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Response characteristics of local vibrations in stay cables on an existing cable-stayed bridge

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Abstract

This paper examines local parametric vibrations in the stay cables of a cable-stayed bridge. The natural frequencies of the global modes are obtained by using a three-dimensional FE model. The global motions generated by (1) sinusoidal excitations using exciter, (2) a traffic loading, and (3) an earthquake are analyzed by using the modal analysis method or the direct integration method. The local vibration of stay cable is calculated by using a model in which inclined cable is subjected to time-varying displacement at one support during global motions. This paper describes the properties of the local vibrations in stay cables under these dynamic loadings by using an existing cable-stayed bridge.

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1. Introduction

Previous studies show that local vibrations of large amplitude are induced in the stay cables (hereinafter abbreviated to ‘cable’) of cable-stayed bridges under wind and traffic loading [1]. This phenomenon has been confirmed in vibration tests on the Hitsuishijima Bridge [2], Yohkura Bridge [1] and Tatara Bridge [3] in Japan. These vibrations are considered to be local parametric vibrations (i.e., dynamic instability) in the cable due to excitation at the support by girder and/or towers’ oscillation. Since multi-cable systems have been widely used in cable-stayed bridges, the natural frequencies of the global modes are easily close to the natural frequencies of the cables and the large-amplitude cables vibration becomes prone to be exhibited. Therefore, further research is necessary on the parametric vibrations of cables in cable-stayed bridges.

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Kovács was the first to point out the possibility of parametric vibrations in the cables [4]. Analyses have been recently carried out in various fields, e.g., in Refs. [5–8]. Generally, large-amplitude vibrations of cables can be induced when the natural frequency of global modes in a cable-stayed bridge is either close to that of the cables (the second unstable region) or twice that of the cables (the principal unstable region). Takahashi and co-workers studied the relationship between the natural frequencies of the global modes and those of cables and the response characteristics of local parametric vibrations in cable-stayed bridges in Japan [9]. In their study, the cables were given periodic time-varying displacements at the supports. However, the study did not explain the local vibrations in the cables of cable-stayed bridges under environmental and service loadings such as wind, earthquake and traffic loading, which contain a broad spectrum of excitation frequencies. There is little literature on the local vibrations of cables that take into account the vibration characteristics of the whole bridge system.

This paper examines in great detail the local parametric vibrations in cables reflecting the vibration characteristics of an existing steel cable-stayed bridge subjected to all forms of excitations including sinusoidal excitations, a traffic loading and an earthquake. Based on a three-dimensional FE model, the global dynamic motion of the bridge is calculated by the modal analysis method in the case of sinusoidal excitations and traffic loading or the direct integration method for the seismic analysis. The analysis of the local vibrations of stay cables, which are subjected to time-varying displacement at the supports during global motions, are done and their properties are discussed.

2. Studied bridge

The bridge analyzed in this paper is a steel cable-stayed bridge in Japan. The bridge has three spans. The main span is 350 m and the side spans are 160 m. The towers are A-shaped, and the cables are a two-plane, multiple system. A general view of the bridge is shown in Fig. 1. The cables are numbered sequentially from the side span to the main span as shown in Fig. 1.

3. Analytical method

The natural frequencies of the global modes are obtained from a three-dimensional FE model of the cable-stayed bridge. Global vibration analysis under dynamic excitations is then performed, and the responses of the global motions are obtained. Finally, the local vibration of the cables is analyzed. Since the responses of local parametric vibrations in the cables cannot be evaluated in the global analysis by using a FE model, a vibration model of the cable including parametric vibration and forced vibration is used. In this model, the inclined cable is subjected to time-varying displacement at one support during global motions.

The dynamic actions considered in this paper are (1) sinusoidal excitations, (2) a traffic loading, and (3) an earthquake.

3.1. Global vibration analysis of a cable-stayed bridge

The three-dimensional FE model developed in this study is shown in Fig. 2. The girder in this model is a single central spine with offset links to the cable anchor points. The towers and piers

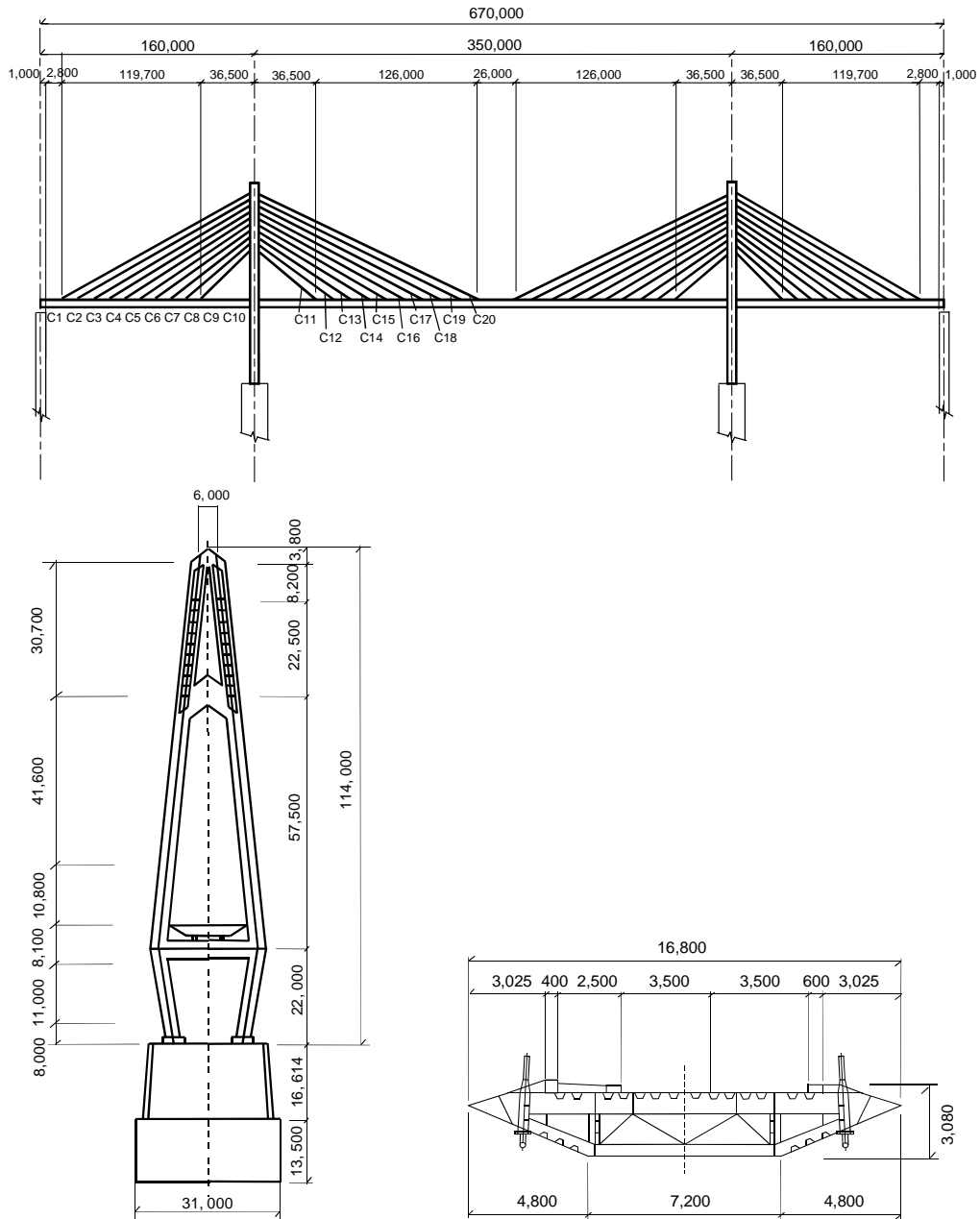


Fig. 1. General view of the cable-stayed bridge (mm).

are modelled using three-dimensional linear beam elements based on the actual cross-section properties. The cables are modelled as linear truss elements with initial tension. The non-linear behavior of cables due to their sags is taken into account by using an equivalent modulus of elasticity [10]. Regarding the boundary conditions, the girder is free to move in the longitudinal

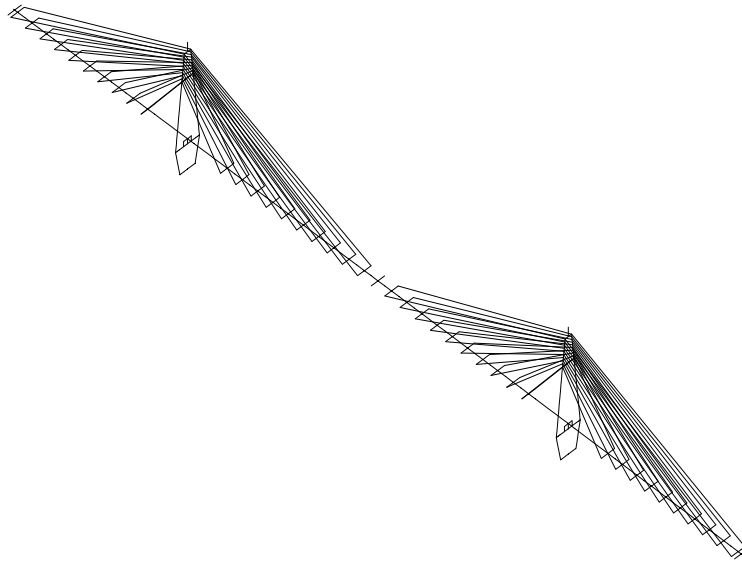


Fig. 2. FE model.

direction and restrained at the supports in the vertical and transverse directions. Only the rotational component around the longitudinal axis is restrained. The tower bases are fixed in all degrees of freedom.

The analysis methods of the global motions of the bridge are the modal analysis method in the case of sinusoidal excitations and the traffic loading, and the Newmark β method ($\beta = 0.25$) of direct integration for the seismic analysis. The modal damping constant of all modes is assumed to 0.02 in the modal analysis. Rayleigh damping is employed in the direct integration and the damping constant of the girder and towers is assumed to be 0.02.

3.2. Local parametric vibration analysis of cables

A model of an inclined cable on a cable-stayed bridge is shown in Fig. 3. The cable in this model is fixed at one end and has time-varying displacements ($X(t)$, $Y(t)$) at the other end. It is assumed that there is no restraint against rotation at the anchorage that is independent of the amplitude of the cable vibration. A local vibration analysis of the cables is carried out by using the calculated relative response between the girder and towers as the displacement input at the cable supports.

The non-linear equation of motion of a flat-sag cable is obtained as follows:

$$m \frac{\partial^2 v}{\partial t^2} - P \frac{\partial^2 v}{\partial x^2} - \Delta P \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v_0}{\partial x^2} \right) = 0, \tag{1}$$

$$\Delta P = \frac{EA}{L} \left\{ u|_{x=L} - u|_{x=0} + \frac{1}{2} \int_0^L \left(\frac{\partial v}{\partial x} \right)^2 dx + \int_0^L \frac{\partial v}{\partial x} \frac{\partial v_0}{\partial x} dx \right\}, \tag{2}$$

where u and v are the displacements in the axial direction (x direction) and the normal line direction (y direction) of the cable as shown in Fig. 3, m is the mass per unit length of the cable, P

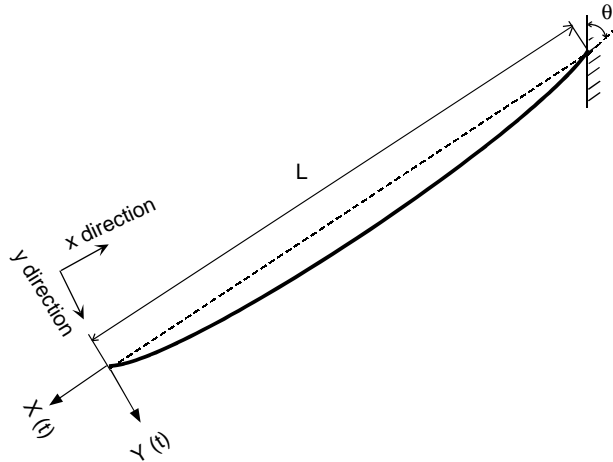


Fig. 3. Model of a stay cable and its boundary conditions.

is the initial tension of the cable, ΔP is the additional tension produced by local vibration in the cable, $v_0 = mg/2P(-x^2 + Lx)$ is the initial shape of the cable, E is Young’s modulus of the cable, A is the cross-sectional area of the cable, L is the span of the cable, g is the gravitational acceleration and t is time.

The following equation describes the assumed responses of the cable, which receives displacement $X(t)$ in the x direction and displacement $Y(t)$ in the y direction at a support:

$$v(x, t) = \left(1 - \frac{x}{L}\right) Y(t) + \sum_{i=1}^{\infty} T_i(t) \sin \frac{i\pi}{L} x, \quad u(x, t) = \left(1 - \frac{x}{L}\right) X(t), \quad (3)$$

where $T_i(t)$ is the time function of the i th mode of the cable.

Substituting Eq. (3) into Eqs. (1) and (2) yields a non-linear equation of motion of a cable. Upon applying a Galerkin method, the following non-linear equation of motion of the first mode considering the damping is obtained:

$$\ddot{T}_1(t) + 2\omega_1 h \dot{T}_1(t) + \omega_1^2 T_1(t) + B_1(t) T_1(t) + B_2 T_1(t)^2 + B_3 T_1(t)^3 = B_4(t), \quad (4)$$

where

$$B_1(t) = \omega_0^2 \left(\frac{X(t)}{X_0} + \frac{1}{2L} \frac{Y^2(t)}{X_0} - \frac{LA_1}{2} \frac{Y(t)}{X_0} \right),$$

$$B_2 = -\omega_0^2 \frac{3LA_1}{\pi} \frac{1}{X_0}, \quad B_3 = \omega_0^2 \frac{\pi^2}{4L} \frac{1}{X_0},$$

$$B_4(t) = \omega_0^2 \left\{ -\frac{2}{\pi\omega_0^2} \ddot{Y}(t) + \frac{4L^2 A_1}{\pi^3} \frac{X(t)}{X_0} + \frac{2LA_1}{\pi^3} \frac{X^2(t)}{X_0} - \frac{2L^3 A_1^2}{\pi^3} \frac{Y(t)}{X_0} \right\},$$

$$X_0 = \frac{PL}{EA}, \quad A_1 = \frac{mg \sin \theta}{P}, \quad \omega_0 = \frac{\pi}{L} \sqrt{\frac{P}{m}},$$

is the natural circular frequency of the string

$$\omega_1 = \frac{\pi}{L} \sqrt{\frac{P}{m}} \left(1 + \frac{8L^3 A_1^2}{\pi^4 X_0} \right),$$

is the natural circular frequency of the cable considering sag, h is the damping constant and θ is the inclined angle of the cable.

In the above equation, the term $B_1(t)$ is the parametric excitation term and $B_4(t)$ is the forced vibration term. The damping constant of the each cable is assumed to be 0.001, based upon experimental data in the bridges of Japan.

The response of a cable subjected to support excitations is obtained by solving Eq. (4) with the Runge–Kutta method.

4. Global modes and local modes

The computed natural frequencies of the global modes of the bridge are shown in Table 1. The natural frequencies of the cables, obtained by using ω_1 of Eq. (4), are listed in Table 2. The dynamic interaction between the bridge and the cables has been analyzed recently by various researchers [11–17]. Table 2 also lists the natural frequencies of local cable vibration obtained by using FE analysis with a multi-element model considering the sag in each stay cable. The natural frequencies of the single cable model obtained from Eq. (4) agree well with those obtained from multi-element FE analysis. It can be said that the separation of cable vibrations from the global vibration is valid in the present calculation.

Fig. 4 describes the relationship between the natural frequencies of the global modes and those of the cables in the bridge. The figure shows the first natural frequencies of the cables (corresponding to the second unstable region) and the doubled natural frequencies (corresponding to the principal unstable region) [5].

Table 1
Natural frequencies and modal shapes

Modal shape	Mode number	Natural frequency (Hz)	Character
Vertical mode	1st	0.310	Symmetric
	2nd	0.423	Asymmetric
	3rd	0.692	Symmetric
	4th	0.815	Asymmetric
	5th	0.931	Symmetric
	6th	1.124	Asymmetric
	7th	1.281	Symmetric
	8th	1.402	Asymmetric
	9th	1.566	Symmetric
Torsional mode	1st	1.192	Symmetric
	2nd	1.979	Symmetric
	3rd	2.081	Asymmetric

Table 2
First natural frequencies of cables (Hz)

Cable no.	Analytical data	Multi-element FE model for cable vibration (16 links for every cable)
C1	0.585	0.592
C2	0.573	0.577
C3	0.749	0.753
C4	0.802	0.802
C5	0.920	0.920
C6	0.986	0.982
C7	1.062	1.062
C8	1.184	1.186
C9	1.164	1.164
C10	1.644	() ^a
C11	1.636	() ^a
C12	1.146	1.147
C13	1.162	1.163
C14	1.039	1.043
C15	0.962	0.959
C16	0.896	0.896
C17	0.781	0.780
C18	0.729	0.729
C19	0.616	0.616
C20	0.637	0.637

^aFrequency is beyond frequency range considered.

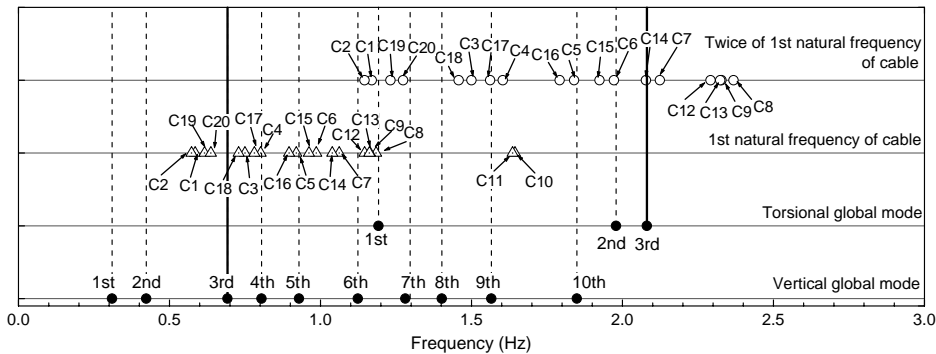


Fig. 4. Relationship between the natural frequencies of the global modes and those of the cables.

Since the first natural frequencies of cables C18, C19 and C20 are in the vicinity of the natural frequency of the 3rd vertical global mode, local parametric vibration in the second unstable region may occur under period loading with this frequency. Similarly, the natural frequencies of the 4th, 5th and 6th vertical modes are close to the first natural frequencies of cables C4 and C17, C5 and C15, and C12. Therefore, local parametric vibration in the second unstable region in these cables may occur.

The natural frequencies of the 6th vertical mode, the 9th vertical mode, the 2nd torsional mode and the 3rd torsional mode are close to twice of the first natural frequencies of cable C2, C17, C6 and C14, respectively. So local parametric vibration in the principal unstable region in those cables may be expected.

5. Local vibration characteristics of cables under sinusoidal excitation

This section discusses the properties of local parametric vibrations in the cables of the cable-stayed bridge under the sinusoidal excitation, which may be induced by an exciter during a vibration test. The amplitude of the exciting force is assumed to be 50 kN.

5.1. Vertical sinusoidal excitation

As shown in Fig. 4, parametric vibrations in cables C18, C19 and C20 may be induced under the 3rd vertical mode of the global modes. Therefore, the frequency of the vertical excitation is set to that of the 3rd vertical global mode (symmetrical, 0.692 Hz), and the excitation point is the center of the main span.

Fig. 5 shows the maximum amplitudes of all cables under excitation. The amplitudes of cables C18, C19 and C20 are greater than those of the other cables. In order to judge the characteristic of those cables, the time responses and spectra of the girder and cable C19 are shown in Fig. 6. The response of cable C19 under forced vibration, which neglects the term $B_1(t)$ of Eq. (4), is also shown to compare with the parametric vibration.

The ratio of the dominant frequency of the girder to that of C19 is approximately 1.0 and the waveform of the parametric vibration is accompanied by beating. So it can be concluded that the parametric vibration in the second unstable region occurs in those cables. Furthermore, the amplitudes under parametric and forced vibration are of the same order.

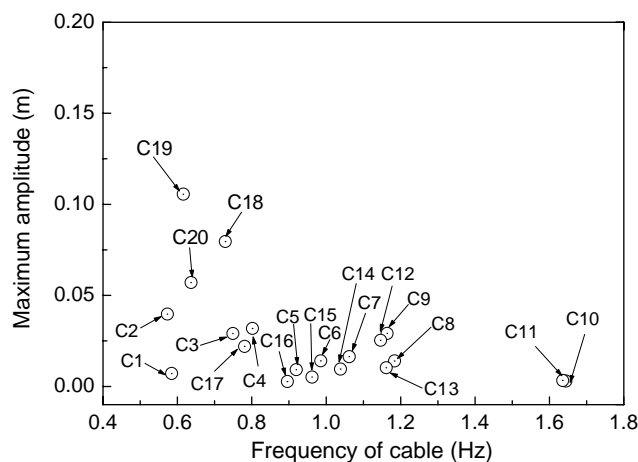


Fig. 5. Maximum amplitudes of the cables under vertical sinusoidal excitation (exciting force = 50 kN, frequency = 0.692 Hz).

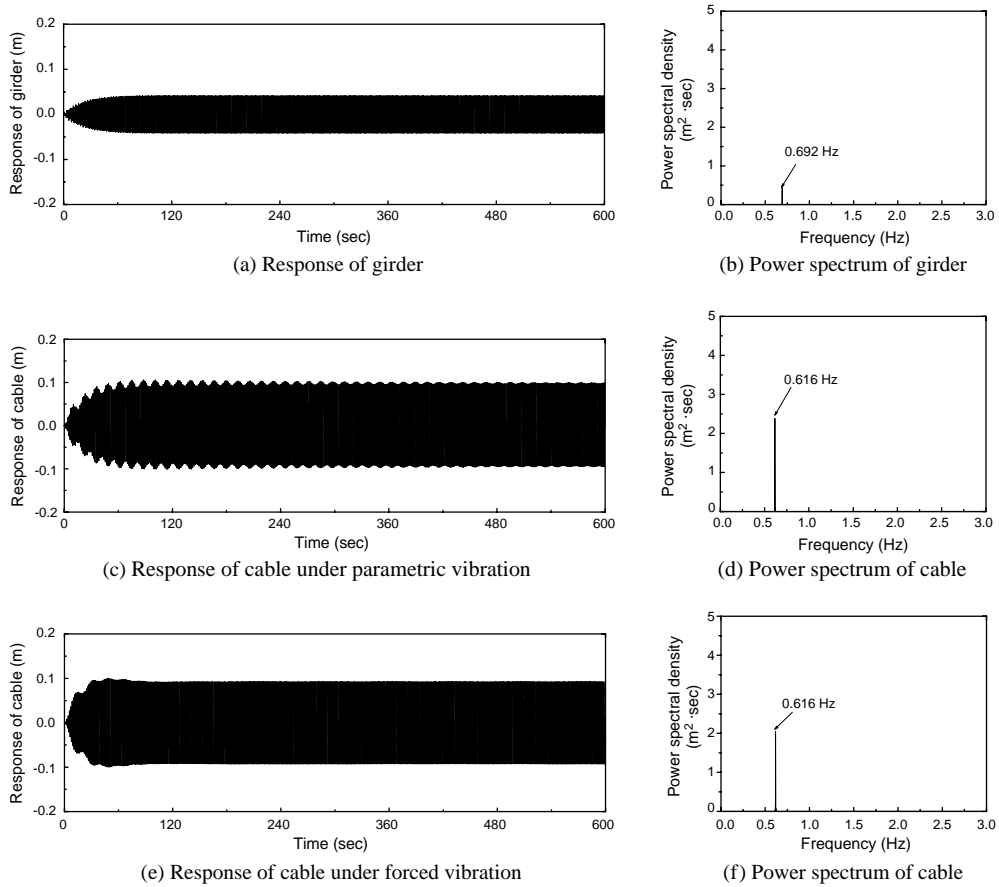


Fig. 6. Time-histories and frequency-domain responses of the girder and cable C19 under vertical sinusoidal excitation (exciting force = 50 kN, frequency = 0.692 Hz).

The cable vibration in the second unstable region has the same property under the frequencies of the 4th vertical mode (0.815 Hz), the 5th vertical mode (0.913 Hz) and the 6th vertical mode (1.124 Hz).

5.2. Torsional–sinusoidal excitation

As shown in Fig. 4, local parametric vibration in the principal unstable region may be expected when the frequency of the bridge is close to the natural frequency of the 6th vertical mode, the 9th vertical mode, the 2nd torsional mode, etc. Analyses are done under sinusoidal excitations of those frequencies. The results are shown in Table 3. Parametric vibration of cable in the principal unstable region occurs only when the excitation frequency is equal to that of the 3rd torsional mode.

Therefore, the following explains the local vibration of cables under the sinusoidal excitation with the frequency of the 3rd torsional global mode.

Table 3
Relation of frequencies and maximum amplitudes between global vibration and related cables

(1) Mode of bridge	(2)Related cable	(1)/(2)	Maximum amplitude	
			Girder (m)	Cable (m)
6th vertical mode	C2	0.573 Hz	1.962	0.0127
9th vertical mode	C17	0.781 Hz	2.005	0.0025
2nd torsional mode	C6	0.986 Hz	2.007	0.0034
3rd torsional mode	C14	1.039 Hz	2.003	0.0096

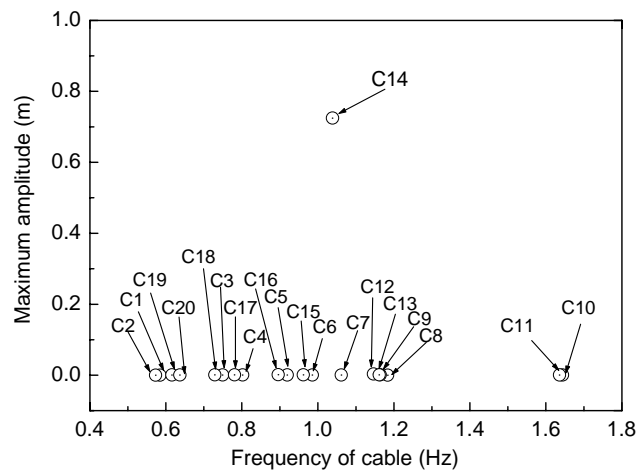


Fig. 7. Maximum amplitudes of the cables under torsional sinusoidal excitation (exciting force = 50 kN, frequency = 2.081 Hz).

Two exciters are installed on each side of the quarter point of the center span and the bridge was shaken in opposite phases with the frequency 2.081 Hz, which is the natural frequency of the 3rd torsional global mode (asymmetrical).

Fig. 7 shows the maximum amplitudes of all cables under torsional–sinusoidal excitation. The response of cable C14 is much larger than those of the other cables. Parametric vibration in the principal unstable region occurs in cable C14 because the first frequency (1.039 Hz) of cable C14 is close to half the frequency of the excitation (2.081 Hz).

Fig. 8 shows the time responses and spectra of the girder and cable C14. A large-amplitude vibration is induced in cable C14 after about 2 min of excitation, while the girder (0.010 m) vibrates steadily. The maximum amplitude of the cable reaches 0.725 m, which is about 70 times greater than that of the girder.

Furthermore, the analysis is carried out under exciting forces of 30 and 40 kN in order to examine the effect of the magnitude of the exciting force. The relationship between the exciting forces and the response of cable C14 is shown in Table 4. Time histories of its response to different exciting forces are shown in Fig. 9. It can be observed that the transient time decreases as the exciting force increases.

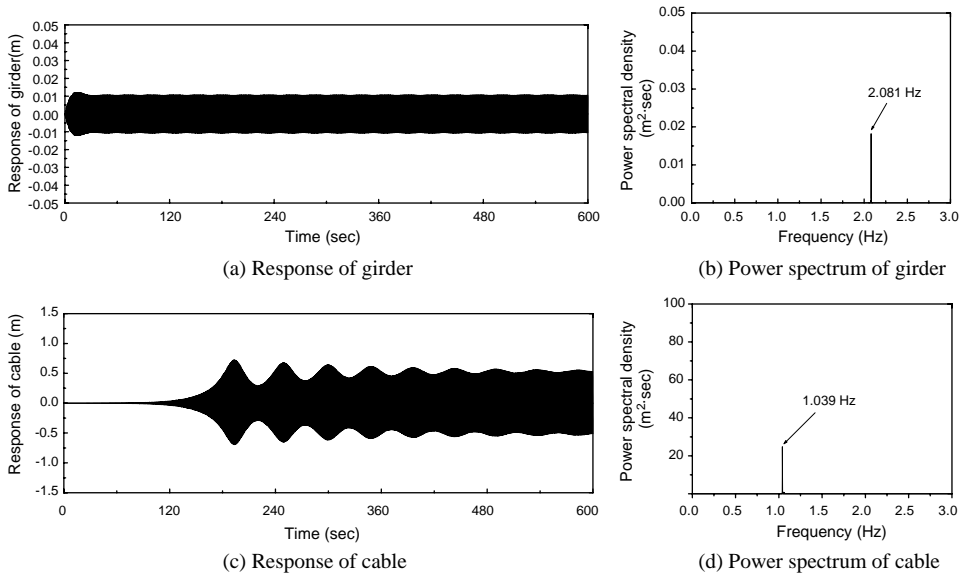


Fig. 8. Time-histories and frequency-domain responses of the girder and cable C14 under torsional sinusoidal excitation (exciting force 50 kN, exciting frequency = 2.081 Hz).

Table 4
Relationship between the exciting force and the response of cable C14

Exciting force (kN)	Exciting torsional moment (kN m)	Maximum amplitude of girder (m)	Maximum amplitude of cable (m)	Time needed to reach maximum amplitude (s)
30	367.5	0.00579	0.607	1223.3
40	490.0	0.00772	0.668	291.1
50	612.5	0.00964	0.725	193.0

The sensitivity of the cable amplitudes to the damping constant is shown in Table 5. The maximum amplitude of the cable decreases and the transient time increases as the damping constant increases. However, the effect of the damping on amplitude is relatively small in the case of parametric vibration in the principal unstable region.

6. Local vibration characteristics of cables under traffic loading

In this section, the local parametric vibrations of cables in the cable-stayed bridge under traffic loading are examined by using the bridge–vehicle–road surface model [18, 19]. The displacement response $y(x, t)$ of the bridge at point x can be expressed as

$$y(x, t) = \phi(x)^T q(t), \tag{5}$$

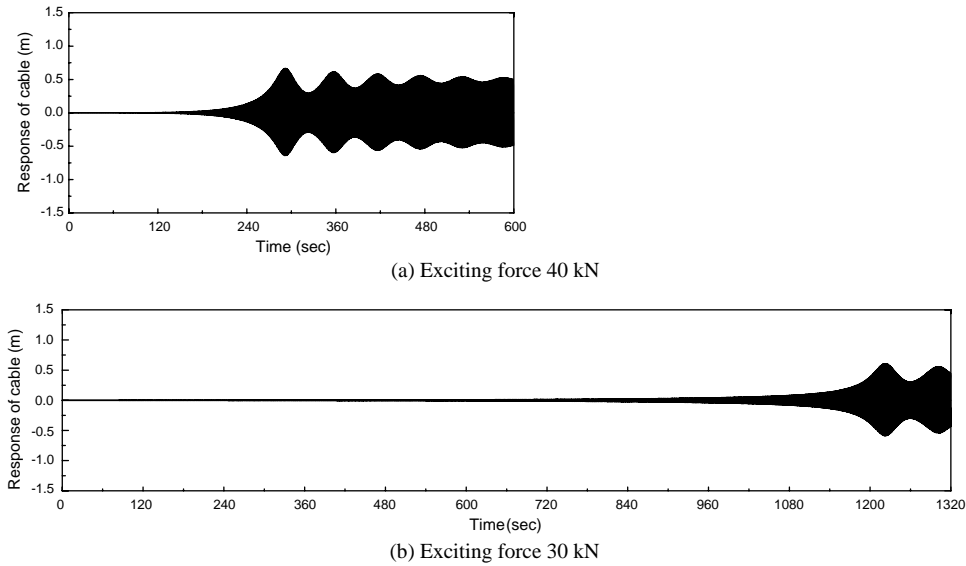


Fig. 9. Effect of the amplitude of the exciting forces on the responses in the principal unstable region of cable C14 (exciting frequency = 2.081 Hz).

Table 5
Relationship between the damping and the response of cable C14 (exciting force 50 kN)

Damping constant	Maximum amplitude of cable (m)	Time needed to reach maximum amplitude (s)
0.001	0.725	193.0
0.002	0.684	229.5
0.003	0.645	280.5
0.004	0.605	365.1
0.005	0.561	527.5

where $\phi(x)^T$ is the modal matrix and $q(t)$ is the generalized co-ordinate to the geometric co-ordinate. The vehicle in the present analysis is a single-degree-of-freedom system as shown in Fig. 10.

Upon applying the modal analysis method, the equation of motion for each mode using the bridge–vehicle–road surface model can be written as

$$\ddot{q}_i(t) + 2h_i\omega_i\dot{q}_i(t) + \omega_i^2q_i(t) = \phi_i^T f_v(t)/m_i, \tag{6}$$

$$\ddot{z}(t) + 2h_0\omega_0\{\dot{z}(t) - \dot{y}(vt, t) - \dot{r}(t)\} + \omega_0^2\{z(t) - y(vt, t) - r(t)\} = 0, \tag{7}$$

$$f_v(t) = -m_s\ddot{z}(t), \tag{8}$$

where $\ddot{q}_i(t)$ is the i th normal co-ordinate, h_i is the damping constant of the i th mode, ω_i is the natural circular frequency of the i th mode, m_i is the generalized mass of the i th mode, m_s is the

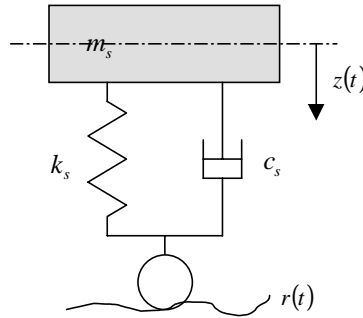


Fig. 10. Model of vehicle.

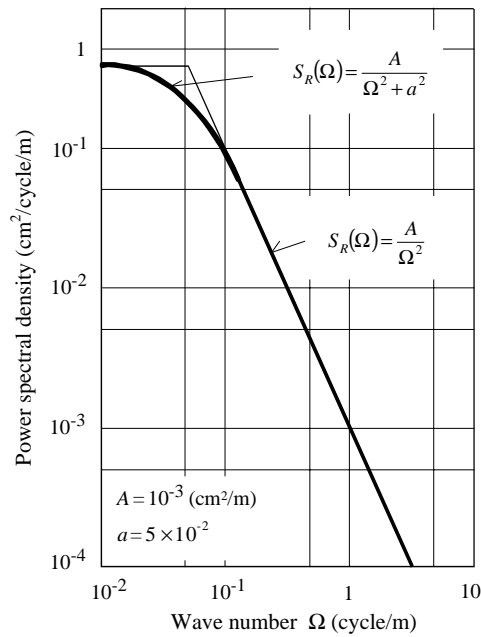


Fig. 11. Power spectrum of road surface roughness.

total mass of the vehicle, $f_v(t)$ is the force from the vehicle, $z(t)$ is the displacement of the vehicle, $r(t)$ is the road roughness, h_0 is the damping constant of the vehicle, ω_0 is the natural circular frequency of the vehicle and v is the speed of the vehicle.

The power spectral density of the road surface roughness, as observed from a expressway in Japan, is shown in Fig. 11 and can be expressed as

$$S_R(\Omega) = \frac{A}{\Omega^2 + a^2}, \tag{9}$$

where Ω is the frequency of the road surface (cycle/m), A is a parameter expressing the level of road surface roughness and a is a parameter based on the observed result. In the present paper, the road surface is assumed to be in the best condition when $A = 0.001$ and $a = 0.05$.

Table 6
Parameters of single-degree-of-freedom vehicle

Weight	m_s (kN)	200
Frequency	f_0 (Hz)	2.60
Damping constant	h_0	0.03

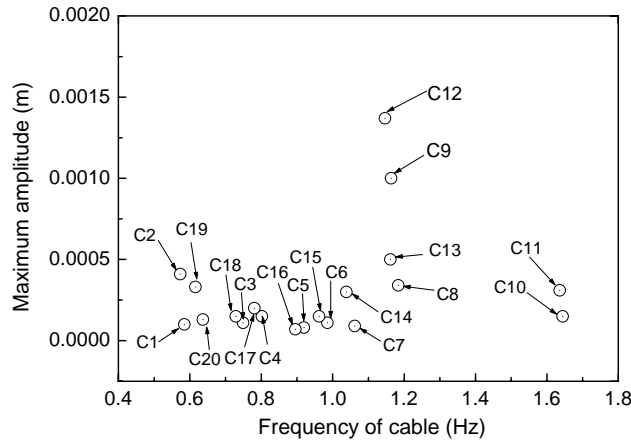


Fig. 12. Maximum amplitudes of cables under a moving vehicle ($V = 30$ km/h).

The response of the bridge under a moving vehicle is obtained by solving Eqs. (6)–(8) with Runge–Kutta method.

The various parameters of the vehicle used in this study are shown in Table 6. The speed of the vehicle is 30 km/h.

Fig. 12 shows the maximum amplitudes of all cables under loading from a traveling vehicle. The maximum responses of cables C9 and C12 are larger than those of the other cables. The natural frequencies of cables C9 and C12 are 1.164 and 1.146 Hz, respectively.

Fig. 13 shows the time responses and spectra of the girder and cable C12. The predominant frequencies of the girder’s response are about 1.50 and 2.45 Hz, while that of cable C12 is 1.146 Hz. It may be assumed that parametric vibration in the principal unstable region is induced in cable C12 because the ratio of the dominant frequency of the girder and that of C12 is approximately 2.0. It can also be observed that the amplitude of the cable is about 3 times as large as that of the girder. However, since the amplitudes of the cables are small, local vibrations will not create any problems.

7. Local vibration characteristics of cables during an earthquake

This section examines the local parametric vibrations of cables in the cable-stayed bridge under the ground motion.

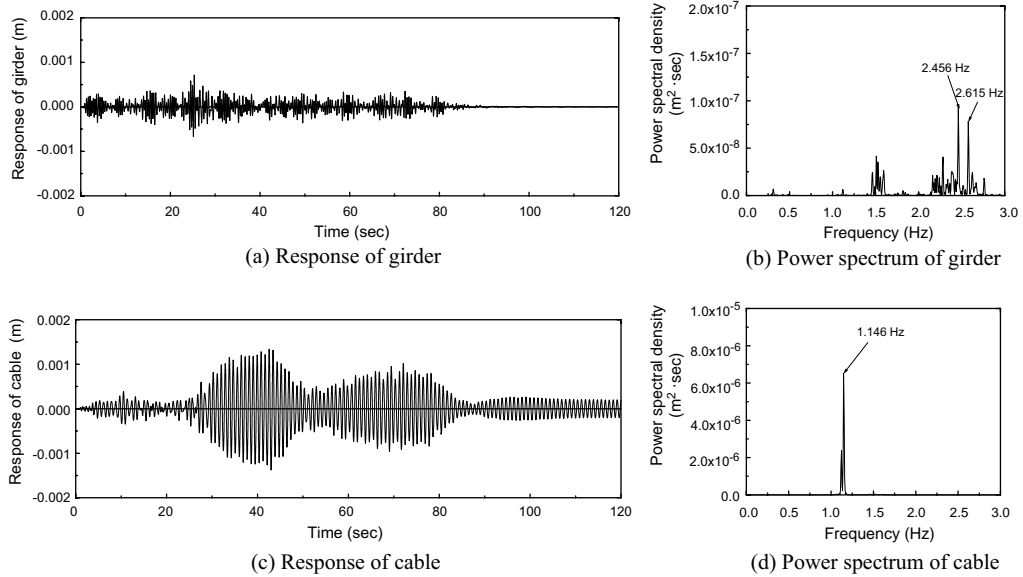


Fig. 13. Time-histories and frequency-domain responses of the girder and cable C12 under a moving vehicle ($V = 30$ km/h).

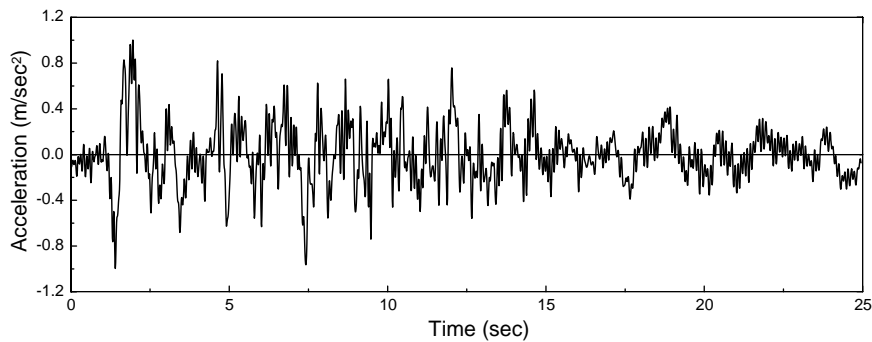


Fig. 14. Input ground motion.

The present analysis uses a moderate ground motion with a maximum acceleration of 1.00 m/s^2 , as shown in Fig. 14 [20]. It is applied in the longitudinal direction of the bridge. The duration time is 25 s. and the time interval of numerical integration is 0.01 s.

Fig. 15 shows the maximum amplitude of all cables in the earthquake. There are no significant differences among the responses of the cables.

Fig. 16 shows the time responses and spectra of the girder and cable C1, the response of which is relatively large. The predominant frequencies of the girder response are the natural frequencies of the global modes, while that of cable C1 under parametric vibration is also the natural

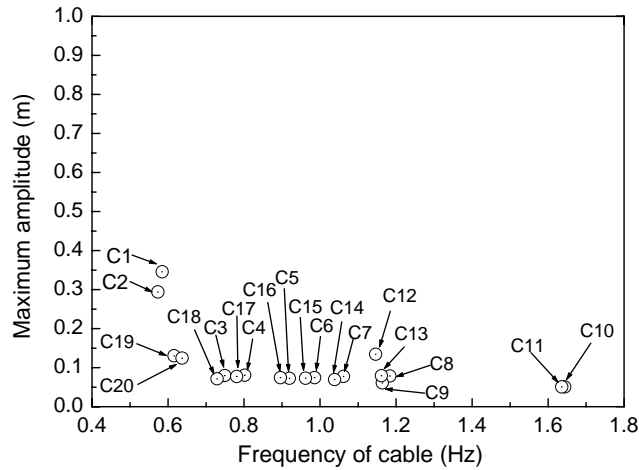


Fig. 15. Maximum amplitudes of cables during the earthquake.

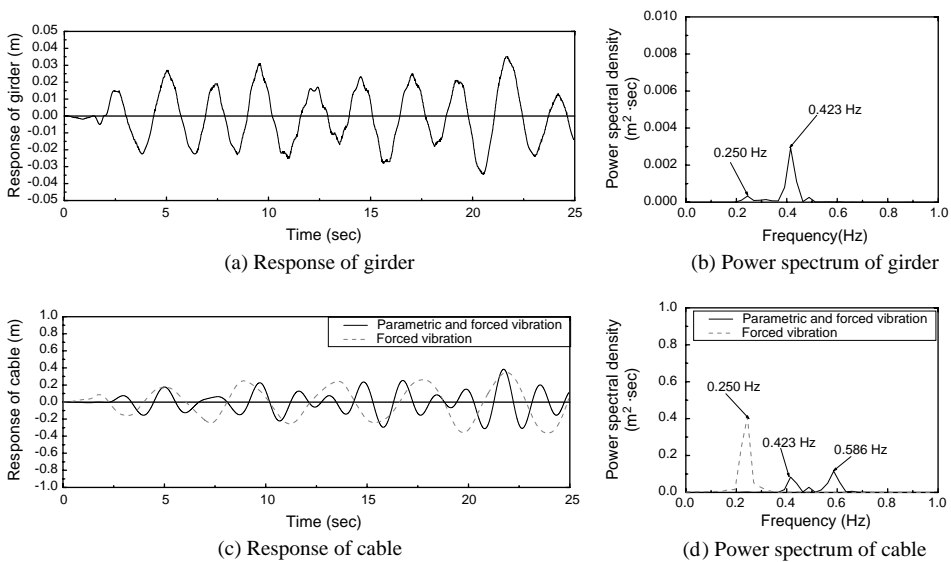


Fig. 16. Time-histories and frequency-domain responses of the girder and cable C1 during the earthquake.

frequency of itself. The waveform of cable C1 under parametric vibration is not accompanied by beating. Therefore, it can be concluded that parametric vibration of the cables does not occur.

8. Concluding remarks

Local parametric vibrations in stay cables of an actual cable-stayed bridge reflecting the vibration characteristics of the bridge subjected to excitations, including sinusoidal excitations, a

traffic loading and an earthquake, are examined and discussed in detail. From the results of the above analysis, the characteristics of local parametric vibrations in the cables of this bridge can be summarized as follows:

- (1) Parametric vibrations in the second unstable region in cables occur under vertical sinusoidal excitation. The amplitude of the cable induced by parametric vibration is of the same order as that induced by forced vibration.
- (2) Parametric vibrations in the principal unstable region in cables occur under torsional sinusoidal excitation, and its amplitudes are large. Parametric vibration in the principal unstable region appears only after a considerable length of time has passed. This length of time depends on the amplitude of the exciting force.
- (3) Parametric vibrations in the principal unstable region in cables occur under traffic loading. However, any local vibration will not produce any problems since the amplitudes of cables are small.
- (4) Parametric vibrations in cables do not occur even if a moderate ground motion with a maximum acceleration of 1.00 m/s^2 is applied in the longitudinal direction of the bridge.

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