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Experimental studies on sound and vibration of a two-tone Chinese Peace Bell

Jianzheng Cheng*, Congqing Lan

Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, P.O.Box 71010, Wuhan 430071, Hubei, China

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Abstract

In order to examine whether the characteristics of the bell are coincident with the design and make some constructive suggestions for the tuning of bell, the frequency characteristics of sound and the vibrational modes of a two-tone Chinese Peace Bell have been studied. With Welch's spectral estimate, the main partials of radiated sound of the bell have been found out. With time–frequency analysis, the changes of the spectrum with time have been shown. By conditional mean frequency (CMF) of the spectrogram of the sound, the change in the pitch of the bell has been discussed. With an accelerator, the vibration distributions on the different points of the bell have been measured. With short time spectrum analysis, the vibrational modes at different frequencies have been reconstructed.

1. Introduction

Since the discovery of a rack of 64 bells in the tomb of the Marquis Yi of Zeng, research and reproductions of the Chinese bell have never been stopped [1–3]. In order to celebrate the arrival of the Y2K, as well as spread Chinese culture, a rack of 108 bells which is known as ''the Chinese Peace Bells'', was made in 1999. These bells not only remained traditional but also displayed some innovations. In order to evaluate whether the sound and the vibration of the bells tally with the design, and to provide some scientific views for designing, making and tuning of bells, the research on the frequency characteristics of the sound and the vibrational modes of the Chinese Peace Bells have been done. In this article, the characteristics of the center bell of the Chinese Peace Bells are discussed. Fig. 1 is a photo of the center bell, Fig. 2 is a plot of the bottom view of the bell. The bell is 360 kg in weight and 1.2 meters high. On the inner side of the side striking points, there are

*Corresponding author.
E-mail address: jzcheng@wipm.ac.cn (J. Cheng).

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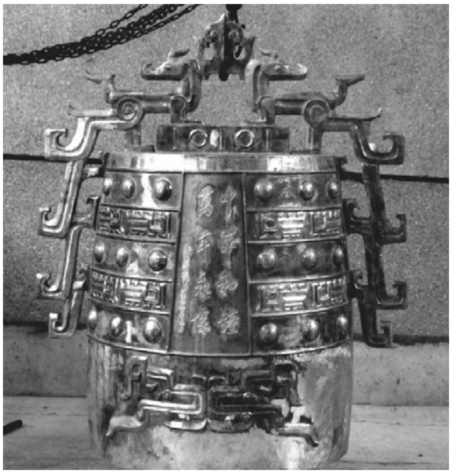


Fig. 1. The photo of the center bell.

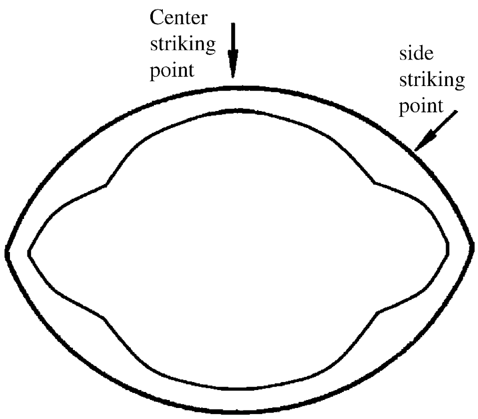


Fig. 2. The bottom view of the center bell.

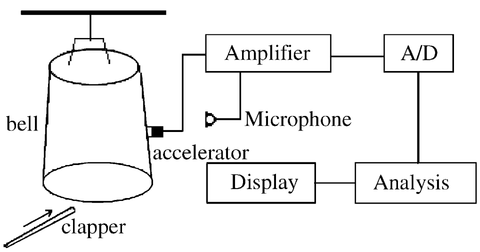


Fig. 3. The set-up of the measuring system.

four raised lip-like regions, which can not only restrain the high-order vibrational modes, but also produce two pitches when the center striking point and the side striking point, are struck respectively.

Fig. 3 is the set-up of the measuring system. When the bell was struck by the clapper, the sounds and the vibrations of the bell were received by the microphone and the accelerator, respectively. After being amplified and digitalized, these signals were stored in a computer. By processing these signals, we could reconstruct the vibrational modes of the bell.

2. Analysis of the characteristics of the sound signal

Some research has revealed that the sound radiated from a bell is not only related to the structure of the bell, but also dependent on the size, shape, and hardness of the clapper, the points at which the bell is struck, and the strength of the blow. Fig. 4 shows a waveform of the sound emitted from the center bell struck at the center striking point with a rubber-head wooden clapper. In the measurement, the response effect of the microphone is considered. It can be seen that the sound reaches the maximum in a short period and then decays gradually. In addition, from the expansion of Fig. 4, we can see the sound signal has a clear period.

2.1. Averaged periodogram of the sound

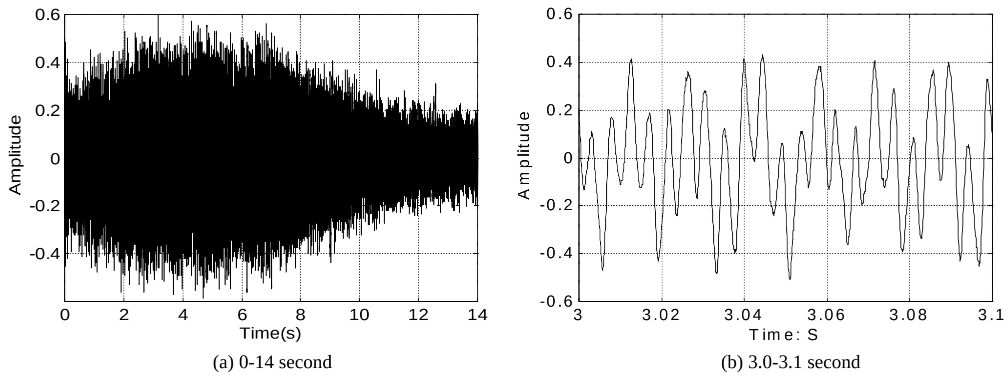
In order to investigate the spectrum structure of the sound of the center bell, we made the spectral estimate with Welch's Method [4], which can be expressed as follows [5]:

$$\tilde{P}_{per}(\omega) = \frac{1}{MUL}\sum_{i=1}^{L}\left|\sum_{n=0}^{M-1} x_N^i(n)W(n)\mathrm{e}^{-\mathrm{j}\omega n}\right|^2,$$

where

$$x_N^i(n) = x_N[n + (i-1)M], \quad 0 \leqslant n \leqslant M-1,\ 1 \leqslant i \leqslant L,$$

$$U = \frac{1}{M}\sum_{n=0}^{M-1} W^2(n),$$



(a) 0-14 second (b) 3.0-3.1 second

Fig. 4. The sound radiated from the center bell.

where $x_N(n)$ is the sound signal of length N and is divided into L sections with length M overlapping each other, $x_N^i(n)$ is the number i section of $x_N(n)$; $W(n)$ is the window function of length M.

Fig. 5 is the averaged periodogram of the signal shown in Fig. 4. From Fig. 5, we can find that there are 68, 222, 353, and 407 partials in spectrum, while 68 Hz is the fundamental partial.

2.2. Time–frequency analysis

For a time-shifting signal, we not only need to know the frequencies included, but also need to know the changes of different frequencies with time. With the help of the short time Fourier transform (STFT), the change pattern of the spectrum can be revealed. The STFT of signal $x(t)$ can be expressed as [5]

$$STFT_x(t,\omega) = \int x(\tau)h^*(\tau - t)\mathrm{e}^{-\mathrm{j}\omega\tau}\,\mathrm{d}\tau,$$

where $h(\tau)$ is a window function which isolates local sections of the signal for Fourier analysis.

Fig. 6 is the plot of STFT of the sound signal in Fig. 4. From Fig. 6, it can be found that the high-frequency harmonics reach the peaks at the beginning and decay exponentially, while the low-frequency harmonics reach their peaks within a few seconds and then decay. This phenomenon might be explained as follows: The non-linear vibrational motion results in the transition from unstable high-frequency vibration to low-frequency vibration and then enhances the low-frequency vibration. Moreover, the amplitudes of 68 and 222 Hz partials wave to a certain extent.

2.3. Instantaneous frequency and the pitch of the bell

A musical instrument has a certain designed pitch and is hoped to attain that pitch as soon as it is played. In theory, the pitch of a musical instrument is related to the instantaneous frequency of the sound radiated by itself. In fact, if there is a frequency predominating to the others in intensity in the spectrum, we can say that the pitch of this musical instrument is decided by this frequency when the subjective factors are neglected. If there are a few frequencies with greater amplitudes,

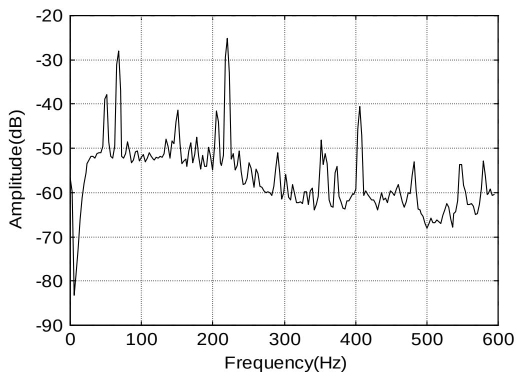


Fig. 5. The Welch's spectral estimate.

the weighted average of these frequencies determines the pitch, expressed as the instantaneous averaged frequency (IAF) [6],

$$\langle \omega \rangle_t = \frac{\int \omega P(t,\omega)\,\mathrm{d}\omega}{\int P(t,\omega)\,\mathrm{d}\omega} = \frac{\int A^2(\tau)\varphi'(\tau)h^2(t-\tau)\,\mathrm{d}\tau}{\int A^2(\tau)h^2(t-\tau)\,\mathrm{d}\tau},$$

where

$$P(t,\omega) = \left|x(t) * h(t)\mathrm{e}^{\mathrm{j}\omega t}\right|^2 = \left|\int x(\tau)h(t-\tau)\mathrm{e}^{-\mathrm{j}\omega\tau}\,\mathrm{d}\tau\right|^2,$$

$$x(t) = A(t)\mathrm{e}^{\mathrm{j}\phi(t)},$$

where $x(t)$ is the signal, $h(t)$ is the impulse response of a low-pass filter modulated by $\mathrm{e}^{\mathrm{j}wt}$ to different frequency band and $P(t,\omega)$ is the spectrogram of the signal $x(t)$.

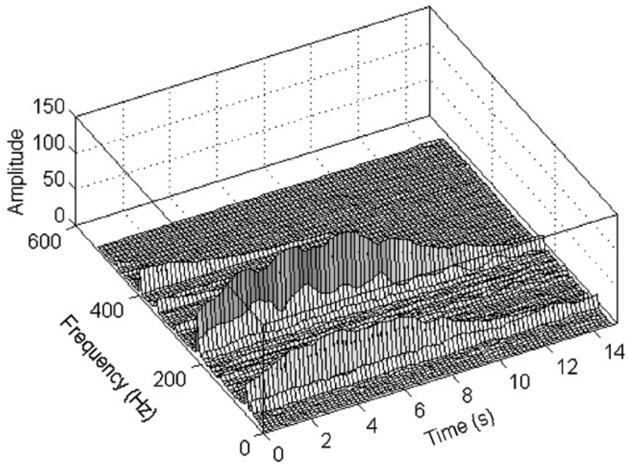


Fig. 6. The time–frequency distribution of the sound signal of the center bell.

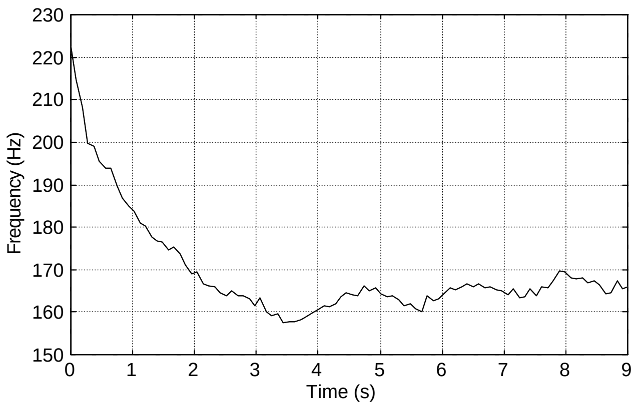


Fig. 7. The instantaneous averaged frequency of the center bell.

Fig. 7 shows the IAF of the sound signal (shown in Fig. 4(a) of the center bell. Since the relative amplitudes of the different frequencies change with time, the IAF is time-shifting. Because of the decay of high partials,the IAF changes from 220 to 160 Hz within 3 s and then the change waves in the range of 160–170 Hz. So the subjective pitch of the bell is not at the fundamental 68 Hz but 160 Hz or so. The reason for the disparity is that the high partials with greater amplitudes, especially 222 Hz, do not decay as fast as expected and the transition time to a pure sound of 68 Hz is too long, thus cannot become a pure sound.

3. Modes of vibration

It is well known that vibration produces sound. When struck by a clapper, a bell is distorted and then vibrates in an exceedingly complex way. In principle, its vibrational motion can be described in terms of a linear combination of the normal modes of vibration whose initial amplitudes are determined by the form of the distortion. The complexity is due to the large number of normal modes of diverse characteristics that contribute to the motion. Fig. 8 shows the nodes at the end and distortion of some normal modes [7]. In theory, the four raised lip-like regions should appear in the nodes or antinodes of each normal modes. For the fundamental mode of striking, at the center they appear in the nodes. Fig. 9 shows the vibrational distributions of several main partials of the bell struck at the center striking point. For symmetry, the vibration is drawn only in half area. By comparing with Fig. 8, the different vibrational distributions of Fig. 9 can be arranged to the different normal modes of Fig. 8. Fig. 9(a) is the mode (2,0)a of the bell, i.e., the fundamental mode for striking the center. In this mode, vibration of the center striking point is strong, while vibration at the side striking points is weak. This mode is usually related to the expected pitch of the bell. Fig. 9(b) is the mode (2,0)b of the bell, i.e., the fundamental mode of side striking, whose distribution is contrary to that of the mode (2,0)a. When the bell is struck at the center striking point, the vibration of the mode (2,0)b exists only in a rather short time, because the much stronger vibration of the mode (2,0)a restricts the vibration of the mode (2,0)b. But if the bell is struck at the side striking points, the mode (2,0)b becomes the main vibrational mode. In some way, the two tones of the bell are produced just by the vibrations of the mode (2,0)a and (2,0)b that are excited by striking the bell at the center striking point and

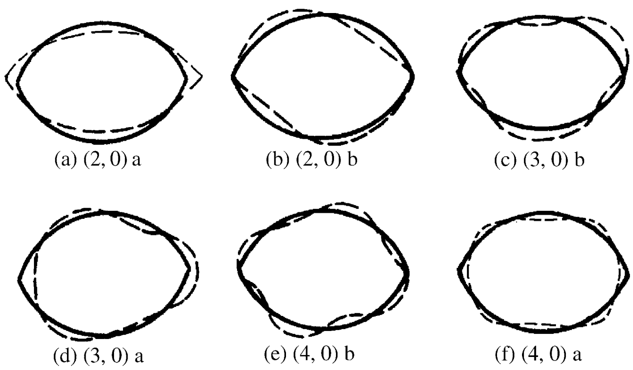


Fig. 8. The nodes at the end of the bell and the distortion of some normal modes.

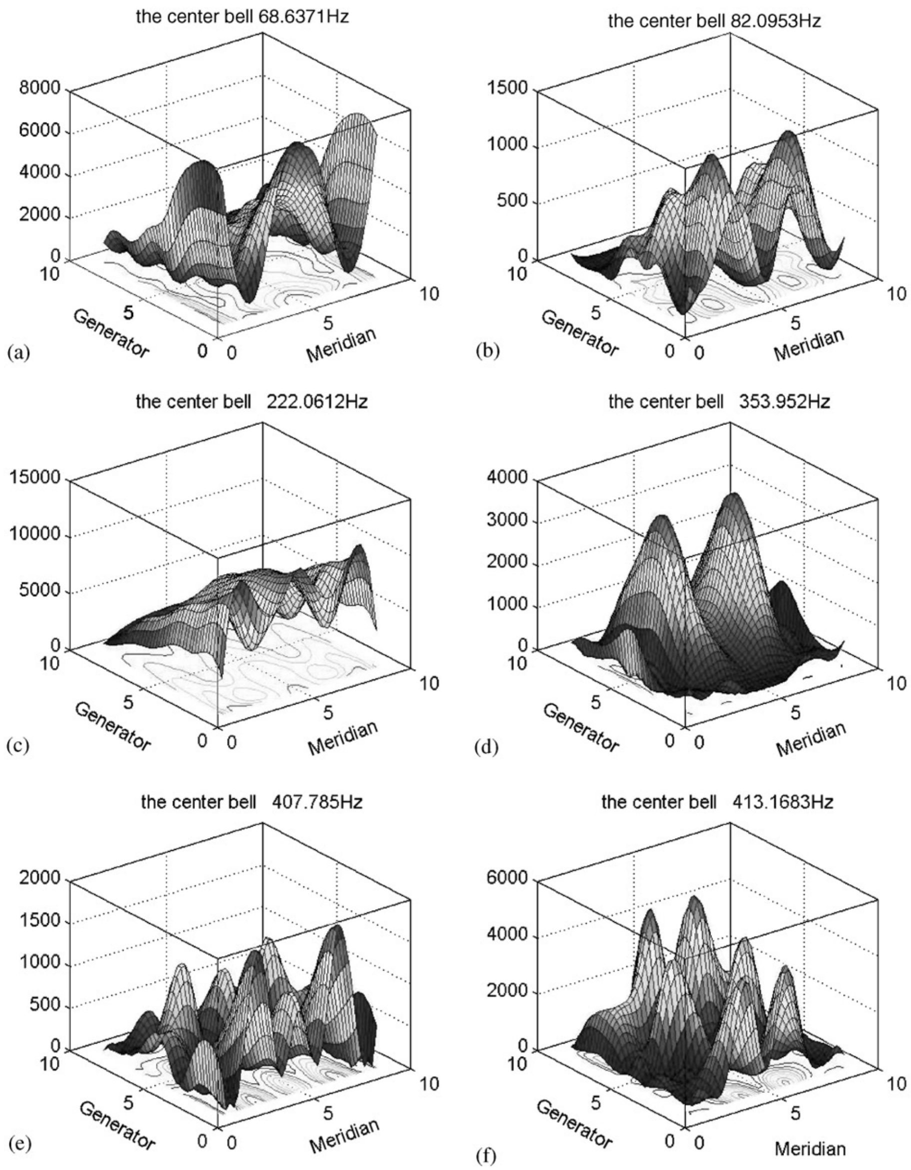


Fig. 9. The vibrational distributions of several main partials of the bell struck at the center striking point. In the generator or meridian direction, 12 cm/unit, in the vertical direction, the value represents the relative amplitude.

the side striking point, respectively. For a performing musical instrument, the insulation between the center striking sound and the side striking sound is required. When a bell is struck at the center striking point, it is hoped to excite the vibration of the mode (2,0)b as little as possible; when a bell is struck at the side striking point, it is hoped to produce the vibration of the mode (2,0)a as little as possible. Fig. 9(c) is the mode (3,0)a whose frequency is 3.2 times that of the mode (2,0)a. From Fig. 6, we can see that the vibration of the mode (3,0)a is much more violent than that of the mode

(2,0)a. Moreover, the vibration of the mode (3,0)a remains the same for a long time. Thus the actual pitch of the bell is much higher. One reason might be that the thickness at the side striking points is not enough to restrain the vibration of the mode (3,0)a. Fig. 9(d) shows the mode (2,1) whose frequency is 5.16 times that of the mode (2,0)a. In this mode, the nodal circle is near the lip of the bell. Fig. 9(e) shows the mode (4,1) whose frequency is nearly the sixth harmonic of the mode (2,0)a. The vibrational amplitude of the mode (4,1) is large at the beginning and then decays very fast. Fig. 9(f) shows mode (2,2) whose frequency is six times that of the mode (2,0)a and five times that of the mode (2,0)b.

4. Conclusion

Owing to the unique inscriptions and grains of the Chinese two-tone bell, theoretical research on its vibration is often very difficult. In the article, the frequency characteristics of the sound of the center bell of the "Chinese Peace Bell", as well as its vibrational modes, have been investigated experimentally. The results show that when the bell is struck at the center striking point, not only the mode (2,0)a is excited, but also the mode (3,0) with a larger amplitude. Since the mode (3,0) exists for a rather long period, it takes a very long period for the sound to become pure, so that the bell designed for a low pitch cannot actually produce the low pitch. Because of the long aftersound, difficulty may be encountered when the bell is performed with rhythm. Based on our research, we suggest that this problem can be solved by adding the thickness of the four raised lip-like regions in the inner side. There may be a sequel that the rule of partial distribution of the bell will be altered and the vibrations of the modes with nodes at these positions, especially the mode (3,0) will be restricted, but as for the mode (2,0)a, there will be little change. So the sound will be more pure and the pitch more stable. Hopefully, this research could give some insight into bell design.

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