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Letter to the Editor

Coupled torsional–lateral vibration analysis of geared shaft systems using mode synthesis

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1. Introduction

Coupling mechanism between torsional and lateral vibrations often influences dynamic characteristics of mechanical systems involving elements like twisted aerofoil sections and unsymmetrical rotors. Surace et al. [1] given the coupling mechanism in twisted blades through an approximate method of analysis. Recently, Al-Berdoor [2] has shown a model for coupled vibrations of unbalanced rotors, by accounting rotor–stator-rubbing forces. In compound-geared shaft–rotor systems, gear backlash and variable speed ratios have considerable influence on the natural frequencies. In addition, the torsional and lateral vibrations are coupled through the forces acting on the gears and hence the motion of the elements in one direction has influence on the motion in other directions. Many authors [3–5] have shown that the predicted dynamic behaviour of a geared shaft system on taking into account the cross-coupling between bending and torsion could be different from that obtained by using a conventional model of the system. Generally, the geared shaft systems are in the form of branched models. Dynamic characteristics of such systems are obtained from many numerical techniques such as transfer matrix methods [6,7], finite element methods [5]. In geared shaft systems, lower modes are of main interest, which can be easily obtained from the alternative analysis techniques like substructure synthesis [8,9] that needs a limited computational core. In the present note, the component mode synthesis technique is employed for obtaining dynamic characteristics of a three-pinion industrial compressor drive system by considering the effects of gear backlash and speed ratio changes. The effect of torsional coupling with bending through a numerical model is also considered in this paper.

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2. Analysis

Any complex geared shaft system can be divided into several simple substructures, which can be easily analyzed one by one disengaging each pair of gears. In general, each substructure consists of a shaft with bearings, gears and other rotating components. Spatial beam elements are often used in dynamic analysis of a substructure. Equations of motion of ith substructure can be expressed as

$$M_i\ddot{X}_i + C_i\dot{X}_i + K_iX_i = F_i(t).$$

Eigenproblem of above equation is solved using conventional methods such as simultaneous iteration scheme. These data will be stored for use during modal synthesis of total system. For convenience, the concept of equivalent system can be employed by considering speed ratios of the gears. Suppose that there are m-substructures in a geared shaft system that their mass and stiffness matrices are determined and dynamic properties are known. In order to set up equations of motion of global system, it is first necessary to arrange all mass matrices $[M_i]$, stiffness matrices $[K_i]$ $(i = 1, 2, 3, ...)$ in a diagonal line, so that

$$[M_0]\{\ddot{X}\} + [K_0]\{X\} = \{F(t)\},$$

where

$$[M_0] = \mathrm{Diag}(M_i),$$
$$[K_0] = \mathrm{Diag}(K_i),$$
$$\{F(t)\} = \{\{F_1\}^{\mathrm{T}}, \{F_2\}^{\mathrm{T}}, \ldots, \{F_m\}\mathrm{T}\}^{\mathrm{T}}.$$

It represents an uncoupled system. Now taking into account, the coupling effects between substructures through gear-mesh stiffness, the equations of motion of the global system

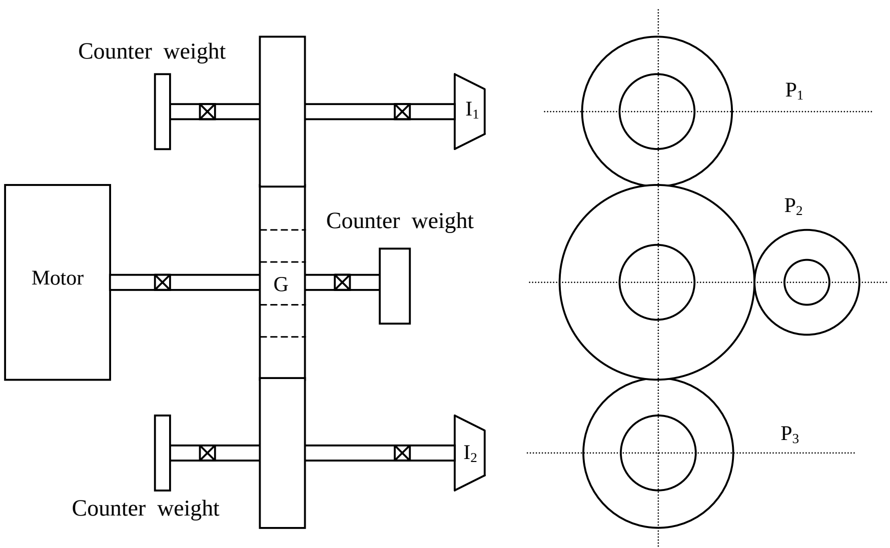

Fig. 1. Schematic of the three-pinion geared system under consideration.

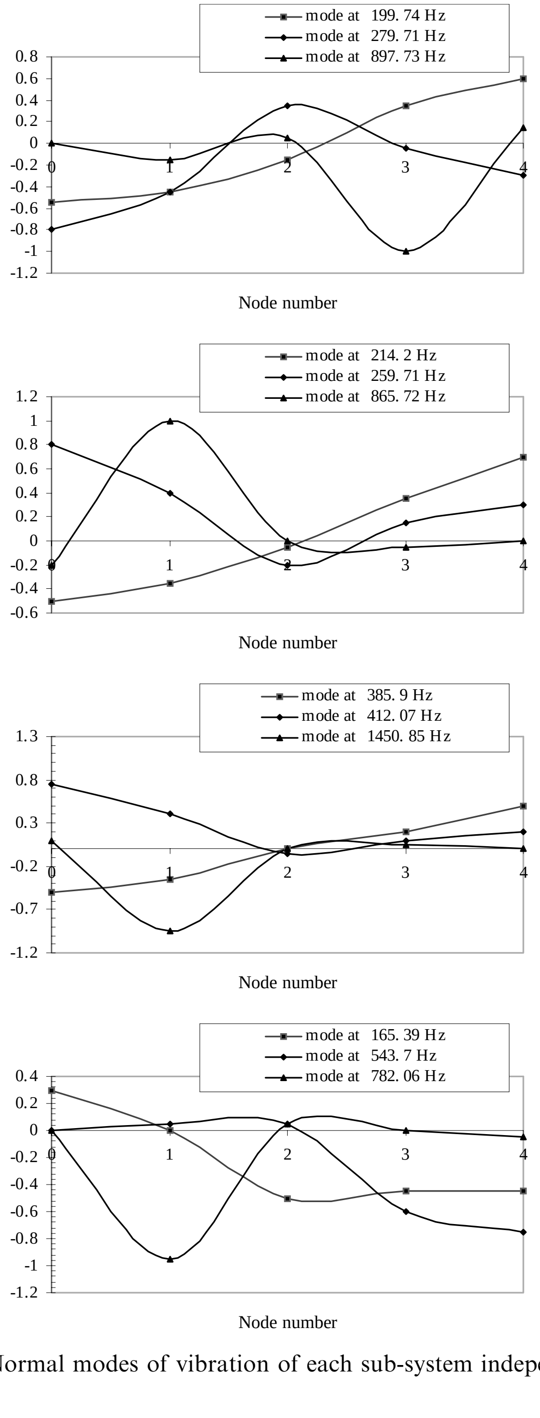

Fig. 2. Normal modes of vibration of each sub-system independently.

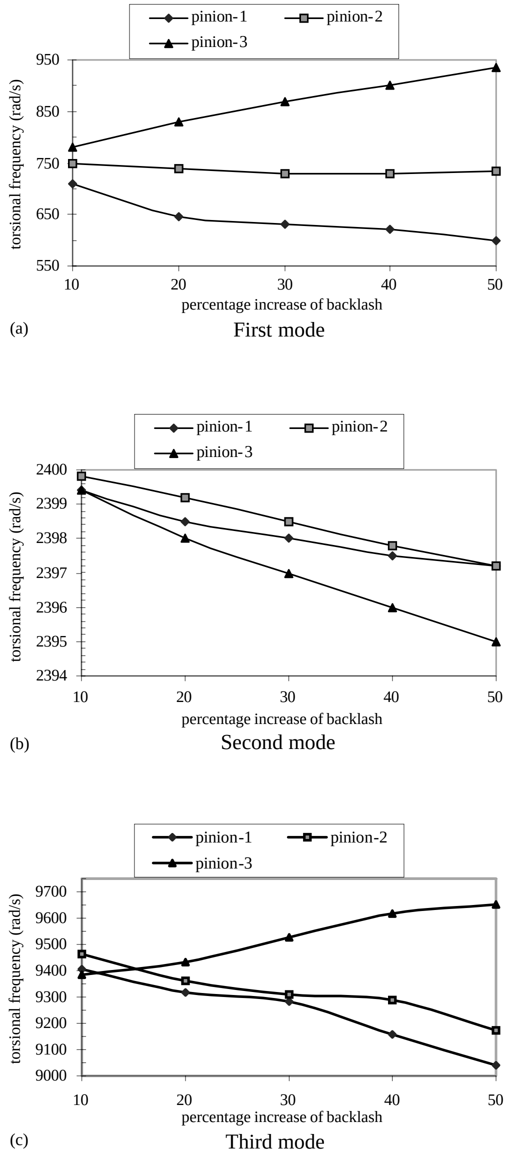

Fig. 3. Variation of torsional frequencies as a function of backlash at the pinions.

becomes

$$[M]\{\ddot{X}\} + [K]\{X\} = \{F(t)\},$$

where $[M] = [M_0]$ and $[K] = [K_0] + [\Delta K]$. Here incremental stiffness matrix $[\Delta K]$ is due to linear spring employed in modelling the meshing process in gear pair [10].

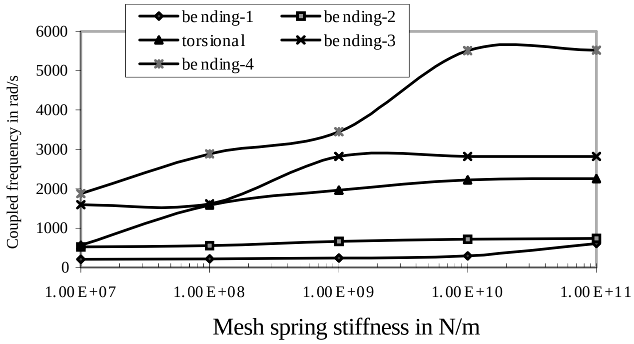

Fig. 4. Effect of gear-mesh stiffness at a fixed gear ratios $GR_1 = 10$, $GR_2 = 14$ and $GR_3 = 20$.

3. Numerical example

Modal synthesis described above has been implemented using a computer-program, which consists of two parts: dynamic analysis of substructures and modal synthesis. A gear transmission system of an industrial compressor unit is taken as an example to demonstrate the technique and to show the coupling effects as well as the influence of parameters like backlash, gear ratio, etc. on natural frequencies. Fig. 1 shows a simple block diagram of the system. Motor drives the input shaft (substructre-1) and bull gear on this shaft floats to enable the three other pinions in substructures 2–4 for transmitting power to the impellers.

The developed user interactive simulation program has feasibility to change the diameters of the shafts, inertia of rotors and gear ratios between the shafts. A two-node, six degree-of-freedom beam element is employed for modelling each shaft. Each of the shafts with rotors and meshing gear pairs is represented through stiffness and mass matrices. The moment of inertia of all the rotors and stiffness of all the shafts are computed with reference to the bull gear shaft and an equivalent model of the entire system is constructed. First three torsional modes of each substructure are shown in Fig. 2.

During periodic operation of gears, backlash level changes from initial value [11], which has a considerable effect on the speed ratio of gears. In the present study, backlash changes are considered to increase by 50% of the initial value in five steps. The corresponding changes in torsional frequencies of overall system are recorded. Fig. 3 shows the variation of first three torsional frequencies of the entire system as a function of backlash in each of the three gear pairs, when considered separately. The sizes of overall assembled stiffness and mass matrices have reduced by more than three times upon using the component mode synthesis method. The coupled lateral–torsional motion is considered with the introduction of meshing spring at each of the contacting gear pairs. Fig. 4 shows the variation of first five modes of the entire system as a function of mesh-spring stiffness, at fixed values of indicated gear-speed ratios.

4. Conclusions

In this letter, substructure synthesis method is employed to analyze the dynamic characteristics of an integrated-geared system consisting of four shafts. Effects of the gear parameters like backlash, gear ratio on torsional frequencies and effect of gear-meshing stiffness on the coupled frequencies of the assembly have been studied. Influence of backlash on the torsional frequencies is dependent on the gear-pair under consideration. The effect has been observed slightly for higher modes. As gear-mesh stiffness increases, all the coupled natural frequencies are also found to increase, but this is more pronounced at higher modes.

Acknowledgements

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