ACADEMIC PRESS

Available online at www.sciencedirect.com
SCIENCE DIRECT

Journal of Sound and Vibration 261 (2003) 761–763

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Author's reply

D.J. Gorman

Department of Mechanical Engineering, University of Ottawa, 770 King Edward Avenue, Ottawa, Canada ON K1N 6N5

Received 18 July 2002; accepted 30 July 2002

This is written in response to the above 'Letter to the Editor', The Journal of Sound and Vibration, as prepared by M.B. Rosales and C.E. Filipich. They will be referred to herein as the 'authors'. Particular attention is drawn to their comments on Refs. [4,5], of their letter. These are publications prepared by the present writer.

Their statement that 'the second frequency corresponding to an isotropic square plate is missing' in Ref. [4] is categorically wrong. The same obviously goes for their similar statement regarding the corresponding eigenvalue of Ref. [5].

Had the authors examined these references more carefully, they would have recognized that eigenvalues tabulated therein are based on a Poisson ratio of 0.333, and not the value of 0.3 which they used in their calculations. It is well known that the Poisson ratio enters the vibration analysis of plates with combinations of free edge conditions in two specific ways. It enters explicitly through formulation of the dimensionless eigenvalue. It also enters implicitly through enforcement of the free edge boundary conditions. The strong dependence of such eigenvalues on the Poisson ratio utilized has been thoroughly discussed in the literature by Leissa. It is why the present writer has, in an earlier publication, prepared eigenvalue tabulations based on each of two different Poisson ratios when plate free edges were involved (see Ref. [1]).

For further enlightenment of the authors (and other interested readers) they might wish to examine Table 1 attached here. This is a counterpart of Table 1 of their Ref. [2].

The first eight eigenvalues (frequency parameters) for a completely free square plate, as reported by the authors in their Ref. [2], are reproduced here for the convenience of the reader. An additional column, column A in the present table, provides corresponding eigenvalues computed by the superposition method. In all computations, a Poisson ratio of 0.3 was utilized. Fifteen terms were utilized in the Levy solution expansions employed in the superposition method. They were found to give much more convergence than that required in order to obtain four significant digit accuracy in computed eigenvalues.

The authors will note that, in fact, the superposition method obtains all eight eigenvalues just as it did in earlier studies when a Poisson ratio of 0.333 was utilized. Furthermore, it will be recalled that all of the energy minimization techniques employed to obtain the results reported by the

E-mail address: dgorman@genie.uottawa.ca (D.J. Gorman).

doi:10.1016/S0022-460X(02)01176-8

Table 1
Tabulation of computed eigenvalues for completely free square plate

Mode	A	B	C	D	E
(1)	13.47	13.47	13.49	13.468	13.46
(2)	19.60	19.61	19.79	19.596	19.56
(3)	24.27	24.28	24.43	24.271	24.25
(4)	34.80	34.82	35.02	34.801	34.77
(5)	34.80	34.82	35.02	34.801	34.77
(6)	61.09	61.13	61.53	61.111	60.98
(7)	61.09	61.13	61.53	61.111	60.98
(8)	63.68	63.72	—	—	63.62

The Poisson ratio equals 0.3. Column A (superposition method). Columns B, C, D, and E, results of WEM method, Leissa crossed-beam study, Leissa shallow shell study, and 50×50 finite element study, respectively, as reported by authors.

authors are known to give eigenvalue 'upper limits', only. Actual eigenvalues will lie below those computed by the energy techniques. This characteristic does not apply to the superposition method. It will be observed that all four-digit eigenvalues computed by this latter method (column A of the table) are equal to, or lower than, those computed by the authors (column B). It is to be expected that as they improve the accuracy of their computations, by utilizing more terms in their expansions, their results will approach those of column A.

It might also be pointed out that all solutions obtained by the superposition method satisfy exactly the governing differential equation throughout the entire domain of the plate. This is a condition that can never be achieved by the method proposed by the authors, or, in fact, any other method employed to produce the results of their Table 1.

The authors also question the 'completeness' of expansion series used by other researchers. In particular, they question the ability of series obtained through differentiation to represent the first and second derivatives, etc., of the basic plate displacements. The differentiability of trigonometric series utilized in connection with Levy-type solutions has already been discussed by the present writer in an earlier publication (see Ref. [1]). Nevertheless, this discussion will be repeated here briefly for the benefit of readers.

Consider a rectangular plate with two opposite edges given simple support. It is agreed that the spatial distribution of its lateral displacement may be written as

$$W(\xi, \eta) = \sum_{m=1}^{\infty} Y_m(\eta) \sin(m\pi\xi), \tag{1}$$

where η and ξ are dimensionless co-ordinates running along the plate edges, the co-ordinate ξ running perpendicular to the simply supported edges. At any particular value of the co-ordinate η plate lateral displacement is expressed as a function of ξ and may be written as

$$X(\xi) = \sum_{m=1}^{\infty} A_m \sin(m\pi\xi), \tag{2}$$

where

$$A_m = 2\int_0^1 X(\xi)\sin(m\pi\xi)\,\mathrm{d}\xi. \tag{3}$$

The Fourier sine series utilized here forms a 'complete orthogonal set'. We assume that the first derivative with respect to ξ of the function $X(\xi)$ is continuous and differentiable. It may therefore be expressed in series form utilizing the 'complete orthogonal' Fourier cosine series as

$$X^{|}(\xi) = \sum_{m=0,1}^{\infty} B_m \cos(m\pi\xi), \tag{4}$$

where the coefficients for $m = 0$, and $m > 0$, are, respectively

$$B_m = \int_0^1 X^{|}(\xi)\,\mathrm{d}\xi \quad \text{and} \quad B_m = 2\int_0^1 X^{|}(\xi)\cos(m\pi\xi)\,\mathrm{d}\xi. \tag{5}$$

It is obvious, in view of the simple support conditions imposed along the plate edges at the extremities of the ξ-axis that, for $m = 0$, $B_m = X(1) - X(0) = 0$.

For $m > 0$, integrating by parts, we obtain

$$B_m = 2X(\xi)\cos(m\pi\xi)|_0^1 + 2m\pi\int_0^1 X(\xi)\sin(m\pi\xi), \tag{6}$$

$$= 2X(\xi)\cos(m\pi\xi)|_0^1 + m\pi A_m. \tag{7}$$

In view of the imposed boundary conditions, it is obvious that we have $B_m = m\pi A_{m.}$ We therefore write for the function $X^{|}(\xi)$,

$$X^{|}(\xi) = \sum_{m=1}^{\infty} A_m \cos(m\pi\xi). \tag{8}$$

This tells us that in view of our choice of series to represent the plate lateral displacement, the series can be differentiated term-by-term. The first derivative of the displacement function thereby obtained is also represented by a 'complete orthogonal' Fourier trigonometric series. Proceeding onward in an identical fashion, it is shown that not only the plate lateral displacement but all of its required derivatives are available in 'complete orthogonal' Fourier series representations through term-by-term series differentiation. This is true of all Levy-type solutions.

References

[1] D.J. Gorman, Free Vibration Analysis of Rectangular Plates, Elsevier North-Holland, New York, 1982.