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Authors' reply

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1. Introduction

First of all, we appreciate the comments of M.B. Rosales and C.P. Filipich who discussed our method of exact series solution for calculating the eigenfrequencies of orthotropic plates with completely free boundaries [1]. Rosales and Filipich mentioned that uniform convergence is lost and that the solution is not 'exact', however, is only of arbitrary precision.

In this reply it is impossible to show that the derived exact series solution converges to the exact solution, since there exists no analytically closed form solution for the problem of a completely free plate. However, we want to show that for an equivalent simpler problem (vibration of a free beam) the exact series solution converges to the analytically closed form exact solution. An exact series solution for structural problems was presented by Wang and Lin [2,3] including the vibration of a completely free beam while the convergence of the exact series solution to the analytically closed form solution was shown by Storch [4]. For other boundary conditions one can refer to Storch [4].

2. Vibration of a free–free beam

Considering the Bernoulli–Euler beam theory, the amplitude W of the harmonic motion of a beam is governed by

$$\frac{d^4 W}{dx^4} - \beta^4 W = 0, \tag{1}$$

where $\beta^4 = \rho\omega^2/EI$, ρ is the mass density, and ω is the circular frequency. If $\cos \alpha_m x$ for $m = 0, 1, 2, \ldots, \infty$ are used as weighting functions, where $\alpha_m = m\pi/L$, the governing differential

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equation can be expressed as

$$\int_0^L \left(\frac{\mathrm{d}^4 W}{\mathrm{d}x^4} - \beta^4 W\right) \cos \alpha_m x \, \mathrm{d}x = 0. \tag{2}$$

Integrating successively from $x = 0$ to L yields

$$[(-1)^m W_1''' - W_0'''] - \alpha_m^2[(-1)^m W_1' - W_0'] + (\alpha_m^4 - \beta^4) W_m^* = 0 \tag{3}$$

for $m = 0, 1, 2, \ldots, \infty$, where $()'$ denotes the derivative with respect to x and

$$W_m^* = \int_0^L W \cos \alpha_m x \, \mathrm{d}x. \tag{4}$$

Representing

$$W = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos \alpha_n x \tag{5}$$

and substituting Eq. (5) into Eq. (3), one obtains A_n using the definition of the transformed quantity, Eq. (4). The general solution of W is found to be

$$\begin{aligned} W = & \frac{1}{\beta^4 L}[W_1''' - W_0'''] \\ & - \frac{2}{L} \sum_{n=1}^{\infty} \{[(-1)^n W_1''' - W_0'''] - \alpha_n^2[(-1)^n W_1' - W_0']\} \frac{1}{\alpha_n^4 - \beta^4} \cos \alpha_n x. \end{aligned} \tag{6}$$

The detailed expression for A_0 and A_n can be readily identified by comparing Eqs. (5) and (6).

According to the boundary conditions for the case of a free–free beam, $W_1''' = W_0''' = 0$, and $W_0' = -W_1'$ and W_1' for symmetric and antisymmetric modes of deformation, respectively. From Eq. (6) one arrives at

$$W = \frac{2}{L} W_1' \sum_{n=1}^{\infty} [(-1)^n + c] \frac{\alpha_n^2}{\alpha_n^4 - \beta^4} \cos \alpha_n x. \tag{7}$$

where $c = 1$ and -1 correspond to symmetric and antisymmetric modes of deformation, respectively. While the general solution given in Eq. (7) satisfies zero shear force at both ends, zero moment corresponding to the second derivative of W with respect to x must vanish at $x = 0$ and L. Second derivative of W with respect to x accounting for the quantities at end points results in

$$\frac{\mathrm{d}^2 W}{\mathrm{d}x^2} = \frac{2}{L} W_1' \left\{ a - \beta^4 \sum_{n=1}^{\infty} [(-1)^n + c] \frac{1}{\alpha_n^4 - \beta^4} \right\}. \tag{8}$$

The frequency equations corresponding to symmetric and antisymmetric modes of deformation, by setting Eq. (8) to zero for $a = 1$ and 0, respectively, are as follows:

$$1 - 2p^4 \sum_{n=1}^{\infty} \frac{1}{16n^4 - p^4} = 0, \tag{9}$$

$$2p^4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4 - p^4} = 0, \tag{10}$$

in which $p = \beta L/\pi$. Using the relation

$$\sum_{n=1}^{\infty} \frac{1}{16n^4 - p^4} = \frac{1}{2p^4} - \frac{\pi}{8p^3}\left(\cot\frac{p\pi}{2} + \coth\frac{p\pi}{2}\right) \tag{11}$$

one can express Eq. (9) in the form

$$p\left(\cot\frac{p\pi}{2} + \coth\frac{p\pi}{2}\right) = 0. \tag{12}$$

The root $p = 0$ corresponds to a symmetric rigid body mode. Using the relation

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4 - p^4} = \frac{\pi}{8p^3}\left(\tan\frac{p\pi}{2} - \tanh\frac{p\pi}{2}\right), \tag{13}$$

Eq. (10) can be expressed in the form

$$p\left(\tan\frac{p\pi}{2} - \tanh\frac{p\pi}{2}\right) = 0. \tag{14}$$

The root $p = 0$ corresponds to an antisymmetric rigid body mode. If one multiplies the two Eqs. (12) and (14) one arrives at the well-known characteristic equation for the free–free frequencies,

$$\cos p\pi \cosh p\pi = 1. \tag{15}$$

3. Concluding remarks

This reply demonstrates that the exact series solution of a completely free beam can be transformed to the well-known characteristic equation for calculating the eigenfrequencies. Extrapolating this information, it can be concluded that for a plate with completely free boundary the exact series solution uniformly converges to the (non-existing) analytical solution. In summary, one can state that the concept of exact series solution converges uniformly and that the solution is exact.

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