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Letter to the Editor

Applying effort constraints on adaptive feedforward control using the active set method

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1. Introduction

In some active noise and vibration control applications, the total power of the control signal or the amplitude of the individual control signal is limited either because the system being controlled does not allow large injected power or the actuators used for control have limited driving capability. A method based on a penalty function has been proposed, and thus transforms the constrained optimization problem into an unconstrained optimization problem by using the Lagrange multipliers, leading to a very simple form, which is effectively the same as the complex multiple error LMS algorithm with leakage [1]. However, this method does not guarantee that the control output remains within any specified constraint and the selection of the value of the leakage coefficient can only be done using a trial and error procedure. An alternative way to solve this problem is to use the ideas of the active set method (a gradient projection method focused on the solution of the Kuhn–Tucker equations), which is widely used in the field of the constrained optimization to solve the non-linear programming problem [2–6]. The work described here is to show the feasibility of the method for applying effort constraints on adaptive feedforward control.

2. Description of the proposed algorithm

To simplify comparisons with the work of Elliot and Baek [1], the algorithm is developed in the frequency domain. The vector of complex signals measured at the error sensors is

$$\mathbf{e} = \mathbf{d} + \mathbf{G}\mathbf{u}, \tag{1}$$

where $\mathbf{d}$ is the vector of the disturbance signal, $\mathbf{u}$ is the vector of the control signal and $\mathbf{G}$ is the complex matrix representing the response of the system under control at the frequency of interest.

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For the total control power constraint, the problem is of the form

$$\text{minimize } J = \mathbf{e}^{\mathrm{H}}\mathbf{e} \quad \text{subject to } \mathbf{u}^{\mathrm{H}}\mathbf{u} \leqslant W_{max}, \tag{2}$$

where H denotes the Hermitian transpose, and W_{max} is the allowed maximum control power. The problem is a non-linear programming problem where both the objective function and the constraint include quadratic terms. For the individual control signal amplitude constraint, the problem is simplified to a quadratic programming problem in the form

$$\text{minimize } J = \mathbf{e}^{\mathrm{H}}\mathbf{e} \quad \text{subject to } |u_i| \leqslant A_{max}, \tag{3}$$

where u_i is the ith element of $\mathbf{u}$ and A_{max} is the allowed maximum individual control signal amplitude.

The proposed iteration algorithm for solving problem (2) is given by

$$\mathbf{u}(k+1) = \mathbf{u}(k) - \mu \mathbf{G}^{\mathrm{H}}\mathbf{e}(k), \tag{4}$$

$$D = \mathbf{u}(k+1)^{\mathrm{H}}\mathbf{u}(k+1), \tag{5}$$

If $D > W_{max}$,

$$\mathbf{u}(k+1) = \mathbf{u}(k+1)(W_{max}/D)^{1/2}, \tag{6}$$

where μ is the convergence coefficient, which should be small enough to maintain stability. The proposed iteration algorithm for solving problem (3) is given by

$$\mathbf{u}(k+1) = \mathbf{u}(k) - \mu \mathbf{G}^{\mathrm{H}}\mathbf{e}(k), \tag{7}$$

$$\text{for all } i, \quad D_i = |u_i(k+1)|. \tag{8}$$

If $D_i > A_{max}$,

$$u_i(k+1) = u_i(k+1)(A_{max}/D_i), \tag{9}$$

where $u_i(k+1)$ is the ith element of the $\mathbf{u}(k+1)$.

The problem with the original steepest decent algorithm is that it sometimes fails to preserve feasibility (satisfying constraints). To remedy this problem, the active set method is used, which is an iterative procedure that involves two phases: the first phase calculates a feasible point (a weight vector satisfies the constraint), the second phase generates an iterative sequence of feasible points that converge to the solution. The search direction for generating the sequence of feasibly points is calculated by projecting the estimated gradient ($2\mathbf{G}^{\mathrm{H}}\mathbf{e}(k)$ here) into the constraint set when a constraint is encountered, which is done by re-scaling the control vector in the algorithm as seen in Eqs. (6) and (9). As the objective function is quadratic and the constraints are strictly convex in both cases, the algorithm should converge to the minimum under the constraints [3, p. 165].

3. Simulation

A single input, single output control system is considered first with a disturbance $\mathbf{d}$ of $-5-3\mathrm{i}$, plant response $\mathbf{G}$ of $1+0.3\mathrm{i}$, and the start point $\mathbf{u}(0)$ of $1+3\mathrm{i}$. The constraint is $A_{max}=5$. The optimal solution without constraints is $\mathbf{u}_{opt} = 5.41 + 1.38\mathrm{i}$ ($|\mathbf{u}_{opt}| = 5.59$), and the objective function J is reduced from 24.1 ($\mathbf{u}(0) = 1+3\mathrm{i}$) to 0.0 ($\mathbf{u}_{opt}$).

For all the simulations, the convergence coefficient μ is set to 0.1. For the leakage algorithm using the transformation method [1], the leakage coefficient (the products of the Lagrange multiplier and the convergence coefficient in Ref. [1]) is 0 if $|u_i| \leqslant 0.9A_{max}$ and increases in proportion to the value of $|u_i|$ above $0.9A_{max}$. The proportional coefficients (leakage) are set to 0.005, 0.0025 and 0.001 corresponding to the resulting objective functions of 1.04 ($\mathbf{u}_{opt} = 4.47 + 1.14\text{i}$, $|\mathbf{u}_{opt}| = 4.61$), 0.36 ($\mathbf{u}_{opt} = 4.86 + 1.23\text{i}$, $|\mathbf{u}_{opt}| = 5.01$) and 0.07 ($\mathbf{u}_{opt} = 5.16 + 1.31\text{i}$, $|\mathbf{u}_{opt}| = 5.32$). For the proposed algorithm, the resulting objective function is 0.37 ($\mathbf{u}_{opt} = 4.85 + 1.23\text{i}$, $|\mathbf{u}_{opt}| = 5.0$). Fig. 1 shows the convergence path of the algorithms plotted against contours of the objective function J, where the shaded area is the feasible set.

As can be seen from Fig. 1, the proposed algorithm can successfully achieve the optimum under the constraints, while for the leakage algorithm, the convergence depends on the value of the

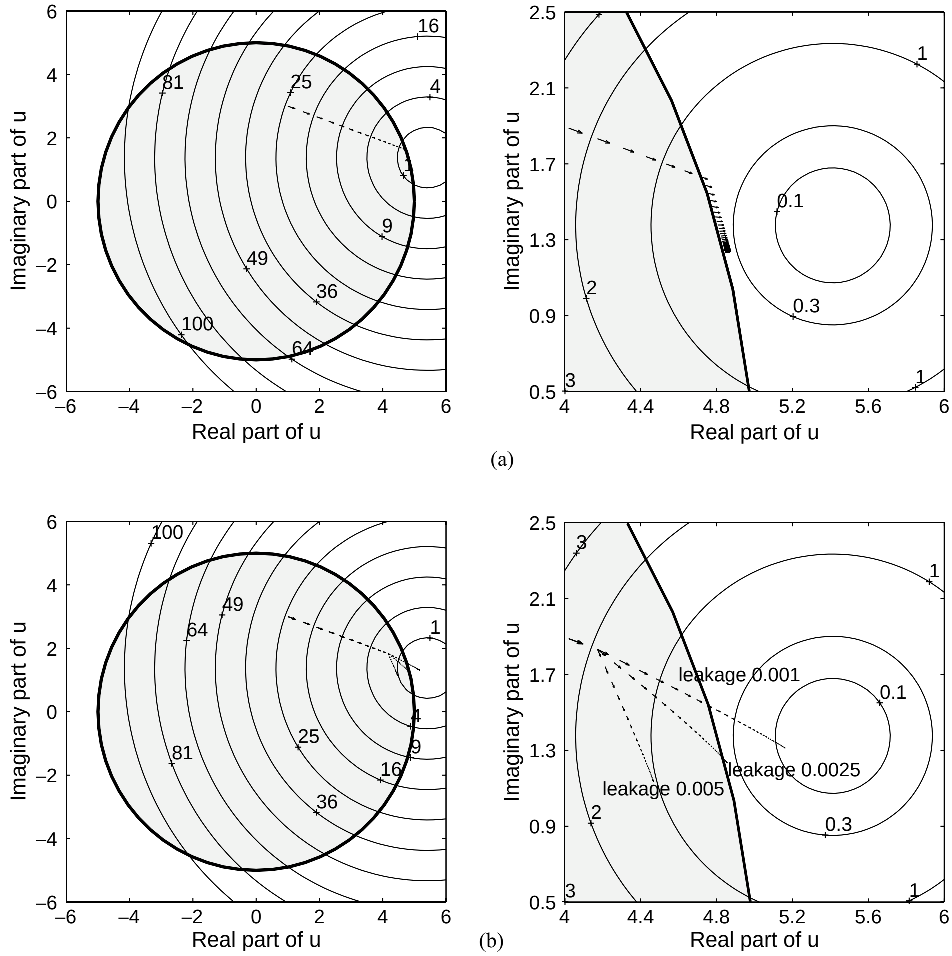

Fig. 1. Convergence paths shown with contours of the objective function for a feedforward adaptive control algorithm operating on a SISO system at a single frequency, in which the control effort is limited to the shaded area: (a) the proposed algorithm and (b) the leakage algorithm.

leakage. If the leakage is too small, the constraint is quite likely to be violated resulting in overloading of the control output. If the convergence coefficient is too large, the convergence often stops early before reaching the optimum, resulting in a poor control performance. The worst problem is that the optimum selection of leakage coefficients depends also on the amplitude of the disturbance, which allows the algorithm to be used in practice only by trial and error. An active noise control system that used two control loudspeakers and four error microphones to reduce freefield sound was also simulated, showing similar results.

4. Experimental work

The feasibility of the proposed algorithm was further verified in the experiments. Fig. 2 shows the block diagram of the experimental set-up where two loudspeakers were placed together; one was used to simulate the primary noise source, and the other was used as the control source. The signal from the signal generator was fed to the power amplifier of the primary loudspeaker and was also fed to the ANC controller as the reference signal. The ANC controller processed the error signal from the error microphone located between the two loudspeakers, and then output the control signal into the power amplifier of the control loudspeaker.

The proposed algorithm was realised on a SHARC EZ-KIT Lite board, which has an Analog Devices ADSP-21061 floating point DSP running at 40 MHz and an Analog Devices AD1847 16-bit Stereo SoundPort Codec providing 2-channel 16 bit A/D converters and 2-channel 16 bit D/A converters. The Codec has on-chip anti-aliasing and reconstruction filters for analog signals and programmable Gain control for the microphone input, so no low-pass filter or pre-amplifier are shown in the Fig. 2.

In the experiments, the maximum allowed output to the control source is 1.0 V_{pp}. Thus the control output is constrained to 0.98 V_{pp}. If the output is greater than 0.98 V_{pp}, the output is clipped to the maximum value, similar to what happens when the actuator is saturated. The primary disturbance is a 400 Hz tone with an amplitude of about 4.5 dB (refer 1.0 $V_{r.m.s.}$). After taking into account the cancellation path transfer function between the control output and the error input, the required control output for completely cancelling the primary disturbance at the error microphone is 1.14 V_{pp}, which is greater than the output constraint. Fig. 3 shows the spectrum of the residual error signal with the proposed algorithm and the leakage algorithm. For

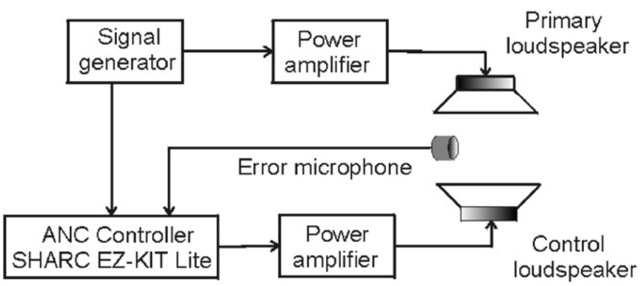

Fig. 2. Block diagram of the experimental set-up.

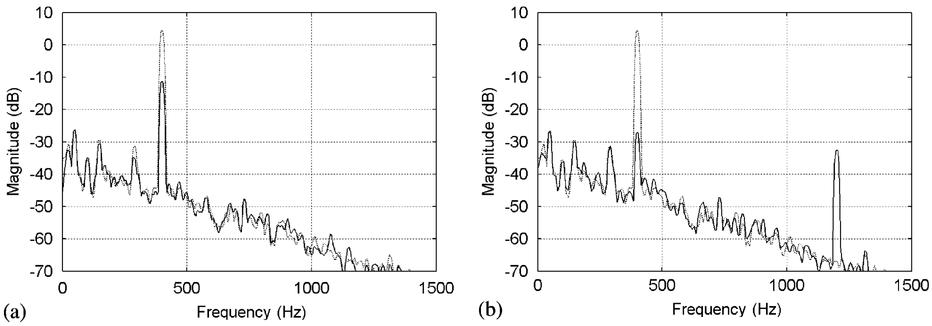

Fig. 3. Error signal spectrum with and without active noise control (dash line): (a) the proposed algorithm and (b) the leakage algorithm with the leakage coefficient being 0.001.

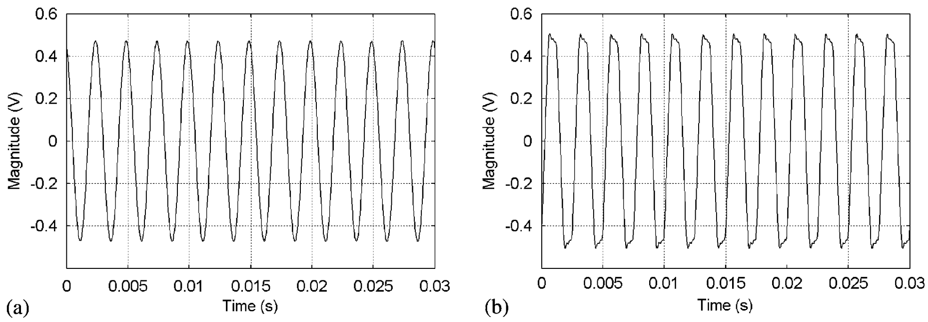

Fig. 4. Control output signals when the active noise control is on (a) the proposed algorithm and (b) the leakage algorithm with the leakage coefficient being 0.001.

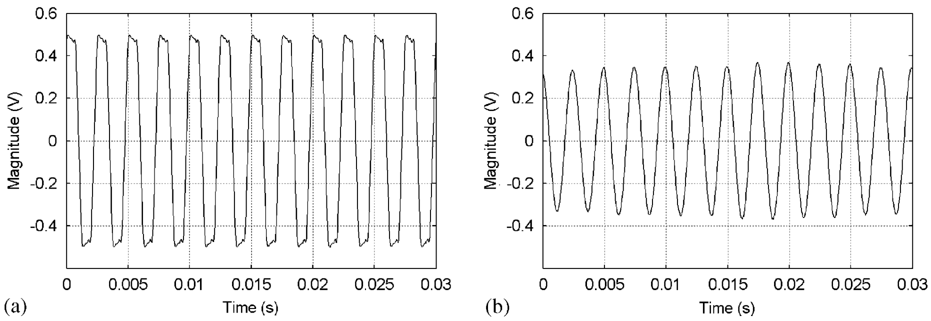

Fig. 5. Control output signals when the active noise control is on for the leakage algorithm: (a) the leakage coefficient is 0.0001 and (b) the leakage coefficient is 0.05.

the leakage algorithm, the leakage coefficient was 0.001, smaller than the optimum number (0.015) that just could prevent the control output from violating the constraint. Fig. 4 shows the time-domain control output signal corresponding to Fig. 3. Fig. 5 shows the time-domain control output signal for the leakage algorithm with different leakage coefficients, and Table 1 summarised the results.

Table 1
Residual error and the control output amplitude for different algorithms

	Just primary	The proposed algorithm	The leakage algorithm with a coefficient of α			
			$\alpha = 0$	$\alpha = 0.001$	$\alpha = 0.015$	$\alpha = 0.05$
Error signal at 400 Hz (dB)	4.5	−11	−39	−27	−11	−3.8
Error signal at 1200 Hz (dB)	−69	−69	−30	−33	−64	−69
Control output (V_{pp})	0	0.98	1.0	1.0	1.0	0.77

It can be seen from Figs. 3–5 and Table 1 that the proposed algorithm can guarantee that the constraint is not violated. When the constraint is met, the control output remains as a sine wave with very little distortion. However, for the leakage algorithm, the value of the leakage coefficient is hard to choose so there is no guarantee that the constraint will not be violated. If the selected leakage coefficient is too large (Fig. 5(b)), the control output may not be sufficiently large to provide good cancellation. If the selected leakage coefficient is too small (Fig. 5(a)), the control output may violate the constraint and saturate the actuator. The resulting non-linearity may bring additional high-frequency noise into the system or cause the system unstable [6]. By listening to the residual noise at the case of Fig. 3 with human ear, the perception is that proposed algorithm is much preferable to the leakage algorithm.

5. Conclusion

The active set method has been used to apply effort constraints in a feedforward active control system. The proposed adaptive algorithm minimizes the sum of the squared error signal subject to the bounded control effort. A comparison of the proposed algorithm with the traditional leakage algorithm shows that the proposed algorithm can successfully achieve the optimum cost function value under the constraints while the performance of the leakage algorithm depends significantly on the selection of the leakage coefficients.

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