Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Sound and Vibration 260 (2003) 949–960
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Letter to the Editor

On regular and irregular vibrations of a non-ideal system with two degrees of freedom. 1:1 resonance

M. Tsuchida[a], K. de Lolo Guilherme[a], J.M. Balthazar[b,c,*], G.N. Silva[a], B.I. Cheshankov[d]

[a] *Departamento de Ciências da Computação e Estatística, Instituto de Biociências, Letras e Ciências Exatas, Universidade Estadual Paulista, UNESP, Rua Cristóvão Colombo 2265, CEP 15054-000 São José do Rio Preto, SP, Brazil*
[b] *Departamento de Estatística, Matemática Aplicada e Computação, Instituto de Geociências e Ciênciais Exatas, Universidade Estadual Paulista, UNESP, CP 178, CEP 13500-230 Rio Claro, SP, Brazil*
[c] *Departamento de Projeto Mecânico, Faculdade de Engenharia Mecânica, UNICAMP, CP 6122, CEP 13083-970, Campinas, SP, Brazil*
[d] *Technical University of Sofia, Institute for Applied Mathematics and Informatics, Sofia 1156, Bulgaria*

Received 5 June 2002; accepted 7 June 2002

1. Introduction

This work concerns special kinds of problems called non-ideal problems, that is, when the excitation is influenced by the response of the system. The ideal problems are the traditional ones. Naturally non-ideal vibrating problems have one more degree of freedom than the ideal ones, due to the action of the non-ideal source (generally DC motor with limited power supply) and the interacting terms. Note that jump phenomenon and the increase in power required by a source operating near resonance are manifestations of a non-ideal source called Sommerfeld effect. The first book devoted to this subject was written by Kononenko [1]; Nayfeh and Mook [2] present a comprehensive review of different theories until 1978, and recently Balthazar and collaborators [3,4] presented a complete review of different theories concerning this subject.

Here we will analyze a non-ideal problem with two degrees of freedom, operating near a resonance by using numerical simulations. The main purpose of this paper is to study the possibilities of the existence of regular and irregular motions in a non-ideal vibrating problem shown in the Fig. 1. This vibrating problem consists of a block of mass m_1, a linear elastic spring with coefficient of elasticity k_1 and a linear damper with viscous damping coefficient c_1. On the body of mass m_1, a non-ideal motor is placed, with a driving rotor of moment of inertia J and an eccentric mass m_0 situated at a distance r from the axis of rotation. By means of a linear spring

*Corresponding author. State University of Sao Paulo at Rio Claro, Dept. of Statistics, Applied Mathematics and Computation, P.O. Box 178, CEP 13500-230, Rio Claro, SP, Brazil.

E-mail addresses: tsuchida@dcce.ibilce.unesp.br (M. Tsuchida), jmbaltha@rc.unesp.br (J.M. Balthazar), gsilva@dcce.ibilce.unesp.br (G.N. Silva), chesh@vmei.acad.bg (B.I. Cheshankov).

doi:10.1016/S0022-460X(02)01128-8

with coefficient of elasticity k_2 and a damper with coefficient of damping c_2, a body of mass m_2 has been attached to mass m_1.

The Lagrange equations of motion may be written as in Ref. [5]:

$$m_1\ddot{x}_1 = -k_1x_1 - c_1\dot{x}_1 + F(x_1, x_2, \dot{x}_1, \dot{x}_2) + m_0r\omega^2 \cos\varphi + m_0r\dot{\omega}\sin\varphi, \tag{1a}$$

$$m_2\ddot{x}_2 = -F(x_1, x_2, \dot{x}_1, \dot{x}_2), \tag{1b}$$

$$J\dot{\omega} = L - H(\omega) + m_0\ddot{x}_1 \sin\varphi, \tag{1c}$$

$$\dot{L} = -aL - b\omega + k_U U(\omega), \tag{1d}$$

where

$$F(x_1, x_2, \dot{x}_1, \dot{x}_2) = k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1),$$

L is the torque generated by the DC motor of limited power supply and $H(\omega)$ is the resisting torque which will be ignored from now on. The parameters a and b are constants depending on the type and power of the DC motor, and $U(\omega)$ is the voltage of the motor. Note that Eqs. (1a)–(1d) include only non-linear members resulting from the interaction between the vibrating system and the DC motor (non-ideal system). Finally, we rewrite Eqs. (1a)–(1d) in an adimensional form [5] defining the new variables

$$\chi_1 = \frac{m_1}{m_0r}x_1, \quad \chi_2 = \frac{m_1}{m_0r}x_2, \quad \bar{\omega} = \sqrt{\frac{m_1}{k_1}}\omega, \quad \lambda = \frac{m_1}{Jk_1}L, \quad \tau = \sqrt{\frac{k_1}{m_1}}t$$

and the constants

$$\eta_1 = \frac{c_1}{\sqrt{k_1m_1}}, \quad \eta_2 = \frac{c_2}{\sqrt{k_1m_1}}, \quad \mu = \frac{k_2}{k_1}, \quad \theta^2 = \mu\frac{m_1}{m_2},$$

$$\rho = \frac{m_0^2r}{Jm_1}, \quad \alpha = a\sqrt{\frac{m_1}{k_1}} \quad \beta = b\frac{m_1}{Jk_1}$$

to obtain

$$\chi_1'' = -\chi_1 - \eta_1\chi_1' + \mu(\chi_2 - \chi_1) + \eta_2(\chi_2' - \chi_1') + \bar{\omega}^2 \cos\bar{\varphi} + \bar{\omega}' \sin\bar{\varphi},$$

$$\chi_2'' = -\theta^2(\chi_2 - \chi_1) - \frac{\theta^2}{\mu}\eta_2(\chi_2' - \chi_1'),$$

$$\bar{\omega}' = \lambda + \rho\chi_1'' \sin\bar{\varphi}, \quad \lambda' = -\alpha\lambda - \beta\bar{\omega} + u.$$

We remark that for minimizing the time of passage through resonance, a synthesis of control based on Tikhonov's regularization was applied in this problem [5].

Note also that the natural frequencies $\bar{\omega}_1$ and $\bar{\omega}_2$ of the non-ideal vibrating system defined by Fig. 1 and Eqs. (1a)–(1d) are given by associated linear system. For this purpose we linearize the governing equations of motion around $\bar{\varphi} = 0$ to obtain the linear system

$$y' = Ay,$$

where

$$y = (y_1, y_1, \ldots, y_6)^{\mathrm{T}} = (\chi_1, \chi_1', \chi_2, \chi_2', \bar{\omega}, \lambda)^{\mathrm{T}}$$

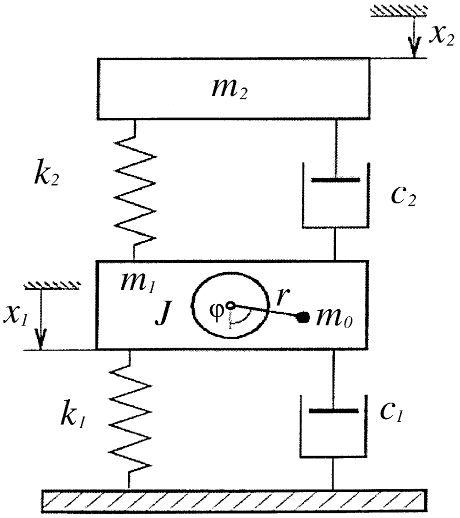

Fig. 1. Vibrational dynamical system consisting of two blocks coupled with springs and dampers. The DC motor with limited power supply and eccentric mass play the part of non-ideal perturbating source.

Table 1
Possible resonances

	Resonances			
	1:1	1:2	1:3	2:3
Parameters				
η_1	0.1581	0.2828	0.2828	0.2828
η_2	0.5885	0.1414	0.0707	0.2432
μ	0.5	0.2	0.1	0.3
θ^2	1.6667	0.3883	0.1348	1.0
α	3.1623	1.4142	1.4142	1.4142
β $(\times 10^4)$	1.0	0.2	0.2	0.2
Frequencies				
$\bar{\omega}_1$	0.8435	0.5465	0.3460	0.7721
$\bar{\omega}_2$	0.8436	1.0950	1.0377	1.1588

and

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -(1+\mu) & -(\eta_1+\eta_2) & \mu & \eta_2 & \frac{2\mu}{\beta} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \theta^2 & \frac{\theta^2}{\mu}\eta_2 & -\theta^2 & -\frac{\theta^2}{\mu}\eta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\beta & -\alpha \end{pmatrix}.$$

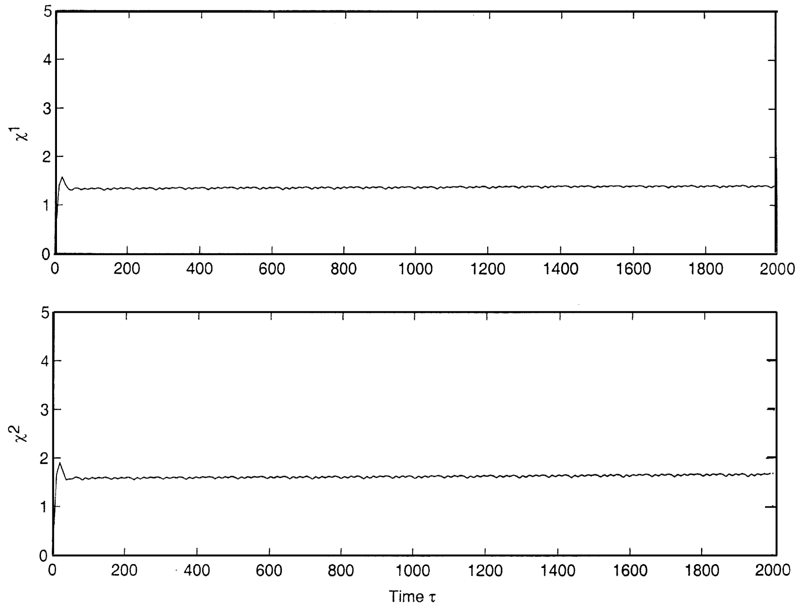

Fig. 2. Profile of maximum amplitude for χ_1 and χ_2 corresponding to $\bar{\omega} = 0.7$ (before passage through resonance). Linear torque model.

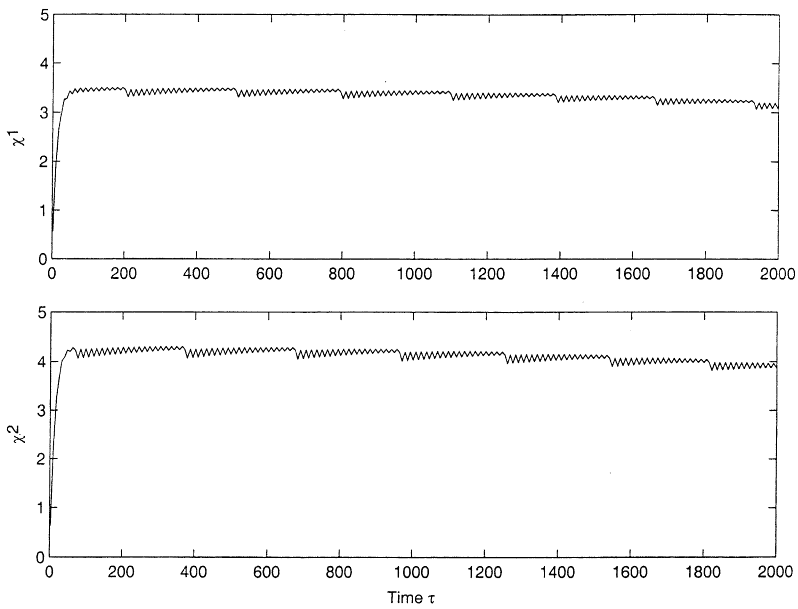

Fig. 3. Profile of maximum amplitude for χ_1 and χ_2 corresponding to $\bar{\omega} = 0.8435$ (passage through resonance). Linear torque model.

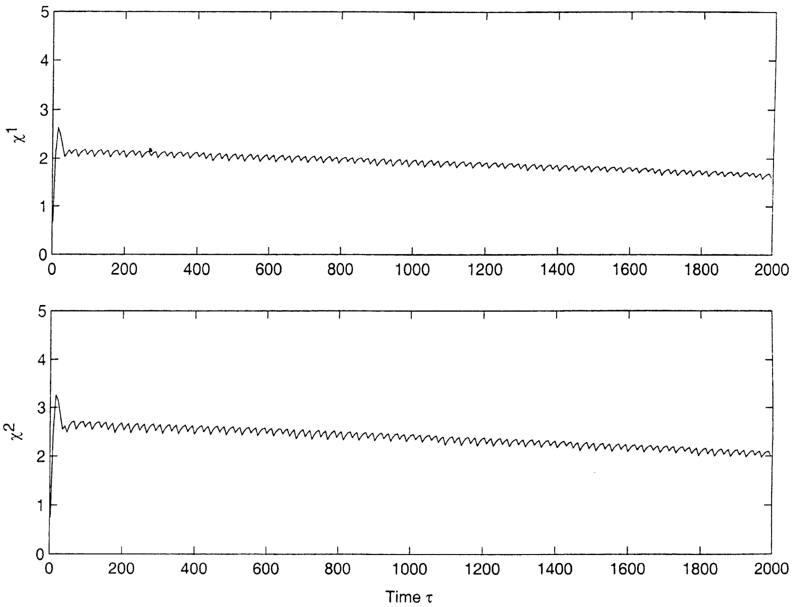

Fig. 4. Profile of maximum amplitude for χ_1 and χ_2 corresponding to $\bar{\omega} = 1.0$ (after passage through resonance). Linear torque model.

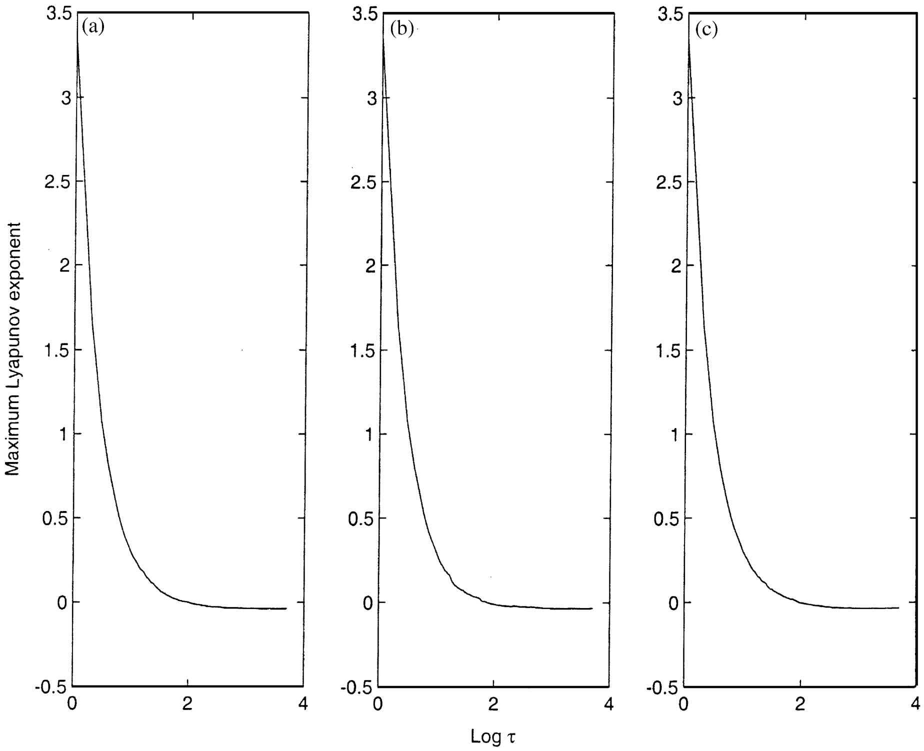

Fig. 5. Graph of maximum Lyapunov exponent against $\log \tau$: (a) $\bar{\omega} = 0.7$, (b) $\bar{\omega} = 0.8435$, and (c) $\bar{\omega} = 1.0$. Linear torque model.

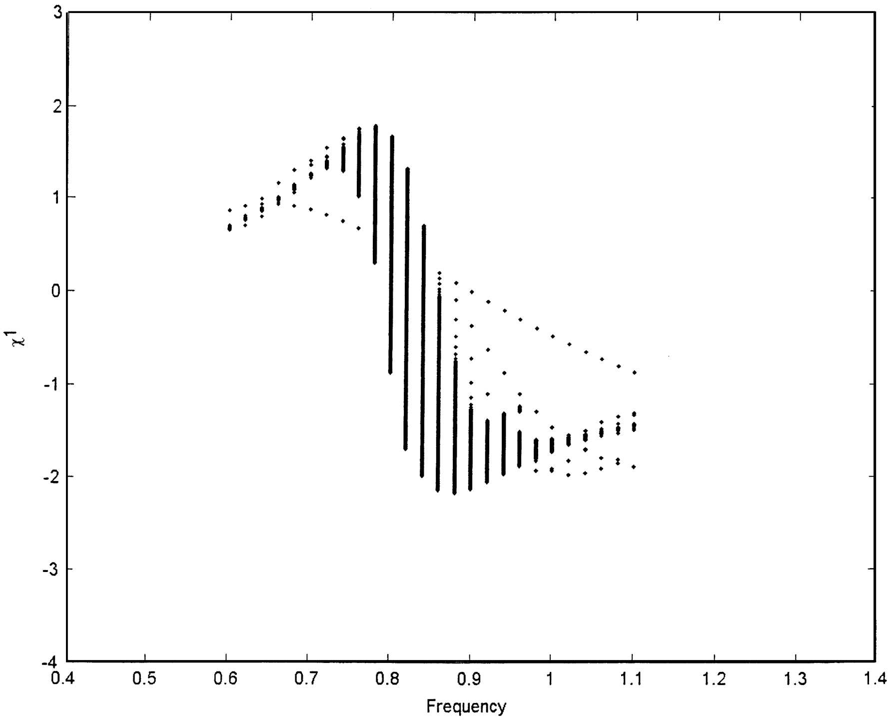

Fig. 6. Surface of section diagram for χ_1 with step size $\Delta\bar{\omega} = 0.02$. The adopted plane is $\bar{\varphi} = 0$ with conditions $\chi_1' > 0$ and $\chi_2' > 0$. Linear torque model.

Choosing adequately the physical parameters of the problem defined by Fig. 1, it is possible to obtain some resonance conditions. In Table 1 we present some possibilities of them, but in this work we analyze only the 1:1 resonance. Another resonance conditions will be reported soon.

We organized this paper as follows. First we discuss some numerical simulations of the vibrating system defined by Fig. 1, using a linear model for the torque L, that is

$$L = a - b\omega, \tag{2}$$

where a is a parameter related to voltage and b is a constant, both depending on the used DC motor. Expression (2) is obtained from Eq. (1d) taking $\dot{L} = 0$. Second, we present the numerical results by using the general expression for the torque, that is, considering $\dot{L} \neq 0$ in Eq. (1d). In this case, we analyze the possibility of existence of regular and irregular motion.

2. On numerical simulations by using linear torque

The goal of this section is to analyze the vibrating problem defined by Fig. 1, taking into account the linear torque defined by Eq. (2). Note that the passage through the resonance is obtained by varying the angular velocity $\dot{\bar{\varphi}}$ of the DC motor. Figs. 2–4 show the profile of the maximum amplitude of χ_1 and χ_2, respectively, before ($\bar{\omega} = 0.7$), during ($\bar{\omega} = 0.8435$) and after

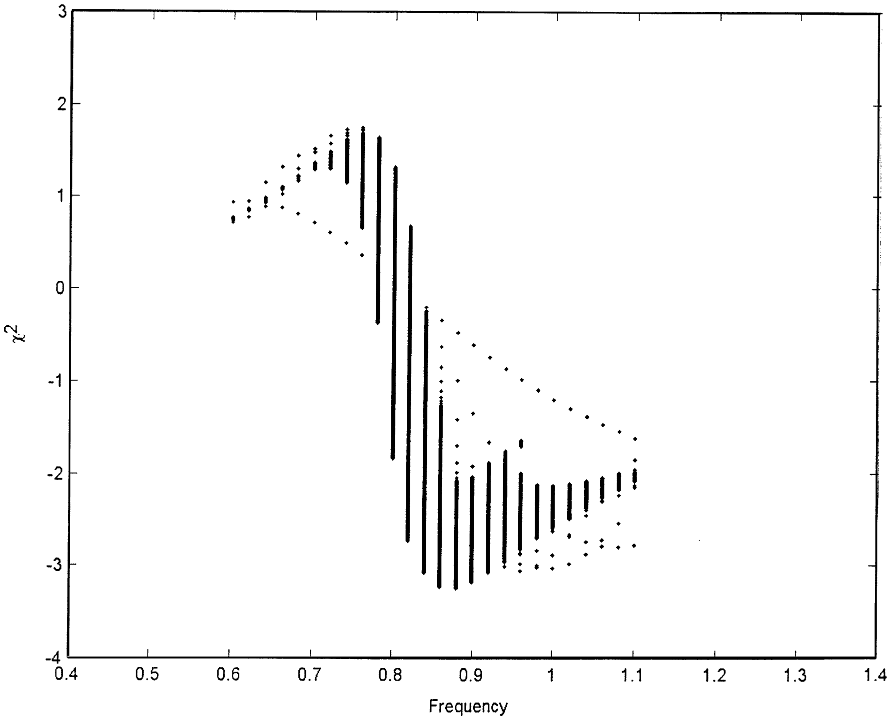

Fig. 7. Surface of section diagram for χ_2 with step size $\Delta\bar{\omega} = 0.02$. The adopted plane is $\bar{\varphi} = 0$ with conditions $\chi_1' > 0$ and $\chi_2' > 0$. Linear torque model.

($\bar{\omega} = 0.1$) the passage through resonance, where the vibration amplitude increases and decreases accordingly but the motion remains regular. The evaluation of the maximum Lyapunov exponent for the same values of $\bar{\omega}$ confirms this affirmation (Fig. 5). The surface of section diagram corresponding to $\bar{\varphi} = 0$ with $\chi_1' > 0$ and $\chi_2' > 0$ are shown in Figs. 6 and 7, respectively, for χ_1 and χ_2. To conclude the discussion, in Fig. 8 we present the average of the maximum amplitude of χ_1 and χ_2 against the frequency $\bar{\omega}$. The analysis of the linear torque model under 1:1 resonance reveals a regular vibration motion, and the maximum amplitude just increases during the passage through the resonance.

Next we perform a similar analysis of the non-linear torque model.

3. On numerical simulations using non-linear torque

In this section we study the vibrating system defined by Fig. 1, taking into account the non-linear torque as defined by Eq. (1d). Fig. 9 shows the profile of stationary amplitudes of χ_1 and χ_2 before the passage through resonance $\bar{\omega} = 0.7$, where the vibration has a regular behaviour. Nevertheless, for $\bar{\omega} = 0.8435$ (passage through resonance) and $\bar{\omega} = 1.0$ (after passage) the vibration becomes irregular as is shown in Figs. 10 and 11, respectively. To reinforce this fact, we present the frequency spectrum (Fig. 12) for χ_1, and maximum Lyapunov exponents (Fig. 13)

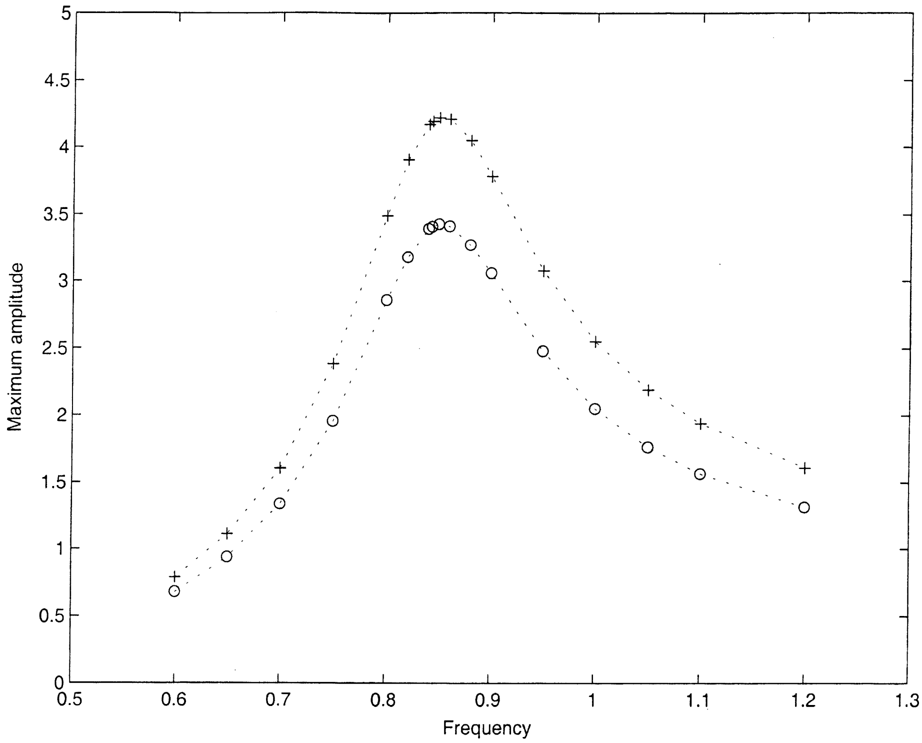

Fig. 8. Average maximum amplitude of χ_1 (o) and χ_2 (+) as function of frequency $\bar{\omega}$. The peaks correspond to the passage through resonance $\bar{\omega} = 0.8435$. Linear torque model.

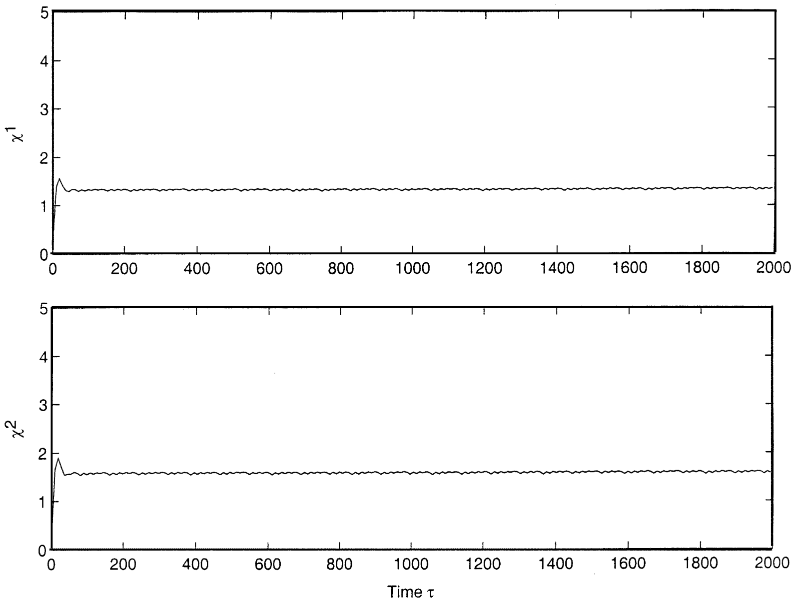

Fig. 9. Profile of maximum amplitude for χ_1 and χ_2 corresponding to $\bar{\omega} = 0.7$ (before passage through resonance). Non-linear torque model.

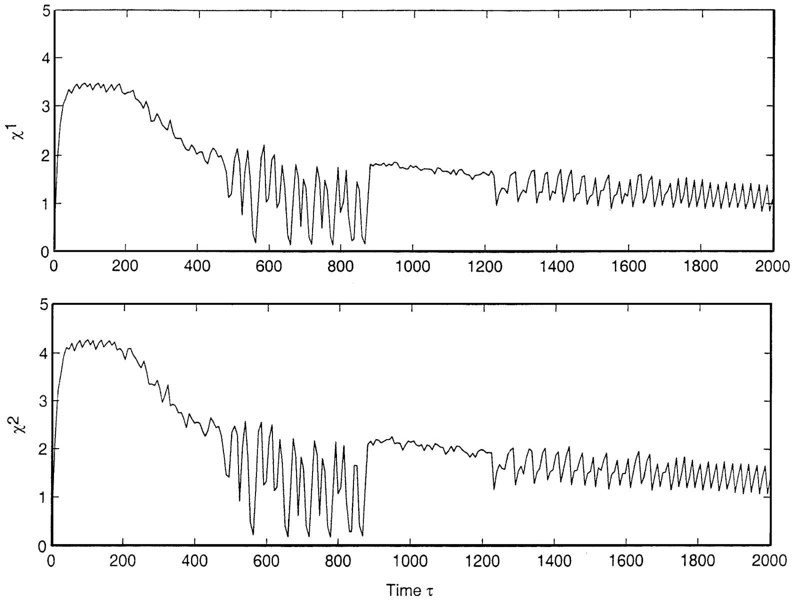

Fig. 10. Profile of maximum amplitude for χ_1 and χ_2 corresponding to $\bar{\omega} = 0.8435$ (passage through resonance). Nonlinear torque model.

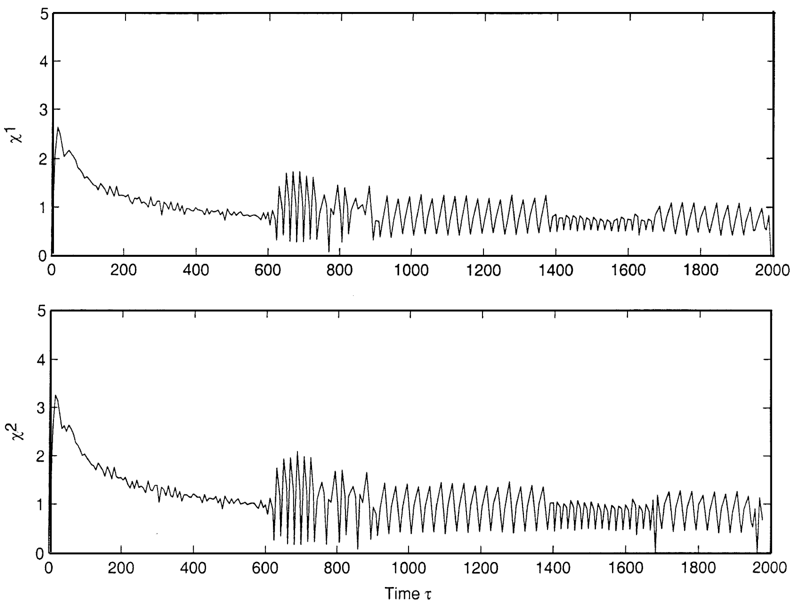

Fig. 11. Profile of maximum amplitude for χ_1 and χ_2 corresponding to $\bar{\omega} = 1.0$ (after passage through resonance). Nonlinear torque model.

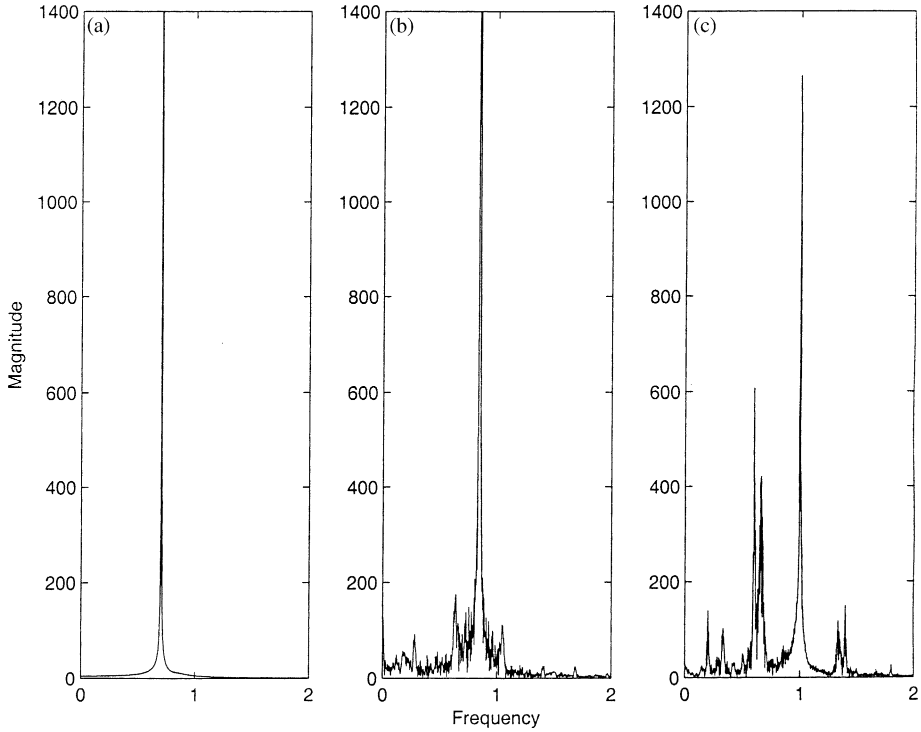

Fig. 12. Frequency spectrum for χ_1: (a) $\bar{\omega} = 0.7$, (b) $\bar{\omega} = 0.8435$, and (c) $\bar{\omega} = 1.0$. Non-linear torque model.

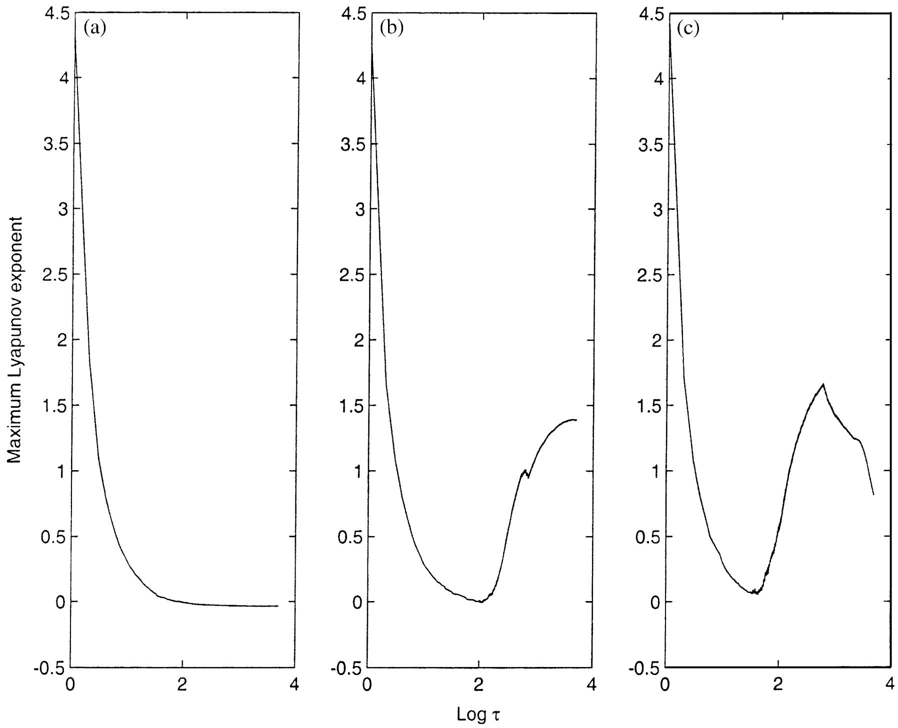

Fig. 13. Graph of maximum Lyapunov exponent against log τ: (a) $\bar{\omega} = 0.7$, (b) $\bar{\omega} = 0.8435$, and (c) $\bar{\omega} = 1.0$. Non-linear torque model.

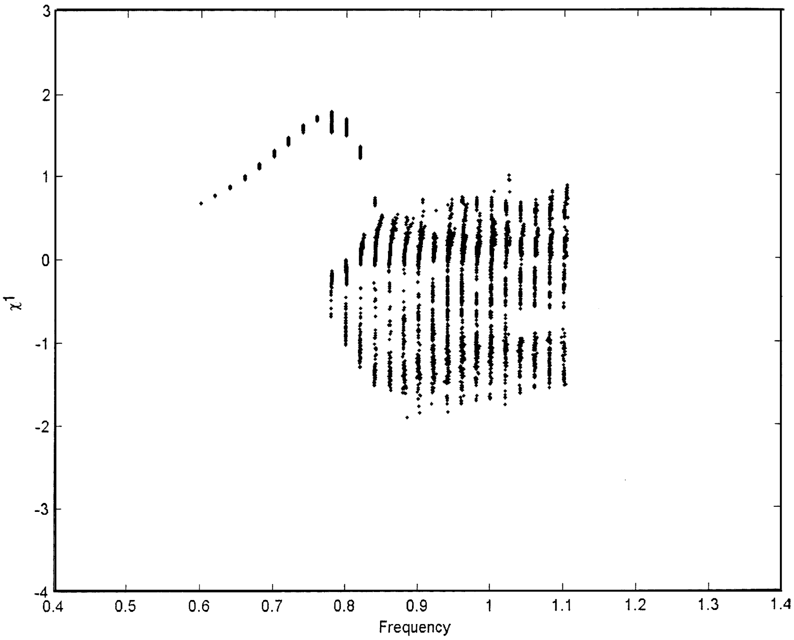

Fig. 14. Surface of section diagram for χ_1 with step size $\Delta\bar{\omega} = 0.02$. The adopted plane is $\bar{\varphi} = 0$ with conditions $\chi_1' > 0$ and $\chi_2' > 0$. Non-linear torque model.

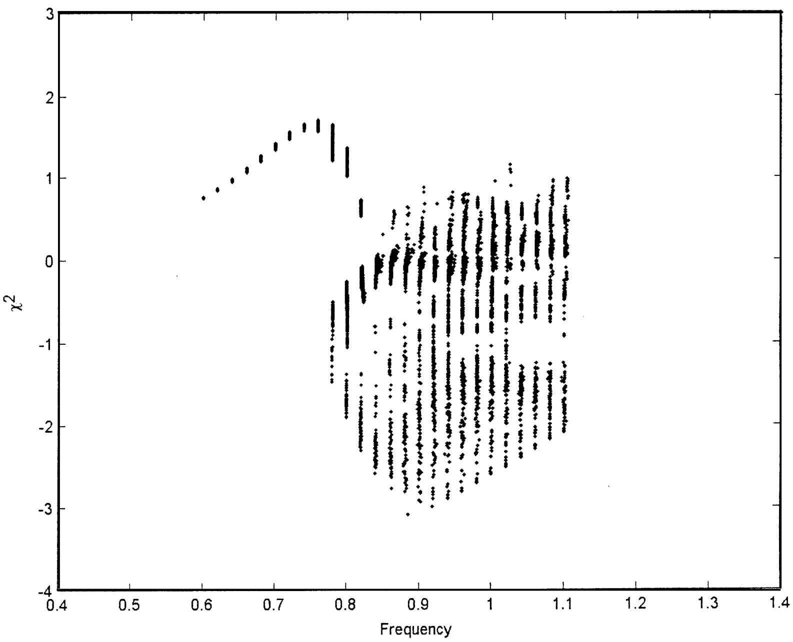

Fig. 15. Surface of section diagram for χ_2 with step size $\Delta\bar{\omega} = 0.02$. The adopted plane is $\bar{\varphi} = 0$ with conditions $\chi_1' > 0$ and $\chi_2' > 0$. Non-linear torque model.

corresponding to before, during and after the passage through the resonance. Finally, the bifurcation diagrams are showed in Figs. 14 and 15. The adopted surface of section is the same as the linear torque model, that is, the plane $\bar{\varphi} = 0$ with both χ_1' and χ_2' positive.

4. Concluding remarks

We analyzed the vibrating problem defined by Fig. 1 by using linear and non-linear torque models and considering the 1:1 resonance. In the case of linear torque we obtained regular behaviour and both regular and irregular motion in the non-linear case. The other resonance conditions showed in Table 1 will be studied in forthcoming papers.

Acknowledgements

The authors thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Pesquisas (CNPq), both Brazilian funding agencies, for the financial supports.

References

[1] V.O. Kononenko, Vibrating Systems with a Limited Power Supply, Illife Books, London, 1969 (in English).
[2] A.H. Nayfeh, D.T. Mook, Nonlinear Oscillations, Wiley, New York, 1979.
[3] J.M. Balthazar, D.T. Mook, R.M.L.R.F. Brasil, H.I. Weber, A. Fenili, D. Belato, J.L.P. Felix, Recent results on vibrating problems with limited power supply, in: J. Awrejcewicz, J. Brabski, J. Nowakowski (Eds.), Sixth Conference on Dynamical Systems Theory and Applications, Lodz, Poland, December 10–12, 2001, pp. 27–50.
[4] J.M. Balthazar, D.T. Mook, H.I. Weber, R.M.L.R.F. Brasil, A. Fenili, D. Belato, J.L.P. Felix, An overview on non-ideal vibrations, Mecannica 330 (7) (2002) 1–9.
[5] J.M. Balthazar, B.I. Cheshankov, D.T. Rushev, L. Barbanti, H.I. Weber, Remarks on the passage through resonance of a vibrating system with two degree of freedom excited by a non-ideal energy source, Journal of Sound and Vibration 239 (5) (2001) 1075–1085.