix} \mathbf{P}^{(2)} - \mathbf{P}^{(1)} \\ \mathbf{P}^{(3)} - \mathbf{P}^{(1)} \\ \vdots \\ \mathbf{P}^{(N)} - \mathbf{P}^{(1)} \end{bmatrix}. \tag{25}$$

This equation determines the coefficients of the operator $\Im$. The source pressure can then be computed using the first, say, of Eq. (24). Note that the coefficient matrix in Eq. (25) has the dimensions $(2K(N - 1)) \times j(1 + 2M)$. Since the number of loads is selected according to Eq. (14), the first dimension will be equal to $j(1 + 2M)$, making the coefficient matrix a square matrix, as desired.

If the equivalent source is known to be time-invariant, then Eq. (17) applies and Eq. (22) reduces to

$$\begin{bmatrix} \widehat{\mathbf{S}}_R \\ \widehat{\mathbf{S}}_I \end{bmatrix} = \begin{bmatrix} \widehat{\mathbf{V}}_{0R} \\ \widehat{\mathbf{V}}_{0I} \end{bmatrix} \mathbf{X}_0 + \begin{bmatrix} \widehat{\mathbf{P}}_R \\ \widehat{\mathbf{P}}_I \end{bmatrix}, \tag{26}$$

which is written briefly as

$$\mathbf{S} = \mathbf{V}_0\mathbf{X}_0 + \mathbf{P}. \tag{27}$$

Eqs. (24) and (25) hold for this case with $\boldsymbol{X}$ replaced by $\boldsymbol{X}_0$ and $\boldsymbol{V}^{(n)}$ replaced by $\mathbf{V}_0^{(n)}$, $n = 1, 2, \ldots, N$, the coefficient matrix in Eq. (25) now being a square matrix of dimension $j \times j$.

3.2. The 'black-box' formulation

In this case, Eq. (13) can be solved one by one. This follows from the fact that every one of these equations contains $1 + 2M$ complex unknowns, which is equal to the number of loads required for the solution (see Eq. (16)). Therefore, given the measurement data for $1 + 2M$ loads, any one of Eq. (13) can be written $1 + 2M$ times and the resulting set of equations can be solved for the complex unknowns in that equation. Repeating this procedure for the remaining equations determines all the complex unknowns in Eq. (13). This formulation is not considered any further in this paper, as the number of loads that are required is generally too large to be of practical value, for example, at least 21 loads are required to measure the first 10 harmonics of the source pressure strength.

In the time-invariant case, which is the usual two-load method, Eq. (17) can be solved either as described above on per equation basis, or by combining the two sets of the equations corresponding to the two loads. The latter is similar to the solution described for Eq. (24), except that now the unknowns vector can be dealt with in complex form.

3.3. Load and harmonic over-determination

The linearity and load-independence hypotheses that underlie the foregoing one-port equivalent source formulations are seldom satisfied exactly in real fluid machinery. When applied to real sources that are non-linear and load dependent to some degree, the modelling errors may be minimized in the least-squares sense by using a number of extra loads, as is the current practice with the two-load and the multiple-load methods. For the present explicit one-port formulation, this consists of writing Eq. (25) for $N + n$ loads, where n is the number of extra loads. Then, Eq. (25) constitutes an over-determined system of algebraic equations and the unknowns vector, $\boldsymbol{X}$, can be solved by taking the pseudo-inverse of the coefficients matrix. The black-box models are over-determined over loads similarly, by writing Eq. (13), or (17), for $N + n$ loads.

The explicit source models, in addition to over-determination over a load space, also allow over-determination over the harmonics in the spectrum of the measurements. This consists of writing the source equations for the first $K + k$ harmonics, where K satisfies Eq. (15) and k denotes the extra number of harmonics. Then, Eqs. (20) and (21) are written for $K + k$ harmonics and Eq. (25) becomes an over-determined set, which can be solved again in the least-squares sense.

4. The explicit two-load method

The use of an explicit one-port source models with N acoustic loads is called, for ease of reference, the explicit N-load method. From a practical point of view, the most significant explicit N-load method is the explicit two-load method. This section presents an application of the explicit two-load method to show the features additionally provided over the usual two-load method when an explicit equivalent source formulation is adopted. Over-determination over a load space is not considered, as this is well treated in the literature with reference to the two-load and multiple-load methods [1], and it is similarly applied with the explicit N-load method and has similar statistical implications.

For the present application, the source operator is assumed to be of the form $\Im = a + bD$ and the source plane data are derived, by the usual discrete Fourier transform methods, from two-microphone pressure measurements on an engine exhaust pipe. The details of the engine and the measurements are not relevant to the present analysis as only the end results are needed for source characterization. The results are based on a particular set of source plane data for two acoustic loads at the fundamental frequency of 10 Hz and the source characteristics are given for the first 20 harmonics.

Fig. 1 shows the source characteristics for the first 20 harmonics, as computed with the usual two-load method. The source impedance is given in normalized form, that is, as $Z_0^{(k)}/\rho c$, and, therefore, it is non-dimensional. In this method, the harmonics of the source characteristics are determined solely by the corresponding harmonics in the source plane data. While this is consistent with the assumption of linearity and time invariance, it means that, if there are interactions between the harmonics due to violation of these assumptions, their effects will be interpreted incorrectly. Bodén and Albertson [3] have noted that the source impedance with a negative real part is physically meaningless for a linear system and should be taken as indication of non-linear interactions between the harmonics. Also, it is known that, the real part of the source impedance may come out negative when a time-variant source is modelled as if it were time-invariant by using the two-load method [2]. Thus, the observed negative real parts of the source impedance in Fig. 1 may be attributed to the inherent non-linearity or time-variance of the equivalent source under consideration.

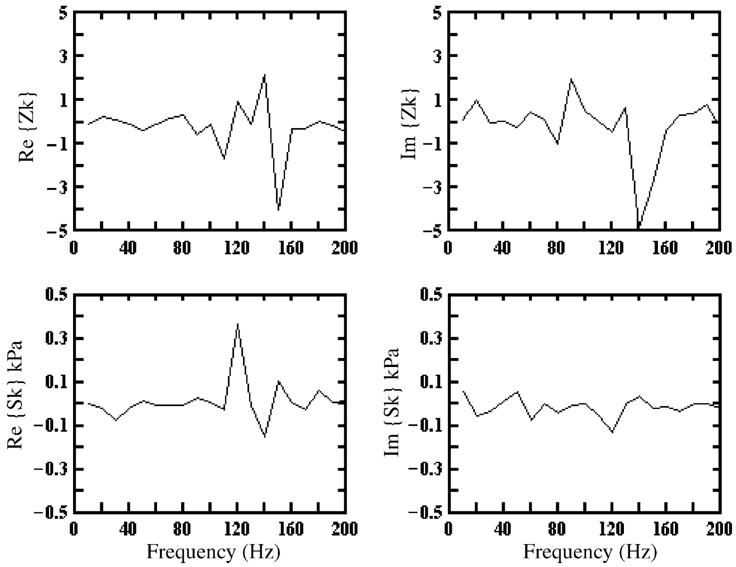

Fig. 1. Source characteristics computed using the two-load method. Re{} and Im{} denote the real and imaginary parts of a complex quantity; S_k denotes the kth harmonic of the complex source pressure strength, Z_k denotes the kth harmonic of the complex normalized source impedance.

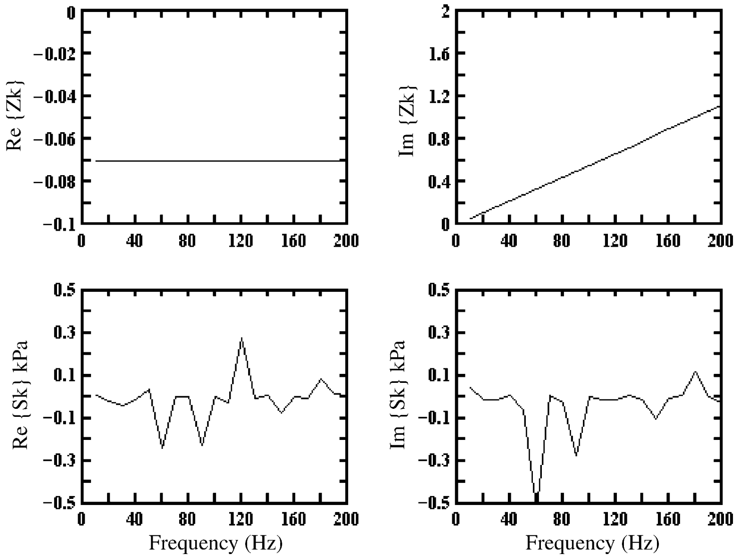

Fig. 2. Source characteristics computed using the explicit time-invariant two-load method with harmonic over-determination over the first 20 harmonics. Labels of axes as in Fig. 1.

Shown in Fig. 2 are the source pressure strength and normalized source impedance as computed using the explicit two-load method with the source operator treated as time-invariant, that is, with the coefficients a and b assumed to be constant. These results are obtained by using harmonic over-determination over the first 20 harmonics in the measurement spectrum. Harmonic over-determination is necessary, as only the first harmonic of the source spectrum can be computed without it for the assumed form of the source operator. Over-determination with a larger number of harmonics was also considered, but this did not effect the characteristics shown in Fig. 2. The source impedance is calculated from

$$Z_0^{(k)} = (\mathrm{i}k\omega)^{\alpha} A_0 + (\mathrm{i}k\omega)^{\beta} B_0 + \cdots, \quad k = 1, 2, \ldots, K \tag{28}$$

and normalized as described above. It is seen that, with harmonic over-determination over the measurement spectrum, the real and imaginary parts of the source impedance are smoothed out to straight lines: The real part of the normalized source impedance is equal to $a/\rho c$ and the slope of the imaginary part is proportional to b, where a and b are determined by linear regression over the measurement spectrum. The real part of the source impedance is close to zero, but still negative, which may be due to traces of interactions between the harmonics. It should be noted that, without harmonic over-determination, the explicit two-load method with a time-invariant source operator with j terms is equivalent to the usual two-load method in its harmonic range, i.e., up to and including the harmonic $K = j/2$.

Shown in Fig. 3 are the source pressure strength and the normalized *effective* source impedance, which is defined by Eq. (29), as computed by using the explicit two-load method with the source operator treated as time-variant. These results were obtained by using $M = 6$ harmonics in spectra of the coefficients $a(t)$ and $b(t)$, and harmonic over-determination over the first 40 harmonics ($K = 40$) in the measurement spectrum. Without harmonic over-determination, the

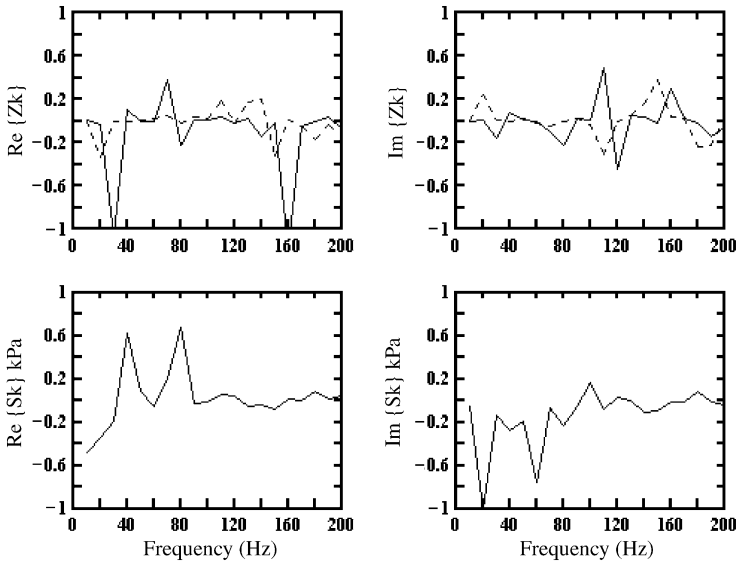

Fig. 3. Source characteristics computed using the explicit time-variant two-load method with harmonic over-determination over the first 40 harmonics and six harmonics in the spectrum of the coefficients of the source operator. Solid: Load 1, dash: Load 2. Labels of axes as in Fig. 1.

source spectrum can be determined to $1 + 2M$ harmonics, but M could not be increased freely due to the impact of ill-conditioning problems, which tend to be more inhibiting without harmonic over-determination. Therefore, in this case, harmonic over-determination is used both to extend the frequency range of the source spectrum and to alleviate ill-conditioning problems with relatively large M. Increasing the extent of harmonic over-determination does not alter the source characteristics in Fig. 3.The effective source impedance is defined, from Eq. (13), as

$$Z_e^{(k)} = Z_0^{(k)} + \sum_{\substack{m=1 \\ m \neq k}}^{M} Z_m^{(k-m)} \frac{V'_{k-m}}{V'_k} + \sum_{m=1}^{M} \breve{Z}_m^{(-k-m)} \frac{V'_{k+m}}{V'_k}, \quad k = 1, 2, \ldots, K. \tag{29}$$

which is normalized as usual. With this definition, Eq. (13) can be expressed in the form of Eq. (17) with $Z_0^{(k)}$ replaced by $Z_e^{(k)}$. The effective source impedance depends explicitly on the source plane measurement data and, therefore, it can be computed for each acoustic load. Fig. 3 shows the effective source impedance in normalized units for both loads. With the two characteristics being markedly different, the equivalent source may be said to be load dependent to some degree.

For smaller values of M, the source characteristics display substantial variations, but these variations tend to diminish as M increases. This indicates the presence of relatively stronger interactions between the harmonics of smaller orders than larger orders. To give an idea of these variations, the source characteristics are shown superimposed in Fig. 4 for $M = 6$, 7 and 8 harmonics in the spectra of $a(t)$ and $b(t)$, the normalized source impedance being for the load that is referred to as the second load in Fig. 3. The characteristics for the source pressure display a fair

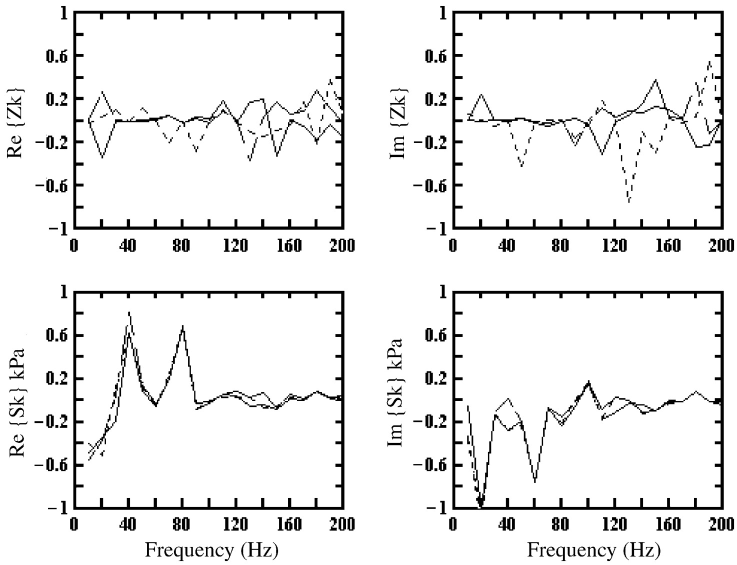

Fig. 4. Effect of the number of harmonics in the spectrum of the coefficients of the source operator on source characteristics computed using the explicit time-variant two-load method with harmonic over-determination over the first 40 harmonics. Solid: $M = 6$, dash: $M = 7$ and broken: $M = 8$. Labels of axes as in Fig. 1.

degree of convergence, however, there are marked variations in the effective normalized source impedance.

5. Conclusion

A unified formulation of equivalent linear one-port source characterizations of duct-borne noise based on two-microphone pressure measurements with external loads has been presented. The main feature of this formulation is the introduction of the linear source operator $\Im$. If this operator is treated as a black-box, one obtains the well-known two-load method for a time-invariant source. For a time-variant periodic source, the black-box approach is not convenient because it requires too many external loads. If, however, the operational structure of the source operator is known, or stipulated, explicit one-port source models that require only few external loads can be developed. This paper has presented the equations governing the measurement of such explicit one-port equivalent source models and demonstrated the features that are additionally available when using these models.

Inherent in one-port source characterization using when external loads is the assumption that the equivalent source is independent of the loads, as well as being linear. These conditions are seldom satisfied exactly in real fluid machinery. Therefore, the ability to test whether a measured source is linear and loadindependent is of importance. The features of the explicit one-port source models may find use as statistical measures of source non-linearity, time-variance and load dependence, but these remain as topics for future study.

In the present analysis, the source operator $\Im$ is assumed to have a known explicit operational structure. If such information is not available, $\Im$ can be considered as a truncated form of a complete power series expansion in D. This is possible for any stable linear network with ideal elements. For example, if the equivalent source network is time-invariant, the operator D can be replaced with the Laplace variable to convert the operator $\Im$ to the driving point impedance, which is a quotient of two polynomials in the Laplace variable and can in general be expanded into a power series. For a time-variant network, the driving point impedance is not generally expressible in such an explicit form due to involved convolutions, but a power series expansion is still possible. The problem here is that the number of terms, j, in the power series representation of the source operator cannot be increased freely, because Eq. (25) becomes severely ill-conditioned as j increases. Thus, it is desirable to keep j small enough, less than 10, say, unless ways are found to prevent the impact of ill-conditioning.

References

[1] H. Bodén, M. Åbom, Modelling of fluid machines as sources of sound in duct and pipe systems, Acta Acoustica 3 (1995) 549–560.

[2] H. Bodén, The multiple load method for measuring the source characteristics of time-variant sources, Journal of Sound and Vibration 148 (1991) 437–453.

[3] H. Bodén, F. Albertson, Linearity tests for in-duct acoustic one-port sources, Journal of Sound and Vibration 237 (2000) 45–65.