ACADEMIC PRESS

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Journal of Sound and Vibration 260 (2003) 417–429

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Liquid flows and vibration characteristics of straight-tube cylindrical shells

M. Willatzen*

Mads Clausen Institute for Product Innovation, University of Southern Denmark, Grundtvigs Allé 150, DK-6400 Sønderborg, Denmark

Received 19 December 2001; accepted 15 April 2002

Abstract

The influence of a moving fluid confined by a solid circular cylindrical shell on the propagation of acoustic waves generated by sources located on the circular cylindrical shell is examined. An expression for the acoustic pressure in a moving fluid is derived including azimuthal asymmetry effects in the general case, where the fluid velocity points along the cylindrical shell axis and can be written as an infinite power series expansion in the radial co-ordinate. Secondly, continuity of pressure and normal velocity at the liquid–shell interface is imposed to (a) derive a set of coupled differential equations governing the possible vibrational modes of the shell and (b) determine dispersion relations, i.e., mode propagation constants β as a function of frequency as well as changes in β values accomodated by flow. In the remaining part of the paper, phase speed changes with flow and transit-time differentials of circular cylindrical shell vibrations are discussed with special emphasis to flow measurement properties.

1. Introduction

In this paper, vibration dynamics of thin cylindrical shells containing a moving fluid are studied. Several works on vibration dynamics, flutter, damping, and stability of thin cylindrical shells containing a quiescent [1–4] or moving [5–10] fluid exist in the literature. In this paper, special emphasis is given to the variation of phase speeds and group speeds of the various modes propagating along the cylindrical shell axis due to the presence of a moving fluid in the general case where the flow velocity can be written as a series expansion in the radial co-ordinate r. To the best of the author's knowledge such a general quasi-analytical analysis in the flow velocity has not

*Tel.: +45-6550-1682; fax: +45-6550-1660.
E-mail address: willatzen@mci.sdu.dk (M. Willatzen).

PII: S0022-460X(02)00940-9

been considered before. The discussion is continued by addressing the possibility of measuring flow by vibrating a cylindrical shell at one point of location and measuring the change in phase of the vibration at some other point located on the tube due to flow of fluid through the tube [11–14]. In particular, the aim is to search for vibrational modes which are most sensitive to a moving fluid, i.e., modes for which flow-induced phase changes and associated transit-time differentials between two points located at different axial positions on the cylindrical shell are as high as possible.

In the first part of the paper, a general differential equation governing pressure variations in a moving fluid confined by cylindrical shell walls is derived [15–18]. The equation obtained is general in the sense that no assumptions as to the profile of the fluid flow in the absence of acoustic excitation are imposed except that the flow profile can be written as a power series expansion in the radial co-ordinate. Following the derivation of the pressure p in the flowing liquid, coupling to the cylindrical shell vibrations is considered. This coupling is a consequence of the continuity of pressure and normal velocity at the boundary between the liquid and the cylindrical shell. In doing this, a system of three linear equations is obtained by use of the shell- and fluid differential equations. The secular equation of the associated linear matrix problem allows determination of dispersion relations, i.e., determination of the functional relationship between axial propagation constants β and (angular) frequency ω of vibrations. The propagation constants β are functions of the fluid flow in the absence of acoustic vibrations and therefore changes in phase speeds $c_p = \omega/\beta$ with flow result. In turn, as mentioned above, phase-speed changes with flow are associated with phase changes with flow and transit-time differentials between two points located at different axial positions on the shell, the measurement of which is the basic principle behind the straight-tube cylindrical shell flow meter [11–14].

2. Theory

This section is partitioned in four subsections: (a) differential equation for the acoustic pressure in a compressible moving fluid; (b) acoustic pressure solution using the Frobenius power-series expansion method; (c) coupling between a moving compressible fluid and vibrations in the cylindrical shell confining the fluid; and (d) implications of a fluid flow on phase speeds/group speeds of cylindrical shell vibrational waves.

2.1. *Differential equation for the acoustic pressure in a compressible moving fluid*

In the following, sound propagation in a moving non-viscous fluid is described. The fluid is confined by walls of cylindrical geometry, but otherwise the characteristics of the walls are unspecified at this point. The flow velocity is assumed to be much smaller than the fluid speed of sound such that a linear model in the flow velocity suffices.

The starting point is the equation of continuity

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

and the Euler equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v} = -\frac{\boldsymbol{\nabla} p}{\rho}. \tag{2}$$

For the velocity, pressure, and density

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{v}', \tag{3}$$

$$p = p_0 + p', \tag{4}$$

$$\rho = \rho_0 + \rho', \tag{5}$$

where $\mathbf{v}_0$, p_0, and ρ_0 denote the (background) flow velocity, pressure, and density in an undisturbed medium, respectively, and it is assumed that the background flow corresponds to a steady incompressible flow situation ($\rho_0 = constant$, $\boldsymbol{\nabla} \cdot \mathbf{v}_0 = 0$). The primed quantities $\mathbf{v}'$, p', and ρ' represent small changes in flow velocity, pressure, and density due to the presence of, for example, low-intensity ultrasound waves in the medium. To first order in the small quantities above, Eqs. (1) and (2) read

$$\frac{\partial p'}{\partial t} + \rho_0 c^2 \boldsymbol{\nabla} \cdot \mathbf{v}' + (\mathbf{v}_0 \cdot \boldsymbol{\nabla})p' = 0, \tag{6}$$

$$\frac{\partial \mathbf{v}'}{\partial t} + \left(\mathbf{v}' \cdot \boldsymbol{\nabla}\right)\mathbf{v}_0 + (\mathbf{v}_0 \cdot \boldsymbol{\nabla})\mathbf{v}' = -\frac{\boldsymbol{\nabla} p'}{\rho_0} + \frac{p'}{(\rho_0 c)^2}\boldsymbol{\nabla} p_0, \tag{7}$$

and use has been made of the isentropic relation

$$p' = \left(\frac{\partial p}{\partial \rho_0}\right)_S \rho' = c^2 \rho', \tag{8}$$

assuming adiabatic and reversible conditions.

Next, the acoustic pressure $p' = p'(r, \theta, z; t)$ is separated in cylindrical co-ordinates r, θ, and z by assuming the functional form

$$p'(r, \theta, z; t) = P_0 f(r) g(\theta) \exp(\mathrm{i}\beta z - \mathrm{i}\omega t), \tag{9}$$

corresponding to monofrequency operation [exp (iωt)] and axially propagating waves [exp (iβz)] where $\mathrm{i} = \sqrt{-1}$. The function $g(\theta)$ can be represented by exp(i$m\theta$), where m is an integer as $g(\theta)$ must be a single-valued function satisfying

$$g(\theta) = g(\theta + 2\pi). \tag{10}$$

Inserting Eq. (9) and using $g(\theta) = \exp(\mathrm{i}m\theta)$ into Eqs. (6) and (7) leads to

$$-\mathrm{i}\omega p' + \rho_0 c^2 \left(\frac{\partial v_r'}{\partial r} + \frac{v_r'}{r} + \frac{\mathrm{i}m}{r} v_\theta' + \mathrm{i}\beta v_z'\right) + \mathrm{i}\beta v_0 p' = 0, \tag{11}$$

$$-\mathrm{i}\omega v_z' + v_r' \frac{\partial v_0}{\partial r} + \mathrm{i}\beta v_0 v_z' = \frac{-\mathrm{i}\beta}{\rho_0} p', \tag{12}$$

$$-\mathrm{i}\omega v_r' + \mathrm{i}\beta v_0 v_r' = -\frac{1}{\rho_0}\frac{\partial p'}{\partial r}, \tag{13}$$

$$-\mathrm{i}\omega v_\theta' + \mathrm{i}\beta v_0 v_\theta' = -\frac{\mathrm{i}m}{\rho_0}\frac{p'}{r}, \tag{14}$$

where use has been made of the assumption that the background flow $\mathbf{v}_0$ points in the axial direction and depends on the radial co-ordinate only:

$$\mathbf{v}_0 = (v_{0r}, v_{0\theta}, v_{0z}) = (0, 0, v_{0z}(r)). \tag{15}$$

Observe that Eqs. (2) and (15) imply that

$$\boldsymbol{\nabla} p_0 = -\rho_0(\mathbf{v}_0 \cdot \boldsymbol{\nabla})\mathbf{v}_0 = 0. \tag{16}$$

Eqs. (13) and (14) can be rewritten as

$$v'_r = \frac{\mathrm{i}}{\rho_0} \frac{1}{(\beta v_0 - \omega)} \frac{\partial p'}{\partial r}, \tag{17}$$

$$v'_\theta = -\frac{m}{\rho_0} \frac{1}{(\beta v_0 - \omega)} \frac{p'}{r}. \tag{18}$$

Inserting Eq. (17) into Eq. (12) yields

$$v'_z = -\frac{1}{\rho_0(\beta v_0 - \omega)^2} \frac{\partial p'}{\partial r} \frac{\partial v_0}{\partial r} - \frac{\beta}{\rho_0(\beta v_0 - \omega)} p'. \tag{19}$$

Upon isolating v'_r, v'_θ, and v'_z in terms of their dependence on acoustic pressure p' and derivatives of p', an ordinary differential equation for p' can be derived (insertion of Eqs. (17)–(19) into Eq. (11)) (see also Refs. [15–18])

$$\frac{\partial^2 p'}{\partial r^2} + \left(\frac{1}{r} - \frac{2\beta}{(\beta v_0 - \omega)} \frac{\partial v_0}{\partial r}\right) \frac{\partial p'}{\partial r} + \left(\frac{(\beta v_0 - \omega)^2}{c^2} - \beta^2 - \frac{m^2}{r^2}\right) p' = 0. \tag{20}$$

Next, the assumption that flow velocities are much smaller than the phase speed is made, i.e., $\beta v_0/\omega < < 1$. This condition is well satisfied in, for example, water/oil transport systems where the flow speed is in the range: 0–10 m/s while the sound speed is at or above 1500 m/s. In this case ($\beta v_0/\omega \ll 1$), Eq. (20) can be approximated by

$$\frac{\partial^2 p'}{\partial r'^2} + \left(\frac{1}{r'} + \frac{2\beta}{\omega} \frac{\partial v_0}{\partial r'}\right) \frac{\partial p'}{\partial r'} + \left(R^2 \frac{\omega^2 - 2\beta\omega v_0(r')}{c^2} - R^2\beta^2 - \frac{m^2}{r'^2}\right) p' = 0, \tag{21}$$

where

$$r' = \frac{r}{R} \tag{22}$$

has been introduced and R denotes the cylinder radius.

2.2. Acoustic pressure solution using the Frobenius power-series expansion method

Before applying the Frobenius method to solve Eq. (21), let it be assumed that the background flow $v_0(r')$ can be expanded in an infinite power series

$$v_0(r') = \sum_{\lambda=0}^{\infty} v_{0\lambda} r'^{\lambda}. \tag{23}$$

Note that this assumption is a weak restriction since most radially dependent velocity profiles can be written as a power series in the radial co-ordinate including flat profiles, parabolic profiles, and logarithmic profiles as suggested by Willatzen [19] and Nikuradse [20].

The Frobenius method is based on the assumption that p' can be written as a series expansion in r' (see, e.g., Ref. [21])

$$p' = \sum_{\lambda=0}^{\infty} a_\lambda r'^{\lambda+k}, \tag{24}$$

where k is unspecified (in general, at this point, k can be any real constant). Insertion of Eqs. (23) and (24) into Eq. (21) and demanding that $a_0 = 1$ gives $k = \pm m$ if terms proportional to r'^{k-2} are equated (corresponding to $\lambda = 0$). The case $k = -m$ can be rejected immediately as a physical allowable solution because p' in that case diverges at $r = 0$

$$\lim_{r\to 0} |p'| = \lim_{r\to 0}\left(r'^{-m} \left|\sum_{\lambda=0}^{\infty} a_\lambda r'^{\lambda}\right| \right) = \lim_{r\to 0} r'^{-m} = \infty, \tag{25}$$

and so $k = +m$ is required. Next, employing the identity principle for infinite power series to terms proportional to $r'^{\lambda+m}$ leads to the following recursion formula:

$$a_0 = 1,$$

$$a_1 = -\frac{m}{(2m+1)}\frac{2\beta}{\omega} v_{01} a_0,$$

$$\begin{aligned} a_{\lambda+2} = & -\frac{2\beta}{\omega}\frac{\lambda+m+1}{\left((\lambda+m+2)^2 - m^2\right)} v_{01} a_{\lambda+1} + \frac{2R^2\beta\omega}{\left((\lambda+m+2)^2 - m^2\right)c^2}\sum_{\lambda'=0}^{\lambda} a_{\lambda'} v_{0\lambda-\lambda'} \\ & + \frac{\left(\beta^2 R^2 - \frac{R^2\omega^2}{c^2}\right)}{\left((\lambda+m+2)^2 - m^2\right)} a_\lambda - \frac{2\beta}{\omega\left((\lambda+m+2)^2 - m^2\right)}\sum_{\lambda'=0}^{\lambda}(m+\lambda')(\lambda+2-\lambda') v_{0\lambda+2-\lambda'} a_{\lambda'} \\ & \text{if } \lambda \geqslant 0. \end{aligned} \tag{26}$$

Thus, a general solution to Eq. (21) is given by Eq. (24) where the coefficients a_λ obey the recursion formula expressed by Eq. (26) and $k = +m$. The parameter β is unspecified at this point as the possible values for β are determined by invoking boundary conditions at the fluid–shell interface (refer to the next subsection).

2.3. Coupling between a moving compressible fluid and vibrations in the cylindrical shell confining the fluid

The equations of motion for the cylindrical shell, according to the Donnell bending theory of shells [22,23], are

$$R^2\frac{\partial^2 U}{\partial z^2} + \frac{(1-v)}{2}\frac{\partial^2 U}{\partial \theta^2} + R\frac{(1+v)}{2}\frac{\partial^2 V}{\partial z\partial\theta} - vR\frac{\partial W}{\partial z} = \tau\frac{\partial^2 U}{\partial t^2}, \tag{27}$$

$$R\frac{(1+v)}{2}\frac{\partial^2 U}{\partial z \partial \theta} + R^2\frac{(1-v)}{2}\frac{\partial^2 V}{\partial z^2} + \frac{\partial^2 V}{\partial \theta^2} - \frac{\partial W}{\partial \theta} = \tau \frac{\partial^2 V}{\partial t^2}, \tag{28}$$

$$(P_f - P_{ext})\frac{\tau}{h\rho_s} + vR\frac{\partial U}{\partial z} + \frac{\partial V}{\partial \theta} - W - \kappa\left[R^4\frac{\partial^4 W}{\partial z^4} + 2R^2\frac{\partial^4 W}{\partial z^2 \partial \theta^2} + \frac{\partial^4 W}{\partial \theta^4}\right] = \tau\frac{\partial^2 W}{\partial t^2}, \tag{29}$$

where

$$\tau = \rho_s \frac{(1-v^2)}{E} R_0^2, \tag{30}$$

$$\kappa = \frac{1}{12}\left(\frac{h}{R_0}\right)^2, \tag{31}$$

and z, r, θ are the cylindrical shell co-ordinates, h is the shell thickness, P_f is the total fluid pressure exerted on the shell inner surface, and P_{ext} is the external pressure acting on the outer surface of the shell. Moreover, ρ_s, v, and E are the mass density of the shell, the Poisson ratio of the shell material, and Young's modulus of the shell material, respectively. The displacement of the shell $\mathbf{d}$ at $\mathbf{r} = (R\cos\theta, R\sin\theta, z)$ is

$$\mathbf{d} = U\mathbf{e}_z + V\mathbf{e}_\theta + W\mathbf{e}_r, \tag{32}$$

where $\mathbf{e}_z$, $\mathbf{e}_\theta$, and $\mathbf{e}_r$ are the position-dependent unit vectors in the cylindrical co-ordinate system. The distance of the displaced shell surface from the axis is

$$r(\theta, z; t) = R + W(\theta, z; t). \tag{33}$$

Assume that all functions $\chi = (U, V, W)$ can be written in the form

$$\chi = \phi(r)\psi(\theta)\exp\left(\mathrm{i}\left[\beta_s z - \omega_s t\right]\right), \tag{34}$$

and note that since

$$\psi(\theta) = \psi(\theta + 2\pi n), \tag{35}$$

for any integer choice of n, it is possible to write, similar to the fluid case:

$$\psi(\theta) = \psi_n(\theta) = \exp(\mathrm{i}n\theta). \tag{36}$$

Introduce for each n the notation:

$$\chi_n = \phi_n(r)\psi_n(\theta)\exp\left(\mathrm{i}\left[\beta_{sn} z - \omega_s t\right]\right), \tag{37}$$

where $\chi_n = (U_n, V_n, W_n)$.

Inserting Eqs. (36) and (37) into Eqs. (27)–(29) yields

$$-\left(\beta_{sn}^2 R^2 + n^2\frac{(1-v)}{2}\right)U_n - n\beta_{sn}R\frac{(1+v)}{2}V_n - \mathrm{i}v\beta_{sn}RW_n = -\tau\omega_s^2 U_n, \tag{38}$$

$$-n\beta_{sn}R\frac{(1+v)}{2}U_n - \beta_{sn}^2 R^2\frac{(1-v)}{2}V_n - n^2 V_n - \mathrm{i}nW_n = -\tau\omega_s^2 V_n, \tag{39}$$

$$(P_f - P_{ext})\frac{\tau}{h\rho_s} + \mathrm{i}v\beta_{sn}RU_n + \mathrm{i}nV_n - W_n\left[1 + \kappa\left(\beta_{sn}^2R^2 + n^2\right)^2\right] = -\tau\omega_s^2 W_n. \tag{40}$$

The pressure difference $P_f - P_{ext}$ can be written as $P_f^{ref} + p' - P_{ext}$, where P_f^{ref} is the reference pressure in the fluid in the absence of acoustic excitation, and p' is the acoustic pressure given by Eqs. (24)–(26) evaluated at $r' = 1$ ($r = R$). This is a consequence of the continuity of pressure at the fluid–shell interface. Furthermore, imposing continuity of normal velocity at the shell–fluid interface gives

$$\frac{\partial W_n}{\partial t} = v_r'(r' = 1), \tag{41}$$

or

$$\begin{aligned} -\mathrm{i}\omega_s W_n &= \frac{\mathrm{i}}{\rho_0} \frac{1}{R(\beta v_0 - \omega)} \frac{\partial p'}{\partial r'}\bigg|_{r'=1} \\ &= -\frac{\mathrm{i}}{R\rho_0\omega} \sum_{\lambda=0}^{\infty} (\lambda + m) a_\lambda \exp(\mathrm{i}m\theta) \exp(\mathrm{i}[\beta z - \omega t]), \end{aligned} \tag{42}$$

where use has been made of

$$v_0(r' = 1) = 0, \tag{43}$$

Eqs. (17), (37), and the expression

$$p'(r', \theta, z; t) = \sum_{\lambda=0}^{\infty} a_\lambda r'^{\lambda+m} \exp(\mathrm{i}m\theta) \exp(\mathrm{i}[\beta z - \omega t]). \tag{44}$$

The boundary condition given by Eq. (43) follows from the fact that the fluid velocity must be zero at the shell–fluid interface for any *real* (viscous) fluid, assuming of course that the cylindrical shell is not moving in the absence of acoustic excitation.

Obviously, the equality of the left- and right sides of Eq. (42) at all times t, angles θ, and positions z leads to the conclusion that

$$m = n, \tag{45}$$

$$\omega = \omega_s, \tag{46}$$

$$\beta = \beta_{sn}, \tag{47}$$

employing Eqs. (36) and (37).

In addition, an expression for W_n follows from Eq. (42):

$$W_n = \frac{1}{R\rho_0\omega^2} \sum_{\lambda=0}^{\infty} (\lambda + n) a_\lambda \exp(\mathrm{i}n\theta) \exp\left(\mathrm{i}\left[\beta_{sn} z - \omega_s t\right]\right). \tag{48}$$

In the following, assume that in the absence of acoustic excitation, the internal pressure equals the external pressure, i.e,

$$P_{ref} = P_{ext}, \tag{49}$$

equivalent to

$$\left(P_f - P_{ext}\right) = p' \quad \text{at } r' = 1. \tag{50}$$

The first term in Eq. (40) can now be written in terms of the displacement W_n:

$$\left(P_f - P_{ext}\right)\frac{\tau}{h\rho_s} = \frac{\tau\rho_0\omega^2 R}{h\rho_s}\frac{\sum_{\lambda=0}^{\infty} a_\lambda(\beta)}{\sum_{\lambda=0}^{\infty}(\lambda+n)a_\lambda(\beta)}W_n. \tag{51}$$

It is now possible to rewrite Eqs. (38)–(40) in matrix form as

$$\begin{pmatrix} \left(\tau\omega^2 - \beta^2R^2 - n^2\frac{(1-\nu)}{2}\right) & -n\beta R\frac{(1+\nu)}{2} & -\mathrm{i}\nu\beta R \\ -n\beta R\frac{(1+\nu)}{2} & \left(\tau\omega^2 - \beta^2R^2\frac{(1-\nu)}{2} - n^2\right) & -\mathrm{i}n \\ \mathrm{i}\nu\beta R & \mathrm{i}n & \tau\omega^2 + \frac{\tau\rho_0\omega^2R}{h\rho_s}\frac{\sum_{\lambda=0}^{\infty} a_\lambda(\beta)}{\sum_{\lambda=0}^{\infty}(\lambda+n)a_\lambda(\beta)} - \left[1+\kappa\left(\beta^2R^2+n^2\right)^2\right] \end{pmatrix}$$
$$\times \begin{pmatrix} U_n \\ V_n \\ W_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{52}$$

The implicit dependence of the coefficients a_λ on β has been highlighted in the matrix equation above [Eq. (52)]. The dispersion relations $\beta_n(\omega) \equiv \beta(\omega)$ can now be found numerically by determining, for each frequency ω, the β values for which the determinant equation

$$\det(\mathbf{A}[\beta]) = 0, \tag{53}$$

is fulfilled, where $\mathbf{A}[\beta]$ is the (3×3) coefficient matrix of Eq. (52).

2.4. Implications of a fluid flow on phase speeds/group speeds of cylindrical shell vibrational waves

Following the determination of dispersion relations, it is a simple task to calculate the phase speed c_{pn}, group speed c_{gn}, and changes in phase speed Δc_{pn} induced by the presence of the background flow v_0. The phase speed is given by

$$c_{pn} = \frac{\omega}{\beta_n}, \tag{54}$$

whereas group speed c_{gn} follows from

$$c_{gn} = \frac{\partial\omega}{\partial\beta_n}. \tag{55}$$

The change in phase speed, accomodated by the presence of a background flow, is now found from

$$\Delta c_{pn} = \frac{\omega}{\beta_n} - \frac{\omega}{\beta_{zn}}, \tag{56}$$

where β_{zn} are the solutions for β_n at zero-flow conditions ($v_0 = 0$). Note that the acoustic pressure p' in the case of a vanishing background flow fulfils the modified Bessel equation [see Eq. (21)], i.e.,

$$p'(r', \theta, z; t) = A_n \mathrm{I}_n\left(\sqrt{R^2\beta_{zn}^2 - \frac{R^2\omega^2}{c^2}}\,r'\right)\exp(\mathrm{i}n\theta)\exp\left(\mathrm{i}[\beta_{zn}z - \omega t]\right), \tag{57}$$

and the matrix element A_{33} becomes

$$A_{33} = \tau\omega^2 + \frac{\tau\rho_0\omega^2 R}{h\rho_s} \frac{1}{\sqrt{\beta_{zn}^2 R^2 - R^2\frac{\omega^2}{c^2}}} \frac{\mathrm{I}_n\left(\sqrt{\beta_{zn}^2 R^2 - R^2\frac{\omega^2}{c^2}}\right)}{\mathrm{I}_n'\left(\sqrt{\beta_{zn}^2 R^2 - R^2\frac{\omega^2}{c^2}}\right)} - \left[1 + \kappa\left(\beta_{zn}^2 R^2 + n^2\right)^2\right]. \quad (58)$$

Consider the case where two transducers $T1$ and $T2$ are mounted on the cylindrical shell and separated by an axial distance L. Firstly, a signal is transmitted from $T1$ and received at transducer $T2$. Secondly, a similar signal is transmitted from transducer $T2$ and received at transducer $T1$. The transit-time difference Δt_n between the two signal exchanges, accomodated by the presence of a background flow, becomes

$$\Delta t_n = \frac{L}{c_{pn}^-} - \frac{L}{c_{pn}^+} = 2\left(\frac{L}{c_{pn}^-} - \frac{L}{c_{pzn}}\right) = \frac{2L}{\omega}\left(\beta_n^- - \beta_{zn}\right), \quad (59)$$

where

$$c_{pn}^{\pm} = \frac{\omega}{\beta_n^{\pm}},$$

$$c_{pzn} = \frac{\omega}{\beta_{zn}}. \quad (60)$$

The notation c_{pn}^+ (β_n^+) denotes the phase speed (axial wave vector) for wave propagation along the direction of flow, while c_{pn}^- (β_n^-) denotes the phase speed (axial wave vector) for wave propagation against the direction of flow. Similarly, c_{pzn} denotes the phase speed for wave propagation at zero background flow conditions. In deriving the second equality of Eq. (59), use has been made of the fact that the change in phase speed Δc_{pn} accomodated by flow changes sign as the flow direction is reversed, i.e.,

$$c_{pn}^{\pm} = c_{pzn} \pm |\Delta c_{pn}|. \quad (61)$$

The transit-time difference is associated with a phase difference $\Delta\phi_n$:

$$\Delta\phi_n = \frac{\Delta t_n}{\omega}, \quad (62)$$

which can be measured and used for flow metering purposes. This is the basic principle behind the straight-tube cylindrical shell flow meter. In the following section numerical results will be presented and possibilities for flow measurement based on excitation of vibrations of a straight-tube cylindrical shell carrying a moving fluid will be investigated.

3. Numerical results and discussions

In this section numerical results are given for the first two axial angular modes $n = 0$ and 1 using the parameter values: $R = 0.05$ m, $E = 1.95 \times 10^{11}$ N/m^2, $\nu = 0.28$, $\rho_0 = 1.0 \times 10^3$ kg/m^3, $h = 0.005$ m, $f = \omega/2\pi = 500$–1500 Hz, and $c = 1.5 \times 10^3$ m/s. This set of data corresponds to the case where water is confined by a cylindrical shell of steel and a transducer positioned on the shell excites acoustic waves in the frequency range 500–1500 Hz. Similar to the previous section values obtained for the axial wave number of mode n at zero flow are designated β_{zn}, while those obtained at a mean flow of $\bar{v}$ are designated β_n. It is assumed that the flow profile is parabolic corresponding to a laminar flow situation:

$$v_0(r) = 2\bar{v}\left(1 - \left(\frac{r^2}{R^2}\right)\right). \quad (63)$$

In Fig. 1 (top), dispersion curves for the case $n = 0$ are shown for the three branches characterized by a real and positive β_{z0} value in the frequency range 500–1500 Hz and $\beta \leqslant 20\,\mathrm{m}^{-1}$. It should be pointed out that real β solutions come in pairs for a given frequency in the sense that if $\beta(\omega)$ is a solution at a frequency ω then $-\beta(\omega)$ is also a solution at a frequency ω (only so at zero mean-flow conditions $\bar{v} = 0$). Also shown in Fig. 1 (bottom) are changes in β_0 values with flow $\Delta\beta_0 = \beta_0 - \beta_{z0}$ for the same three branches shown in Fig. 1 (top). The $\Delta\beta_0$ values are computed at a mean flow $\bar{v} = 1\,\mathrm{m/s}$.

In the case $n = 1$, only one real positive branch exists for $\beta \leqslant 20\,\mathrm{m}^{-1}$ using the same parameter values as in the case $n = 0$. This branch (ω, β_1) is shown in Fig. 2 (top) for a mean flow: $\bar{v} = 0$.

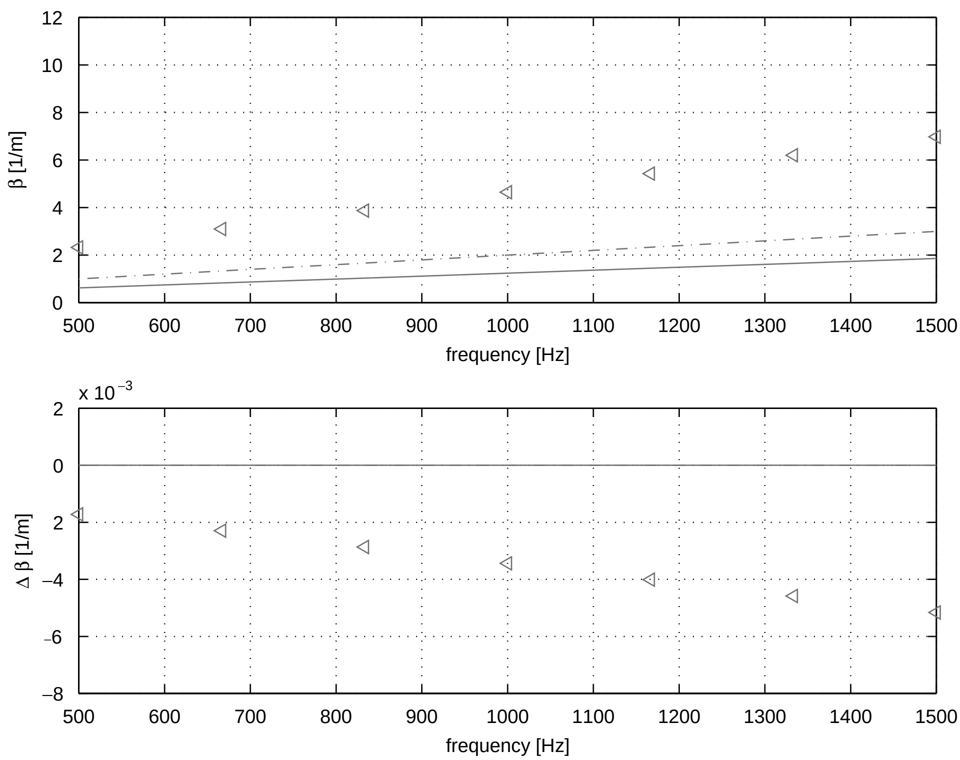

Fig. 1. Axial propagation constants β_n plotted against frequency for the axial angular mode $n = 0$. Four branches exist in the β_n interval: $[0; 20]\,\mathrm{m}^{-1}$. Parameter values are as specified in Section 3. The four curves shown correspond to the four branches.

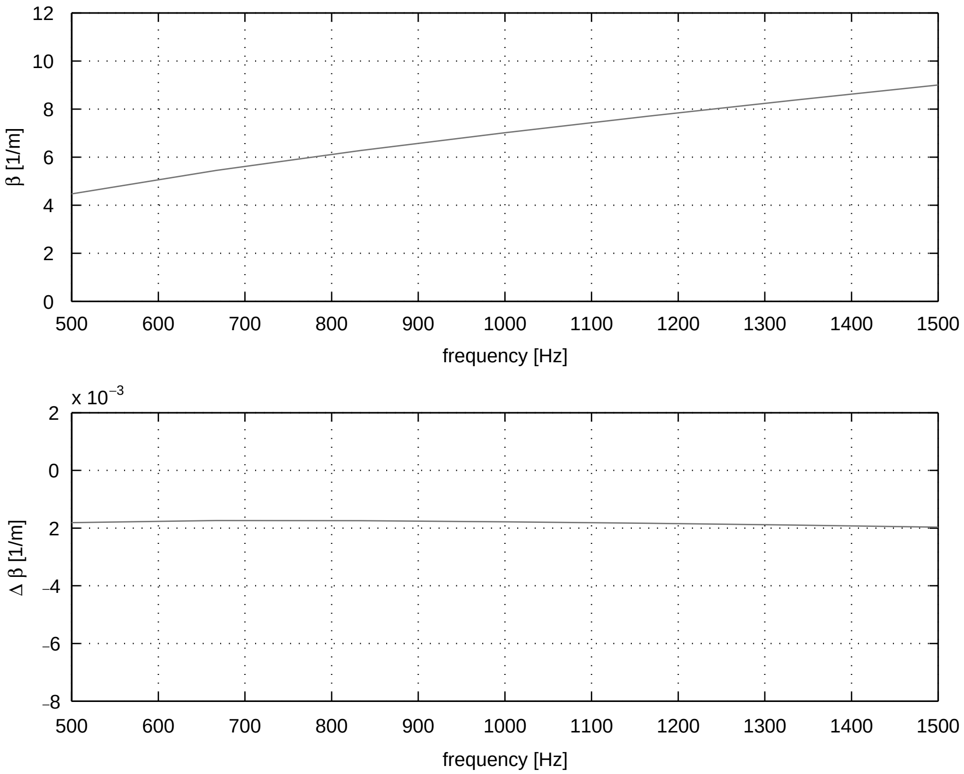

Fig. 2. Axial propagation constants β_n plotted against frequency for the axial angular mode $n = 1$. One branch exists in the β_n interval: $[0; 20]\,\mathrm{m}^{-1}$. Parameter values are as specified in Section 3.

Table 1
Calculated β_{zn}, $\Delta\beta_n$, Δc_{pn}, and Δt_n for the axial angular mode $n = 0$ using the parameter values specified in Section 3 Four branches exist in the β_n range: $[0; 20]\,\mathrm{m}^{-1}$.

$n = 0$ Branches/parameters	β_{zn} (m^{-1})	$\lvert\Delta\beta_n\rvert$ (m^{-1})	$\lvert\Delta c_{pn}\rvert$ (m/s)	$\lvert\Delta t_n\rvert$ (s)
Branch 1	1.239	3.151×10^{-7}	1.290×10^{-3}	5.014×10^{-11}
Branch 2	1.998	0	0	0
Branch 3	4.652	3.437×10^{-3}	0.999	5.471×10^{-7}

The values given correspond to the frequency: $f = 1000\,\mathrm{Hz}$ and $L = 0.5\,\mathrm{m}$.

Changes in β_1 accomodated by the presence of a parabolic flow with a mean flow equal to $\bar{v} = 1\,\mathrm{m/s}$ $[\Delta\beta_1 = \beta_1 - \beta_{z1}]$ are shown in Fig. 2 (bottom).

A similar calculation for the case $n = 2$ reveals that no real, positive β values exist for $\beta \leqslant 20\,\mathrm{m}^{-1}$.

In Tables 1 and 2, β_n, $\Delta\beta_n$, Δc_{pn}, and Δt_n values are calculated corresponding to $n = 0$ and 1, respectively. The values given for Δc_{pn} and Δt_n are computed using Eqs. (60) and (59), respectively. Note, in particular, that one of the three β_0 branches (in the case $n = 0$) couple strongly to flow as reflected by the changes in phase speeds associated with these branches

Table 2
Calculated β_{zn}, $\Delta\beta_n$, Δc_{pn}, and Δt_n for the axial angular mode $n = 1$ using the parameter values specified in Section 3. One branch exists in the β_n range: $[0; 20]\,\mathrm{m}^{-1}$.

$n = 1$ Branches/parameters	β_{zn} (m^{-1})	$\lvert\Delta\beta_n\rvert$ (m^{-1})	$\lvert\Delta c_{pn}\rvert$ (m/s)	$\lvert\Delta t_n\rvert$ (s)
Branch 1	7.021	1.783×10^{-3}	2.272×10^{-1}	2.837×10^{-7}

The values given correspond to the frequency: $f = 1000$ Hz and $L = 0.5$ m.

($\Delta c_{p0} = 0.999\,\mathrm{m/s} \approx \bar{v}$). Another β_0 branch couples only weakly to flow $\Delta c_{p0} \approx 0.0013\,\mathrm{m/s} \ll \bar{v} = 1\,\mathrm{m/s}$. The remaining branch is completely decoupled from the liquid background flow as $\Delta c_{p0} = 0\,\mathrm{m/s}$. This latter branch corresponds to the case of pure circumferential vibrations, i.e., $V \neq 0$ and $U = W = 0$.

The branch corresponding to $n = 1$ (Fig. 2) is coupled to flow as well with $\Delta c_{p1} = 2.272 \times 10^{-1}\,\mathrm{m/s}$ at a frequency of 1 kHz.

Results given in Tables 1 and 2 and Figs. 1 and 2 reveal that a propagating wave exists that couples strongly to the liquid background flow in the peristaltic case $n = 0$ and less strongly in the case $n = 1$. Strong coupling here refers to waves for which $\Delta c_{pn} \approx \bar{v}$. Such waves are, potentially, useful in relation to devices for measuring mean flow as they correspond to relatively large transit-time differences.

4. Conclusions

A general differential equation for acoustic pressure including azimuthal asymmetry effects in a moving fluid confined by cylindrical shell walls is derived. Continuity of pressure and normal velocity applied to the fluid–shell interface allows the influence of a moving fluid on dispersion relations $\beta(\omega)$ to be examined. It is assumed, in the absence of acoustic excitation, that fluid velocity points along the cylindrical shell axis and can be written as an infinite series expansion in the radial co-ordinate. Special interest is given to the determination of changes in phase speeds of shell vibrations and associated transit-time differentials between two points located at different axial positions along the shell due to a fluid flow. Finally, implications for flow-measurement properties are addressed.

References

[1] M. Amabili, Free vibration of partially filled, horizontal cylindrical shells, Journal of Sound and Vibration 191 (1996) 757–780.
[2] M. Amabili, F. Pellicano, M.P. Paidoussis, Nonlinear dynamics and stability of circular cylindrical shells containing flowing fluid. Part III: truncation effect without flow and experiments, Journal of Sound and Vibration 237 (2000) 617–640.
[3] T. Mazuch, J. Horacek, J. Trnka, J. Vesely, Natural modes and frequencies of a thin clamped–free steel cylindrical storage tank partially filled with water: FEM and measurement, Journal of Sound and Vibration 193 (1996) 669–690.
[4] J. Horacek, M. Kruntcheva, Finite element modelling of the damping of cylindrical shells vibrating in contact with an annular fluid region, Journal of Sound and Vibration 203 (1997) 723–727.

[5] M.P. Paidoussis, Dynamics of tubular cantilevers conveying fluid, Journal of Mechanical Engineering Science 12 (1970) 85–103.
[6] M.P. Paidoussis, J.-P. Denise, Flutter of thin cylindrical shells conveying fluid, Journal of Sound and Vibration 20 (1972) 9–26.
[7] Y.L. Zhang, J.M. Reese, D.G. Gorman, Initially tensioned orthotropic cylindrical shells conveying fluid: A vibration analysis, Journal of Fluids and Structures 16 (2002) 53–70.
[8] M. Amabili, F. Pellicano, M.P. Paidoussis, Non-linear dynamics and stability of circular cylindrical shells containing flowing fluid. Part IV: large-amplitude vibrations with flow, Journal of Sound and Vibration 237 (2000) 641–666.
[9] M. Heil, The stability of cylindrical shells conveying viscous flow, Journal of Fluids and Structures 10 (1996) 173–196.
[10] A. Selmane, A.A. Lakis, Vibration analysis of anisotropic open cylindrical shells subjected to a flowing fluid, Journal of Fluids and Structures 11 (1997) 111–134.
[11] J. Hemp, Flowmeters and reciprocity, Quarterly Journal of Mechanical Applications and Mathematics 41 (1988) 503–520.
[12] G. Sultan, Single straight-tube Coriolis mass flowmeter, Flow Measurement and Instrumentation 3 (1992) 241–246.
[13] J. Hemp, The weight vector theory of Coriolis mass flowmeters, Flow Measurement and Instrumentation 5 (1994) 247–253.
[14] R. Cheesewright, C. Clark, The effect of flow pulsations on Coriolis mass flow meters, Journal of Fluids and Structures 12 (1998) 1025–1039.
[15] P. Mungur, H.E. Plumblee, Propagation and attenuation of sound in a soft walled annular duct containing shear flow. NASA SP-207, pp. 305–322.
[16] N.K. Agarwal, M.K. Bull, Acoustic wave propagation in a pipe with fully developed turbulent flow, Journal of Sound and Vibration 132 (1989) 275–298.
[17] G.R. Gogate, M.L. Munjal, Analytical solution of sound propagation in lined or unlined circular ducts with laminar mean flow, Journal of Sound and Vibration 160 (1993) 465–484.
[18] M. Willatzen, Sound propagation in a moving fluid confined by cylindrical walls—a comparison between an exact analysis and the local-plane-wave approximation, Journal of Sound and Vibration 247 (2000) 719–729.
[19] M. Willatzen, Perturbation theory applied to sound propagation in flowing media confined by a cylindrical waveguide, Journal of Acoustical Society of America 109 (2001) 102–107.
[20] J. Nikuradse, Gesetzmässigkeiten der turbulenten Strömung in glatten Rohren, Forschung auf dem Gebiete des Ingenieurwesens Forschungsheft, Vol. B, 356 (3) (1932).
[21] G. Arfken, Mathematical Methods for Physicists, 3rd Edition, Academic Press, Orlando, FL, 1985, pp. 113–115.
[22] L.H. Donnell, A discussion of thin shell theory, Proceedings of the Fifth International Congress of Applied Mechanics, Wiley, New York, 1938, pp. 66–70.
[23] A.W. Leissa, Government Printing Office. Vibration of Shells, NASA SP-288, Washington, DC, 1973.