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Non-linear coupled slosh dynamics of liquid-filled laminated composite containers: a two dimensional finite element approach

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Abstract

This paper brings into focus some of the interesting effects arising from the non-linear motion of the liquid free surface, due to sloshing, in a partially filled laminated composite container along with the associated coupling due to fluid–structure interaction effects. The finite element method based on two-dimensional fluid and structural elements is used for the numerical simulation of the problem. A numerical scheme is developed on the basis of a mixed Eulerian–Lagrangian approach, with velocity potential as the unknown nodal variable in the fluid domain and displacements as the unknowns in the structure domain. The FE formulation based on Galerkin weighted residual method along with an iterative solution procedure are explained in detail followed by a few numerical examples. Numerical results obtained by the present investigation for the rigid containers are first compared with the existing solutions to validate the code for non-linear sloshing without fluid–structure coupling. Thereafter the computational procedures are advanced to obtain the coupled interaction effect of non-linear sloshing in laminated composite containers.

1. Introduction

Solution of liquid sloshing problem is challenging in the field of mechanics. This fascinating non-linear phenomenon which is characterized by the oscillation of the unrestrained free surface of the liquid in a partially filled container due to external excitation, is a difficult mathematical problem to be solved analytically as well as numerically. This is because, not only the dynamic

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boundary condition at the free surface is non-linear but also the position of the free surface varies with time in a manner not known *a priori*. The problem becomes more complex when the contained structure is flexible, resulting coupled interaction between the container and the contained fluid. In such a situation, neither the fluid domain nor the structural domain can be solved independently of the other due to the unknown interface forces. Such dynamic interaction between a partially filled flexible container and its contained fluid, due to oscillation of the unrestrained fluid free surface, is generally referred as "coupled slosh dynamics", since the interaction which couples the dynamics of the liquid with that of the contained structure is primarily due to sloshing of the fluid.

Coupled slosh dynamics constitute an important class of problems in many engineering disciplines. These problems simultaneously bring together some of the critical aspects associated with both fluid dynamics and structural dynamics. Each of these two areas is complex by itself and when considered together, the situation becomes even more complex. Its resolution is, therefore, of significant practical importance to aerospace, civil, mechanical, nuclear, marine and many other industries. The challenge to solve such a fascinating problem has attracted the attention of engineers, mathematicians, and other scientific researchers over the past few decades. With the development of space technology as an active programme of the present day research, the interest in solving these problems has increased manifold. This interest has further been enhanced due to the advent of newer structural materials such as light weight, high-strength composites. Composites with their high specific strength and stiffness, resistance to corrosion and flexibility in tailoring the design, find a wide range of applications in engineering industries, in the form of vessels, pipes, tanks, etc. Laminated composite tanks containing liquid propellant aboard the modern launch vehicles is one typical example of such applications. It is expected that the liquid propellant tank structures in future space vehicles will be more flexible and lightweight with the use of advanced composites, which can result in significant increase in payload, range and speed, manoeuverability, fuel efficiency and safety. However, in such situations, the dynamical problems will be more complex because of considerable structural flexibility, moving liquid masses and control interactions. The time varying mass properties and varying structural response are expected to generate enormous complexities because of the multidisciplinary interactions.

During the formulation of coupled problems involving interaction of two or more non-linear physical systems, it is frequently assumed that the action of a particular non-linear subsystem can be accurately represented by linearized models. The sloshing of the fluid mass contained within elastic containers is one such subsystem. Although fluid sloshing is intrinsically non-linear, linear representation of the sloshing, whether analytical or experimental are often used to predict the resulting coupled fluid–structure interaction. The literature reports a variety of analytical techniques for formulating such linear slosh models for simple geometries. All linear analyses share a common presumption that the slosh motion of the fluid free surface is much smaller than the dimension of the container. Therefore, a linear coupled fluid–structure model will remain valid so long as either the external disturbances are small or the structure and slosh natural frequencies are well separated. It may not be possible to satisfy both requirements within the constraints of design, even with attachment of slosh management devices such as baffles and stiffeners. It may further be noted that if the disturbance of the free surface is small in magnitude in comparison to the liquid depth and the wavelength, the free surface conditions may be linearized. This has the inherent advantage that the free surface boundary is fixed in time, which simplifies the numerical solution procedure considerably. On the other hand, in the case of a non-linear transient free surface problem, the dynamic boundary condition at the free surface is non-linear and also the position of the free surface varies with time. The non-linearity of boundary conditions and the presence of a moving boundary complicate the numerical algorithm to a large extent. In such a situation, due to time-dependent non-linear boundary conditions, the liquid potential and the position of the free surface are unknown variables, and therefore, both are to be determined as a part of the solution. Therefore, solution of problem relating to liquid sloshing and its coupled interaction with the contained structure is quite challenging in the field of mechanics.

The literature reports a variety of analytical and experimental techniques for evaluating various parameters associated with the sloshing of liquid in rigid and flexible isotropic containers. The importance of non-linear coupling between the liquid and the container was earlier demonstrated by Abramson [1] who suggested for experimental observations at a high level of activity, until a suitable non-linear theory is advanced to account adequately all the associated phenomena. Faltinsen [2] presented analytical results for a two-dimensional rigid rectangular tank considering the non-linear effects of large amplitude liquid motion. Analysis on the non-linear oscillation of an inviscid, incompressible fluid in a two-dimensional, rigid, open container subjected to forced pitching oscillation was carried out by Nakayama and Washizu [3] using the pseudo-variational principle and the finite element method. An incremental method was used in their analysis to account for the non-linearity. Liu and Ma [4] presented a coupled fluid–structure finite element method for the seismic analysis of liquid-filled systems considering the linearised free surface sloshing effect. Washizu et al. [5] obtained the time histories of slosh height of the liquid in a two-dimensional rigid open container subjected to lateral translation taking the free surface non-linearity into account. Three numerical techniques namely, the conventional finite element method based on a variational principle, the boundary element method and the modified fluid-in-cell method were applied for the purpose of analysis. The dynamics of a linear spacecraft model coupled with the non-linear low-gravity slosh of a fluid in a cylindrical tank was investigated by Peterson et al. [6]. Coupled, non-linear equations of motion for the fluid–spacecraft system were derived through an assumed mode Lagrangian method. Through an approximate perturbation solution of the equations of motion, it was shown that the non-linear coupled system response involves fluid–spacecraft model resonances not predicted by either a linear or non-linear uncoupled slosh analysis. Okamoto and Kawahara [7] carried out both numerical and experimental research on the large amplitude sloshing waves in a two-dimensional rigid container. Wu and Taylor [8] proposed two approaches using finite element technique for the analysis of non-linear transient water waves in two-dimensional rigid container. Numerical results were given for the vertical wave maker problem and for a transient wave in a rectangular container. Chen et al. [9] presented an implicit finite difference approach to simulate large amplitude sloshing motion of liquid subjected to harmonic and earthquake base excitations in two-dimensional rigid tanks. Sloshing of liquid in an arbitrary rigid vertical cylindrical tank subjected to both harmonic and irregular base motions was described by Isaacson and Ryu [10]. Initially, the boundary value problem for the case of an inviscid fluid and a harmonic base motion was solved on the basis of linearized potential flow theory. The solution was based on an eigenfunction expansion method in which a boundary integral equation involving a suitable Green function was solved for each mode. The solution was then extended to the case of energy dissipation of a real fluid, by assuming this to occur at the free surface. Extensions to a stationary random base motion, to the time domain response to a specified base acceleration record and to the estimation of maximum forces using a modal analysis and involving earthquake response spectra are then indicated. Koh et al. [11] analyzed the fluid–structure interaction effects in a liquid-filled rectangular isotropic container employing a variationally coupled BEM–FEM approach with linearized free surface boundary conditions. In a recent investigation Pal et al. [12] solved the coupled slosh dynamic problems in liquid-filled laminated composite containers employing free surface linearized boundary conditions.

From a critical review of literature, it is thus observed that most of the studies on the liquid-filled containers are concerned with rigid tanks. Studies on the sloshing effect of liquid in flexible containers are limited to those made of isotropic materials. The structural flexibility and the free surface slosh effects are not properly addressed in those studies. To the best knowledge of the present authors, analytical or numerical solutions of large amplitude sloshing problems in partially filled flexible laminated composite containers with associated coupled interaction are not available in the open literature.

Acknowledging the above facts, an attempt is made in this investigation to study the non-linear free surface oscillation of the liquid inside elastic containers using finite element technique. The present numerical scheme is developed on the basis of a mixed Eulerian–Lagrangian approach, with velocity potential as the unknown nodal variable in the fluid domain and displacements as the unknowns in the structure domain. Firstly, the problem is defined as a non-linear initial boundary value problem by the use of governing differential equation and boundary conditions based on a mixed Eulerian–Lagrangian approach. Next the problem is formulated using Galerkin's principle, which provides the basis for the present discretization. A finite difference based iterative time stepping technique is employed to advance the solution in the time domain. From the solution of the velocity potential and velocity vector at a given time step at a given iteration, the resulting values of the hydrodynamic pressure in the fluid domain and the updated position of the free surface at the start of the next iteration are obtained. A new finite element mesh is then generated, corresponding to the updated geometry and the above procedures are repeated in the subsequent time steps. During each such operation within a time step, the finite element equations of motion for both the structure and fluid domain are numerically integrated in an iterative manner using Newmark's predictor-multicorrector algorithm [13] to obtain the coupled response of the container.

2. Finite element formulation for fluid domain

The present formulation is first described in detail for a two-dimensional rectangular container. The solution methodology is then extended for solving liquid-filled axisymmetric containers which are special cases of 3-D containers with fluid elements on 2-D planes. Fig. 1 shows the problem geometry for a partially liquid-filled rectangular container. The Cartesian co-ordinate system $O-XZ$ is defined with center of the base being the origin and Z points vertically upwards. For an axisymmetric container, the fluid elements are in the $R-Z$ plane, where Z is the axis of symmetry and R is the radial co-ordinate axis. The depth of the contained liquid is h_l. The contained liquid is assumed to be incompressible and inviscid resulting in an irrotational flow field. The base of the

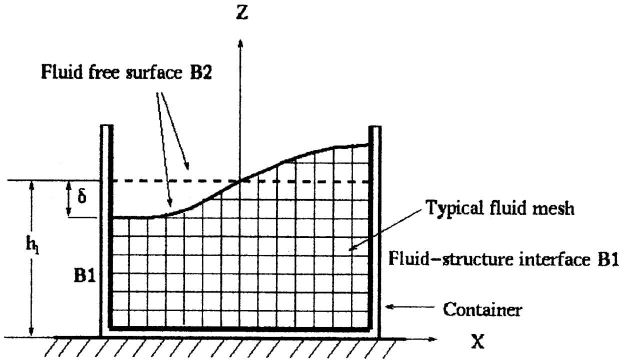


Fig. 1. Problem geometry of fluid-filled rectangular container.

container is assumed to be rigid. The container is subjected to an arbitrary horizontal base excitation.

The equation governing the liquid motion under an arbitrary horizontal base excitation may be expressed in terms of relative Cartesian co-ordinate (x, z) and the relative velocity potential ϕ as follows:

$$\nabla^2\phi = 0 \tag{1}$$

with $\phi = \phi(x, z, t)$ is satisfied in the fluid domain Ω

At the fluid–structure interface boundary B_1

$$\frac{\partial\phi}{\partial n} = v_n, \tag{2}$$

where $\partial/\partial n$ denotes differentiation in the direction normal to the surface of the structure boundary in contact with the fluid and v_n is the common velocity of the fluid and boundary surface in the direction normal to the surface.

$$\text{In the case of rigid walls}\,\frac{\partial\phi}{\partial x} = v_n\,\text{at the wall boundaries} \tag{3}$$

and

$$\frac{\partial\phi}{\partial z} = 0\ \text{at}\ z = 0\ \text{(i.e. at the rigid tank base).} \tag{4}$$

The kinematic and dynamic boundary conditions on the free surface B_2 are

$$\frac{\partial\delta}{\partial t} + \frac{\partial\phi}{\partial x}\frac{\partial\delta}{\partial x} - \frac{\partial\phi}{\partial z} = 0 \tag{5}$$

and

$$\frac{\partial\phi}{\partial t} + \tfrac{1}{2}(\nabla\phi)^2 + x\ddot{x}_g + g\delta = 0. \tag{6}$$

Here $\ddot{x}_g$ is the horizontal ground acceleration and δ is the free surface displacement.

Noting that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\partial \phi}{\partial x} \text{ and } \frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial \phi}{\partial z}, \tag{7}$$

Eq. (6) may be written in the Lagrangian form as

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = g\delta - \tfrac{1}{2}(\nabla\phi)^2 + x\ddot{x}_g. \tag{8}$$

The hydrodynamic pressure of the seismically excited liquid may be obtained from Bernoulli's equation as

$$\frac{p(x,z,t)}{\rho_f} = -\left(\frac{\partial \phi}{\partial t} + \tfrac{1}{2}(\nabla\phi)^2 + x\ddot{x}_g + g\delta\right). \tag{9}$$

Here ρ_f is the fluid density. Eqs. (1)–(6) define the initial and the boundary value problem which are the Laplace equation with non-linear boundary conditions imposed on the free surface. Here the non-linearity manifests itself in two ways :

(i) The elevation of the moving free surface is not known a priori at any given time instant.
(ii) The boundary conditions on the free surface (i.e.,Eqs. (5) and (6)) contain second order differential terms.

The above formulation shows that the problem at each time step is of the mixed Neuman and Dirichlet type. The mathematical formulation for the present case is similar to those reported by Wu and Taylor [8]. To obtain a numerical solution, the following functional is to be considered:

$$\Pi = \frac{1}{2}\int_{\Omega} \nabla\phi\nabla\phi \,\mathrm{d}\Omega - \int_{B_1} \phi v_n \,\mathrm{d}s - \int_{B_2} \frac{\partial \phi}{\partial n}(\phi - f_1)\,\mathrm{d}s, \tag{10}$$

where f_1 is the potential on the free surface at any instant of time t.

The variational statement

$$\delta\Pi = 0 \tag{11}$$

yields

$$\begin{aligned}
&\int_{\Omega} \nabla\delta\phi\nabla\phi \,\mathrm{d}\Omega - \int_{B_1} \delta\phi v_n \,\mathrm{d}s - \int_{B_2} \frac{\partial \delta\phi}{\partial n}(\phi - f_1)\,\mathrm{d}s - \int_{B_2} \frac{\partial \phi}{\partial n}\delta\phi \,\mathrm{d}s \\
&\quad = \int_{\Omega} \nabla(\delta\phi\nabla\phi)\,\mathrm{d}\Omega - \int_{\Omega} \delta\phi\nabla^2\phi \,\mathrm{d}\Omega - \int_{B_1} \delta\phi v_n \,\mathrm{d}s - \int_{B_2} \frac{\partial \delta\phi}{\partial n}(\phi - f_1)\,\mathrm{d}s - \int_{\mathrm{B}_2} \frac{\partial \phi}{\partial n}\,\delta\phi \,\mathrm{d}s \\
&\quad = -\int_{\Omega} \delta\phi\nabla^2\phi \,\mathrm{d}\Omega + \int_{B_1} \delta\phi\left(\frac{\partial \phi}{\partial n} - v_n\right)\mathrm{d}s - \int_{B_2} \frac{\partial \delta\phi}{\partial n}(\phi - f_1)\,\mathrm{d}s = 0.
\end{aligned} \tag{12}$$

Now discretizing the fluid domain into finite number of quadrilateral elements, the nodal values of the velocity potential may be written as

$$\phi = \sum_{1}^{n} N_j(x,y,z)\phi_j(t) \tag{13}$$

in which $N_j\ (x, z)$ are the shape functions, $\phi_j(t)$ are the time-dependent nodal values of the velocity potential ϕ and n is the number of nodes.

Substituting Eq. (13) into Eq. (10) and then using Eq. (11) leads to the following:

$$\int_\Omega \nabla N_i \sum_{j=1}^{n} \phi_j \nabla N_j \,\mathrm{d}\Omega - \int_{B_1} N_i \frac{\partial \phi}{\partial n}\,\mathrm{d}s - \int_{B_2} \frac{\partial N_i}{\partial n}\left(\sum_{j=1}^{n} \phi_j N_j - f_1\right)\mathrm{d}s$$
$$- \int_{B_2} N_i \sum_{j=1}^{n} \phi_j \frac{\partial N_j}{\partial n}\,\mathrm{d}s = 0. \tag{14}$$

Eq. (14) may be written in matrix form as

$$[A]\{\phi\} = [B], \tag{15}$$

where

$$A_{(ij)} = \int_\Omega \nabla N_i \nabla N_j \,\mathrm{d}\Omega - \int_{B_2} \frac{\partial (N_i N_j)}{\partial n}\,\mathrm{d}s, \tag{16}$$

$$B_{(i)} = \int_{B_1} N_i v_n \,\mathrm{d}s - \int_{B_2} \frac{\partial N_i}{\partial n} f_1 \,\mathrm{d}s. \tag{17}$$

The velocity potential in the domain may be obtained from Eq. (15). It is to be noted that the non-linear free surface boundary conditions are dependent on the velocity parameters. Although it is convenient to obtain the velocity by differentiating the shape functions with respect to the co-ordinates, the shape functions usually do not guarantee the continuity of its derivatives at the boundaries of the elements. On the other hand accurate prediction of the velocity is essential to avoid excessive accumulated error in the time stepping procedure. To accomplish the same the following formulation is adopted.

The velocity vector $\bar{U} = u_i + v_j$ may be written in terms of element shape functions as

$$\bar{U} = \sum_{j=1}^{n} U_j N_j(x, y, z). \tag{18}$$

To impose the relationship $\nabla\phi = \bar{U}$, the Galerkin method is used to approximate it in the form

$$\int_\Omega N_i(\nabla\phi - \bar{U})\,\mathrm{d}\Omega = 0. \tag{19}$$

Eq. (19) may be modified using Eq. (13) as

$$\int_\Omega N_i B_j \phi_j \,\mathrm{d}\Omega = \int_\Omega N_i N_j \bar{U}_j \,\mathrm{d}\Omega. \tag{20}$$

In matrix form this may be written as

$$[C]\{u\} = [D_1]\{\phi\}, \tag{21}$$

$$[C]\{v\} = [D_2]\{\phi\}, \tag{22}$$

where

$$[C] = \int_{\Omega} N_i N_j \mathrm{d}\Omega,$$

$$[D_1] = \int_{\Omega} N_i \frac{\partial N_j}{\partial x} N_j \,\mathrm{d}\Omega,$$

$$[D_2] = \int_{\Omega} N_i \frac{\partial N_j}{\partial z} N_j \,\mathrm{d}\Omega.$$

Here u_i and v_i are the components of the velocity vector $\bar{U}_j$ at node "j". From the resulting values of velocity, the updated position of the free surface at the start of the next time step may be obtained using Eq. (7) and the updated velocity potential on the free surface is obtained from Eq. (8). A new finite element mesh is generated corresponding to the updated geometry.

3. Finite element formulation for structure domain

The container (structure domain) is modelled as a flexible thin walled laminated composite shell-of revolution using 2-noded conical shell finite elements with four degrees of freedom (d.o.f.) per node. The d.o.f.s correspond to meridional displacement (u), circumferential displacement (v), radial displacement (w) and meridional rotation. The detail constitutive equations and finite element formulation for the container are presented earlier by Pal et al. [12]. The same is not described here for the sake of brevity.

The discretized form of the governing equation of motion for the liquid–container system may be written as

$$[M_s]\{\ddot{d}\} + [K_s]\{d\} = \{R\}, \tag{23}$$

where $[M_s]$ and $[K_s]$ are the structural mass and sfiffness matrices for the container. These matrices are symmetrical and highly banded. Vectors $\{\ddot{d}\}$ and $\{d\}$ are the generalized nodal accelerations and displacements of the container wall, respectively. $\{R\}$ is the generalized force vector and for the coupled liquid–elastic system this may be written as

$$\{R\} = \{F_e\} + \{P\}, \tag{24}$$

where $\{F_e\}$ represents the external nodal forces and $\{P\}$ represents the nodal forces exerted on the container wall due to the pressure arising from the oscillation of the liquid.

4. Solution procedure

The most important aspect in the non-linear model is the treatment of the moving boundary. In the present investigation Lagrangian method is used to trace the moving boundary. In the Lagrangian approach, the mesh moves with the fluid, so that element distortions are likely to occur. To restructure a distorted Lagangian mesh a smoothing procedure is needed. This is explained in the following sections.

4.1. Time marching scheme

A finite difference based iterative time stepping technique such as

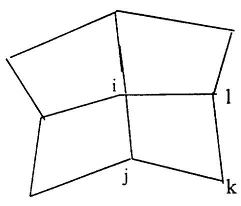
$$
\begin{aligned}
x|_{t+\Delta t} &= x|_t + \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_t \Delta t,\\
z|_{t+\Delta t} &= z|_t + \left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)_t \Delta t,\\
\phi|_{t+\Delta t} &= \phi|_t + \left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)_t \Delta t
\end{aligned}
\tag{25}
$$

is employed to advance the solution in the time domain. Once the solution at time "t_0" has been found, the potential on the new free surface profile at subsequent time $(t_0 + \Delta t)$ may be obtained using Eqs. (7) and (8). This provides the free surface condition for the next time step and the problem may be solved again.

From the solution of the velocity potential and velocity vector at a given time step at a given iteration, the resulting values of the hydrodynamic pressure in the fluid domain and the updated position of the free surface at the start of the next iteration are obtained. A new finite element mesh is then generated, corresponding to the updated geometry and the above procedures are repeated in the subsequent time steps.

4.2. Mesh smoothing

The new set of quadrilateral finite elements produced by the mesh updating procedure described in the previous section are not always well shaped, particularly in the regions where the element size is varying sharply. Mesh smoothing and mesh modification techniques are employed to improve the quality of the quadrilateral elements. In the process of mesh smoothing, the connectivities of the elements and nodes are fixed, but the nodes are repositioned to produce quadrilaterals with somewhat improved shapes. The technique that is used in the present investigation is similar to the well-known Laplacian smoothing [14] which repositions the nodes such that each internal node is at the centroid of the polygon formed by its surrounding elements. The new position of an internal node "i" is computed as



$$
x_i' = \frac{1}{4M}\sum_{a=1}^{M}(x_j + 2x_k + x_l)_a, \qquad z_i' = \frac{1}{4M}\sum_{a=1}^{M}(z_j + 2z_k + z_l)_a, \tag{26}
$$

where M is the number of elements sharing node "i". After having relocated all of the internal nodes, the element shapes are checked to see if all interior angles of every element are between 20° and 160°. If not, the nodes are repositioned according to Eq. (26) till the required convergence is satisfied. The elemental nodes i, j, k, l are named in an anticlockwise manner as per the numbering of elemental shape functions.

4.3. Coupled interaction

A new finite element mesh is then generated, corresponding to the updated geometry and the above procedures presented earlier are repeated in the subsequent time steps. During each such operation within a time step, the finite element equations of motion for both the structure and fluid domain are numerically integrated using Newmark's predictor-multicorrector algorithm [13]. Then the interaction effect is studied by transferring structure normal acceleration to the fluid domain and the fluid pressure to the structure domain at the fluid–structure interface in an iterative manner till a desired level of convergence is achieved simultaneously, for both the pressure in the fluid domain and the displacement of the structural wall. Thus, at any time t, both the hydrodynamic pressure p_i and the structural displacement d_i are iterated till

$$\left|\frac{p_{i+1_t} - p_{i_t}}{p_{i_t}}\right| \leqslant \varepsilon \quad \text{and} \quad \left|\frac{d_{i+1_t} - d_{i_t}}{d_{i_t}}\right| \leqslant \varepsilon$$

is achieved simultaneously, where "i" being the number of iteration and ε being any small preassigned value.

Numerical results obtained by the present investigation for the rigid containers are first compared with the existing solutions to validate the code for non-linear sloshing without fluid–structure coupling. Thereafter the computational procedures are advanced to obtain the coupled interaction effect of non-linear sloshing in elastic containers. Simulation results of liquid sloshing induced by earthquake and harmonic base excitations are compared with those of the linear theory and the limitation of the latter in assessing the non-linear coupled response of flexible container is addressed.

5. Numerical results

Example 1. *Slosh response — forced lateral oscillation.*

To validate the present computational algorithm, the transient non-linear problem of a liquid-filled rectangular container in 2-D is considered. The similar problem is solved by Washizu et al. [5] using the BEM approach and is available in the literature. The container is having a base width of 1.8 m and is filled with water up to a depth of 0.6 m. It is subjected to a sinusoidal forced horizontal acceleration of the type given as

$$a_x(t) = -X_0\omega^2 \sin \omega t \quad \text{for } t \geqslant 0, \tag{27}$$

where X_0 and ω are the amplitude and the frequency of the forced horizontal displacement, respectively. The parameters X_0 and ω are 0.002 m and 5.5 rad/s, respectively.

Fig. 2 shows the finite element mesh profile of the fluid domain at different time steps before applying the mesh smoothing technique. The dashed lines represent the original grid and the dotted lines the deform grid. The severity of the mesh distortion may be observed after 6.0 s. This

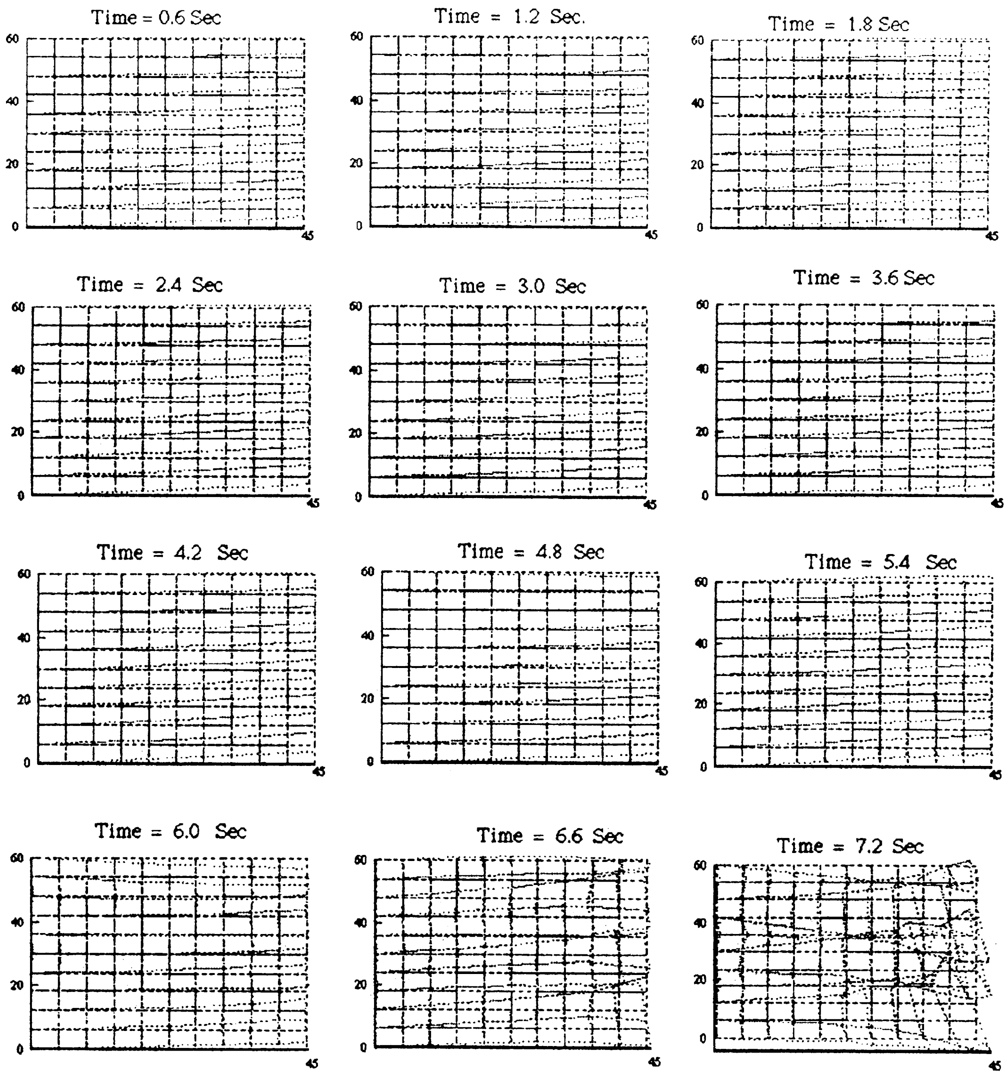


Fig. 2. Finite element mesh profile of the fluid domain at different time steps.

kind of distortions leads to numerical instability. To overcome this problem a smoothing technique (node repositioning) is adopted. After smoothing and remapping the mesh at each iteration of the time integration, the numerical computations were advanced to obtain the required results. The result is shown in Fig. 3 which shows the time history of the free surface displacement at the outer wall of the container for both linear and non-linear free surface boundary conditions. The present numerical results compare well with the BEM solution of Washizu et al. [5]. The velocity vector in the fluid domain is evaluated at each time step for establishing the new position of the nodes. Thus at any time step, the velocity vector at all the fluid nodes can be plotted to see the distribution of velocity inside the fluid domian. A typical case has been shown for two different time steps in Fig. 4.

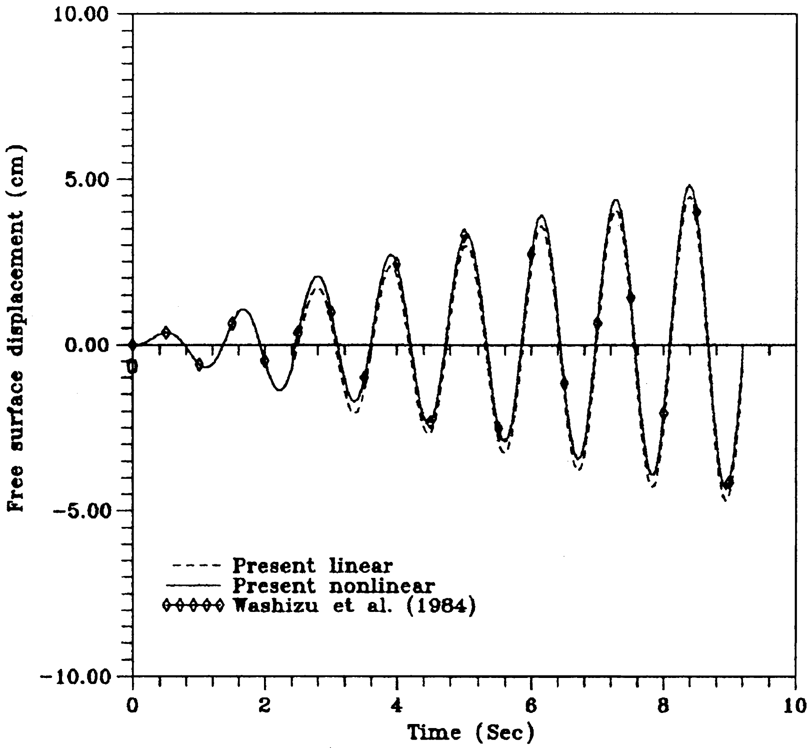


Fig. 3. Time history of the free surface displacement at the outer wall of a rectangular tank due to forced sinusoidal base excitation.

Example 2. *Slosh response — forced pitching oscillation.*

Next the liquid motion in a container, subjected to forced pitching oscillation is addressed. The free surface time history of the liquid in a two-dimensional rectangular container, which is forced to pitch sinusoidally about the origin with an angular velocity is shown in Fig. 5. It is to be emphasized that in this example, the present FEM results are compared with those of Nakayama et al. [3], who had earlier solved the problem using the non-linear concept. The container is assumed to be subjected to the following forced sinusoidal pitching oscillation $\theta(t)$ about the origin O in Fig. 1:

$$\theta(t) = \Theta \cos \omega t \quad \text{for} \quad t \geqslant 0, \tag{28}$$

where Θ and ω are the amplitude and the frequency of the forced oscillation, respectively. For the present problem their value are $\Theta = 0.8^\circ$ and $\omega = 5.5\,\text{rad/s}$. The present numerical result for both linear and non-linear free surface boundary conditions are shown in the Fig. 5. It may be observed that there is a small upward shifting of the amplitude for the non-linear solution. The trend of the present results agree well with those of Nakayama et al. [3] in respect of the amplitude and periodicity of the response.

Example 3. *Slosh response –arbitrary base excitation.*

The seismic ground motion records usually have high level of excitation ($>$0–2 g) and the motion is non-harmonic. Numerical instabilities may occur at different stages of a sloshing

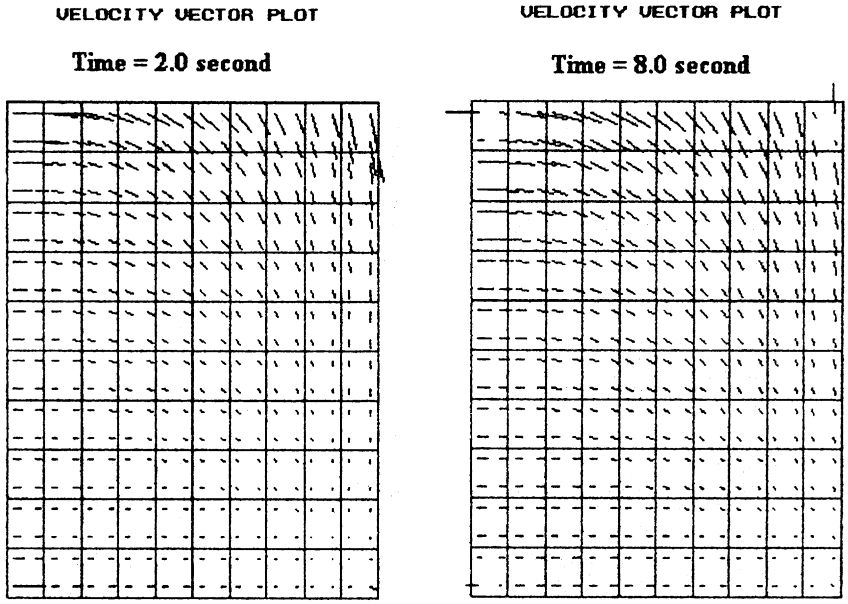


Fig. 4. Velocity vector plot at different time steps for the liquid in a rectangular container under sinusoidal lateral base excitation.

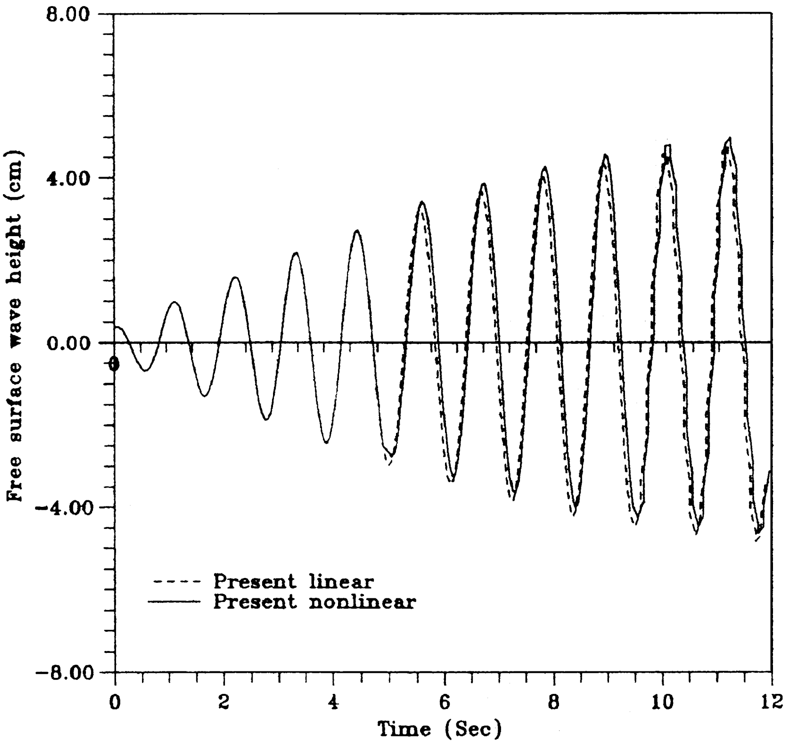


Fig. 5. Time history of the free surface displacement near the wall of a rectangular container due to forced pitching oscillation.

simulation, depending on the algorithm used in discretizing the wave equation on the free surface which range from excessive computational errors to the conditional stability of the algorithm involved. The latter leads to the restriction on grid resolution and the reduction in time steps. In an attempt to investigate the non-linear slosh response in rigid containers due to seismic excitation, the behaviour of the liquid motion in an annular cylindrical container (4.0 m outside diameter, 2.0 m inside diameter and depth of water 1.0 m), subjected to arbitrary base excitation is solved. The S90W component of EI Centro accelerogram of 1940 Imperial Valley Earthquake records (shown in Fig. 6), is used as input ground acceleration into horizontal direction. The fluid domain is discretized in to 20×20 mesh of four noded quadrilateral elements. The problem is solved using both the linear and non-linear model. Fig. 7 illustrates the present numerical results for both the cases. It may be observed from this figure that the non-linear sloshing amplitude have an upward shifting about the time axis and the growth of the positive amplitude is much faster than that of the negative amplitude. This may be due to the fact that the presence of the surface wave considered in the non-linear analysis, enhances the generated slosh displacement more towords positive side, due to random nature of external excitation. The behaviours of the linear and non-linear sloshing response are seen to be significantly different. A time period elongation is also observed in case of non-linear slosh response.

Example 4. *Coupled response – liquid filled isotropic container*.

The dynamic coupled response of a simply supported, liquid-filled isotropic circular cylindrical container, with an inner rigid wall and outer flexible wall, under a non-axisymmetric horizontal base acceleration of $\ddot{d} = \sin 500\,t$ is considered in this example. This problem has earlier been solved by Ball et al. [15] with linearized free surface boundary condition and using series solution concept for the fluid domain and the finite element modelling for the shell. This sample problem on fluid structure interaction has been chosen to establish the efficacy of the present methodology, where in, finite elements are used to model both the fluid and structural medium with inclusion of fluid free surface non-linearity effects. The parameters considered for this problem are:

Outside radius of the container: 2.54 m (100.0 in),
Inside radius of the container: 1.27 m (50.0 in),
Thickness of the container wall: 0.0254 m (1.0 in),
Height of the liquid-filled container: 2.54 m (100.0 in),
Elastic modulus $E_{11} = E_{22} = 6.89 \times 10^7$ kPa (10^7 psi),
Mass density of conatiner material $\rho_s = 2770.5\,\mathrm{kg/m^3}$ ($0.1\,\mathrm{lb/in^3}$),
The fluid inside the tank is water with density $\rho_f = 1000\,\mathrm{kg/m^3}$.

The effect of free surface non-linearity on the mid-span radial displacement of the container is shown in Fig. 8. The dynamic response have been obtained using a time step of 0.0005 s for comparison with the results of the linear solution available in the source mentioned above. The trend of the present linear result obtained with a pressure-free liquid surface boundary condition compares well with the published results in respect of the periodicity and amplitude of response. However, a marked difference between the present linear and non-linear response is observed. The response amplitude in the later case shows an upward shift about the time axis with time period elongation. The response appears to be of typical beating in nature, since the forcing function

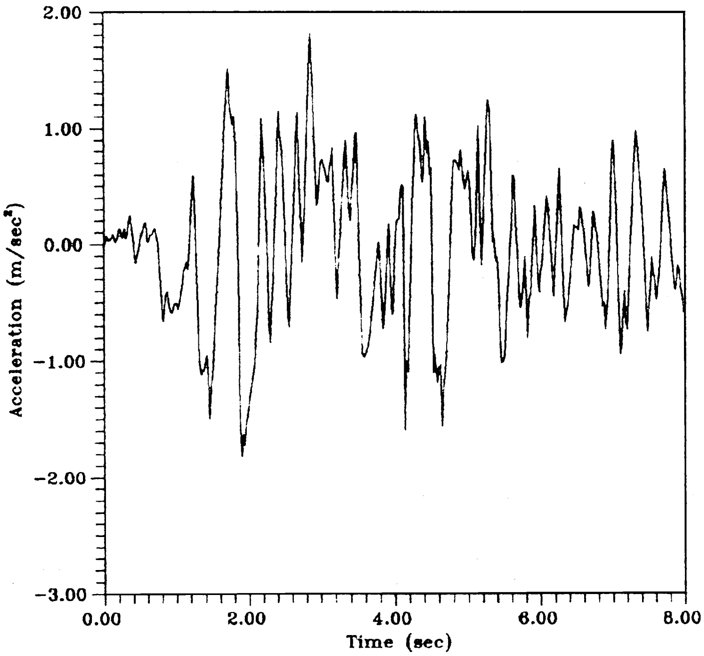


Fig. 6. Time history of the free surface displacement near the outer wall of an annular cylindrical container due to earthquake base excitation.

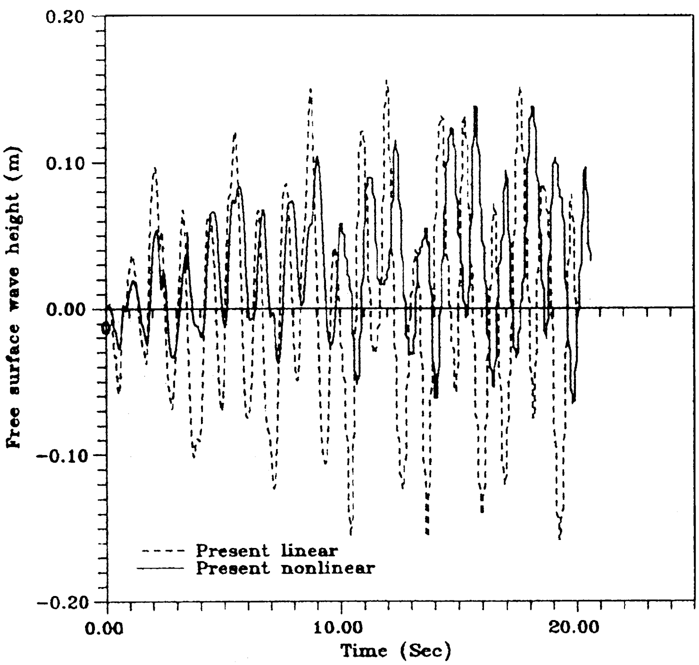


Fig. 7. S90W component of El Centro accelerogram: 1940 Imperial Valley earthquake.

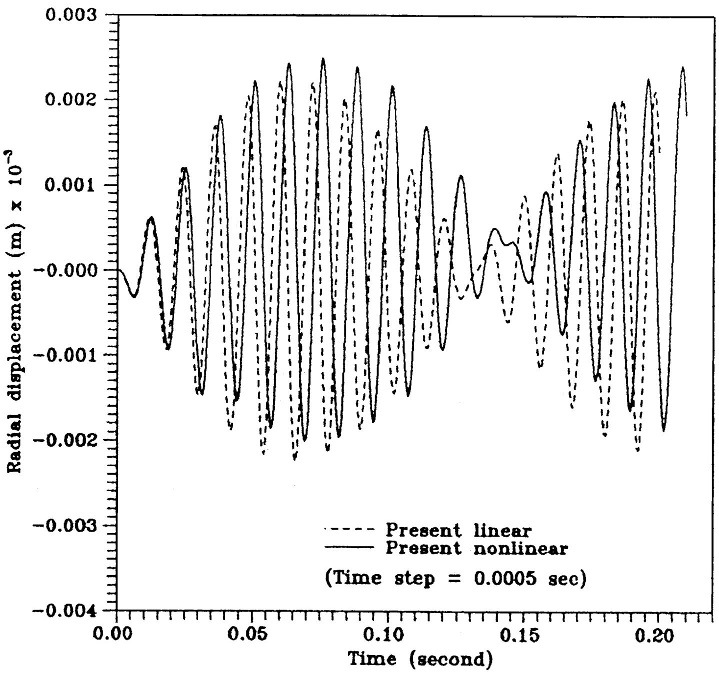


Fig. 8. Effect of free surface non-linearity on the mid-span radial deflection of the fluid-filled isotropic cylindrical tank under non-axisymmetric horizontal base acceleration.

frequency (500 rad/s) is close to the natural frequency of the liquid-tank system (474.5 rad/s, calculated earlier by Ball et al. [15]).

Example 5. *Coupled response – liquid filled laminated composite container.*

Coupled response analysis of a liquid-filled composite cylindrical container is presented in this example. The geometrical and material parameters considered for this problem are the same as that of Wung [16], except for the length and thickness, which have been taken as 0.5207 m (20.5 in) and 0.001 m (0.03937 in), respectively. The container is assumed to be fixed at the base and free at the top and is filled with water up to 0. 508 m (20.0 in). The other parameters are:

$E_{11} = 52.0$ GPa (7.5×10^6 psi),
$E_{22} = 13.80$ GPa (2.0×10^6 psi),
$G_{12} = G_{13} = G_{23} = 8.6$ GPa (1.25×10^6 psi),
$\nu_{12} = \nu_{21} = 0.25$,
$\rho_s = 10.09 \times 10^6$ N s^2/m^4 (1.0 lb s^2/in^4),
$\rho_f = 1000$ kg/m^3,
Radius = 0.508 m (20.0 in),
Laminations [0°/90°] and [45°/ − 45°].

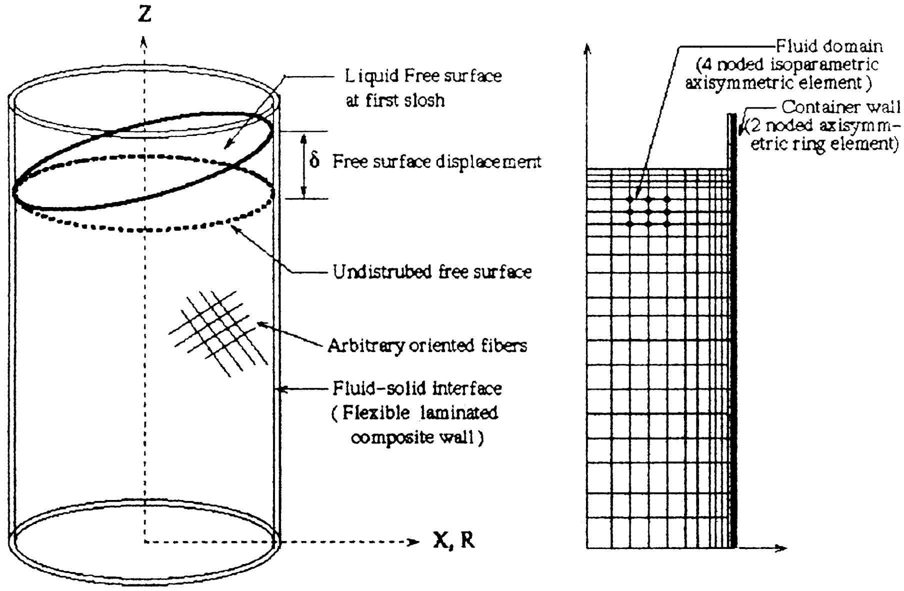


Fig. 9. Problem geometry and typical finite element discretization of liquid-filled laminated composite container.

It is subjected to a sinusoidal base acceleration, with unit amplitude and frequency of 0.9237 Hz, corresponding to containers first slosh mode. The problem geometry and finite element discretization for the container and the contained liquid is illustrated in Fig. 9. The discretization of the appropriate domains have been made on the basis of convergence studies made earlier [12] during the solution of structure dynamics and fluid transient problems. Accordingly, the fluid domain has been discretized with 10×20 mesh (i.e., 200 fluid elements) and the cylindrical tank with 20 shell elements. A finer mesh has been taken near the free surface, where sloshing effects are more pronounced. A similar discretization has also been made nearer to the wall where interaction effects are dominant. This flexible liquid-filled tank is first solved along with its corresponding rigid model to examine the effect of tank flexibility on the sloshing response. This is illustrated in Fig. 10, where the time history of the free surface slosh displacement near the tank wall is plotted. A significant change in the amplitude of sloshing response of the composite and its corresponding rigid tank is observed in this case. However, effects of two different fibre orientations considered in this example, do not seem to induce any significant influence on the results of free surface slosh wave height of the flexible tank. The mechanical coupling effect seems to be reduced in the presence of liquid. Figs. 11 and 12 illustrate the effect of free surface non-linearity on the response of the tank wall for two different lamination schemes. The periodicity and amplitude of the response for both linear and non-linear solutions are seen to be same at the initial stages. However, they differ significantly as the excitation continues. A significant shift in the response amplitude with respect to the time axis is observed for situation with free surface non-linearity. The changes observed in the response between the flexible and rigid tank model as well as the linear and non-linear model are important from the container design point of view.

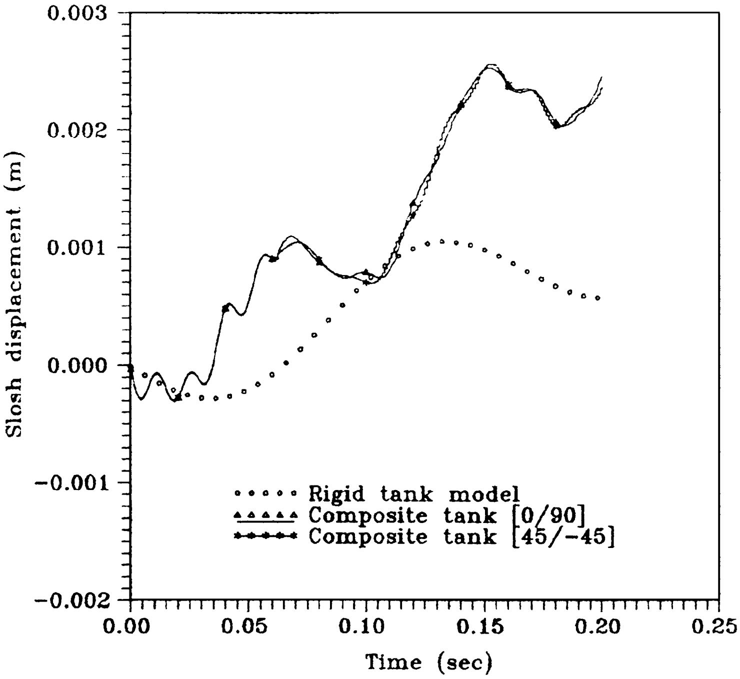


Fig. 10. Effect of tank flexibility on the sloshing response of a liquid-filled composite and its corresponding rigid tank.

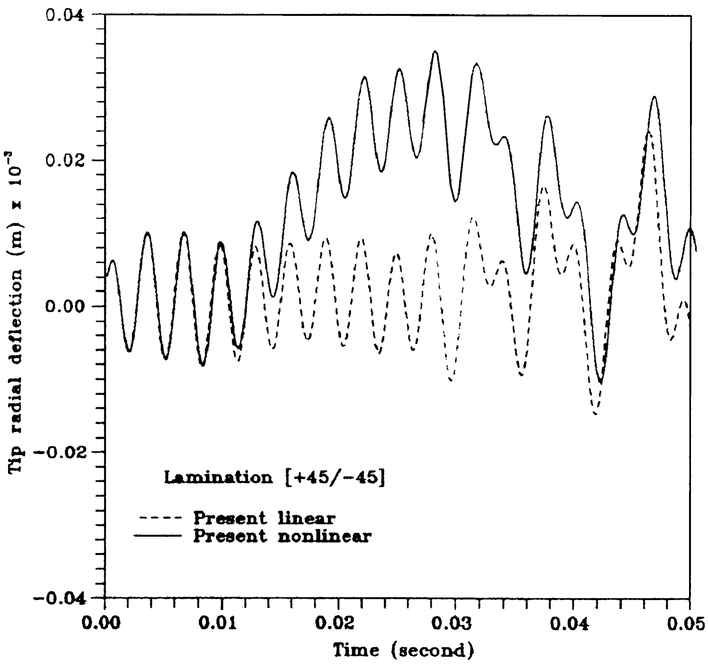


Fig. 11. Effect of free surface non-linearity on the time response of radial defection at free edge of a clamped free liquid-filled laminated composite (angle-ply) cylindrical tank.

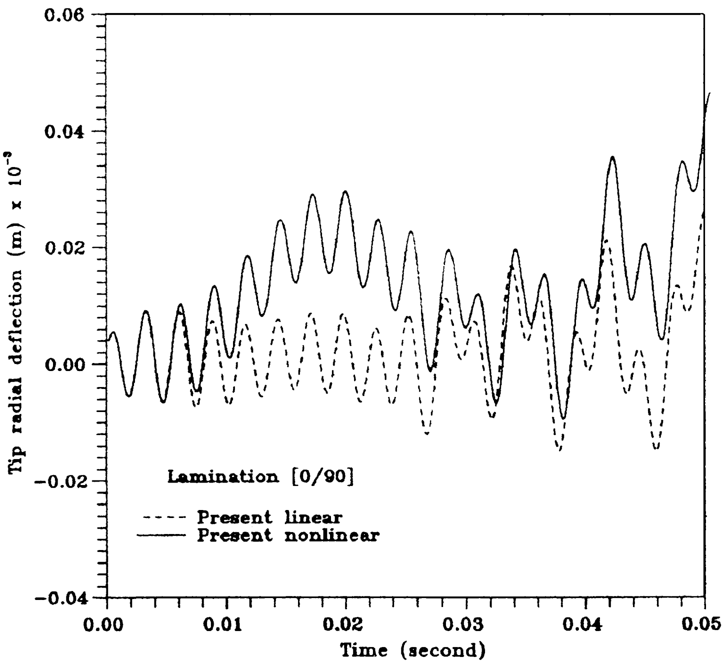


Fig. 12. Effect of free surface non-linearity on the time response of radial deflection at free edge of clamped free liquid-filled laminated composite (cross-ply) cylindrical tank.

6. Conclusions

Most of the liquid-filled containers in general and liquid-fueled space vehicles in particular are, by no means rigid. The coupling of various possible liquid responses with the elastic deformations of a container structure must be considered in the overall analysis of container dynamics. A new avenue in the field of coupled slosh dynamics has emerged due to advent of light weight composites having high specific strength, resistance to corrosion and flexibility in design tailoring, which are used in many weight-sensitive applications in many engineering industries. Considerable structural flexibility, liquid free surface non-linearity, moving liquid masses together with varying structural response generate enormous complexities because of multidisciplinary interactions.

An attempt is made in this paper to address this complex problem and to demonstrate the effects of free surface non-linearity on the coupled response of liquid-filled laminated composite container. Finite element method is used for the numerical simulation of the problem. A numerical scheme is developed on the basis of a mixed Eulerian–Lagrangian approach, with velocity potential as the unknown nodal variable in the fluid domain and displacements as the unknowns in the structure domain. Accordingly, the fluid domain is discretized using four noded quadrilateral fluid finite elements and the structure domain with two noded conical shell finite elements. Numerical results obtained by the present investigation for the rigid containers are first compared with the existing solutions to validate the code for non-linear sloshing without fluid–structure coupling. Thereafter the computational procedures are advanced to obtain the coupled interaction effect of non-linear sloshing in flexible laminated composite containers. Simulation results of liquid sloshing induced by earthquake and harmonic base excitations are compared with those of the linear theory and the limitation of the latter in assessing the non-linear coupled response of flexible container is addressed.

A significant change in the amplitude of sloshing response of the composite and its corresponding rigid tank model is observed which indicates the importance of tank flexibility on the design of flexible liquid-filled containers. The behaviours of the linear and non-linear sloshing response are seen to be significantly different with increase in amplitude and time period elongation in later case. The merit of the present investigation is the studies on non-linear coupled interaction of sloshing liquid with flexible liquid-filled laminated composite containers. To the best knowledge of the present authors, results on similar cases are not available in the open literature.

Appendix A. Nomenclature

E_{11}, E_{22}	structure elastic moduli
$\{Fe\}$	nodal forces exerted on the container wall
G_{12}, G_{13}, G_{23}	structure shear moduli
$\{P\}$	pressure arising from the oscillation of the liquid
$\{d\}$	displacement vector
g	acceleration due to gravity
h_l	liquid depth in the container
h	thickness of container wall
p	hydrodynamic pressure
u	meridional displacement
v	circumferential displacement
v_n	boundary velocity
w	radial displacement
δ	fluid free surface displacement
ϕ	fluid velocity potential
v_{12}, v_{21}	the Poisson ratio
ρ_f	fluid density
ρ_s	structural density

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