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Vibration localization of simplified mistuned cyclic structures undertaking external harmonic force

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Abstract

Critical fatigue problems often occur in mistuned cyclic structures since their forced vibration responses are often much larger than those of perfectly tuned structures. Therefore, it is of great importance to predict the forced vibration responses of mistuned cyclic structures for their safe and reliable designs. In this paper, a simplified model for mistuned cyclic structures is chosen to investigate vibration localization phenomena. The effects of mistuning, stiffness coupling, and damping on the variations of maximum forced vibration responses of the model are examined through numerical study. It is found that strong vibration localization occurs under certain relations among mistuning, stiffness coupling, and damping. It is also found that the maximum forced vibration response asymptotically increases as the number of repeated subcomponents of the cyclic structure increases.

1. Introduction

Cyclic structures can be found in several engineering systems. Aircraft rotor and turbine blades are typical examples of such systems. Repeated subcomponents of cyclic structures are usually manufactured identically. However, there always exist small, random differences among the subcomponents due to manufacturing tolerances, in-operation wear, and so forth. These differences, which are usually called mistuning, often cause significant increase in the forced

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vibration responses of cyclic structures. The maximum forced vibration response of a subcomponent of a mistuned cyclic structure is often much larger than that of a perfectly tuned cyclic structure. Thus mechanical energy stored in a subcomponent of a mistuned cyclic structure is much different from those stored in other subcomponents. This is called the vibration localization of a mistuned cyclic structure. In order to avoid unexpected premature failures of a cyclic structure, the effect of mistuning on the vibration localization needs to be investigated thoroughly.

Cyclic structures consist of a subset of periodic structures. The vibration localization occurred in a periodic structure has been the subject of a number of theoretical studies [1–4]. Ewins [1] showed that the maximum forced response increases with increasing mistuning up to certain level, but further increase in mistuning results in lower forced response amplitudes. Afolabi's investigation [5] concluded that the blade with the most mistuning is likely to be the blade with the largest amplitude. Griffin and Hoosac [6] showed that blades with cantilever frequencies close to coupled blade-disk frequencies usually respond with the greatest amplitude. These different and somewhat conflicting conclusions may have originated from the different models and parameter values used in the studies. Wei and Pierre [7] gave a physical explanation for most of these discrepancies, and introduced intentional mistuning into the design of bladed disks in order to reduce the maximum forced response [8]. Recently, Lin and Mignolet [9] investigated the effect of damping mistuning of the bladed disks, and found that damping mistuning is potentially more dangerous for the forced responses. In general, all the previous studies show that mistuning may result in undesirable effects on the forced vibration response of periodic structures. However, the combined effects of mistuning, coupling, and damping have not been investigated in the previous studies. Furthermore, the influence of the number of subcomponents on the vibration response has not been investigated thoroughly yet.

In this paper, the combined effects of mistuning, stiffness coupling, and damping on vibration localization of cyclic structures are investigated. A coupled pendulum system, which represents coupled cyclic structures, is chosen for the investigation. The simplicity of the system makes it possible to provide an effective way to understand the characteristics of the vibration localization phenomena involved in cyclic structures. The primary objective of the present study is to find the relations among mistuning, stiffness coupling, and damping that cause strong vibration localization in coupled cyclic structures. The maximum (or minimum) forced vibration amplitudes of the cyclic structures are examined around their natural frequencies. The influence of the number of subcomponents of the cyclic structures on the maximum (or minimum) forced vibration amplitude is also investigated. These investigations employing the simplified model will provide easy and clear explanation of vibration localization phenomena. The conclusions drawn from this study employing the simple model are true not only for the simple model itself but also for general cyclic structures. Therefore, they can be used as valuable guidelines for safe and reliable designs of general coupled cyclic structures.

2. Simplified model of coupled cyclic structure

Cyclic structures have repeated subcomponents that have identical structural topology including geometry, stiffness coupling, damping and so forth. The mistuning of a cyclic structure results from small structural irregularities such as length difference. Fig. 1 shows a planar coupled pendulum system which has the same mass m, the same torsional stiffness k_r, and slightly different lengths l_i. Each pendulum is coupled by two identical translational springs of modulus k_t. The distance from a hinge point to the point where translational springs are attached is a. Even though damping symbols do not appear in Fig. 1, linear viscous proportional damping force (with damping constant c) is assumed to act on each pendulum mass. The resulting damping force is given in Eq. (1) shown below. Each pendulum mass is also excited by external sinusoidal harmonic force having magnitude F_0 and frequency Ω. The equations of motion of this pendulum system can be derived as follows:

$$\begin{bmatrix} ml_1^2 & 0 & . & 0 \\ 0 & ml_2^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & ml_n^2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ . \\ \ddot{\theta}_n \end{Bmatrix} + \begin{bmatrix} cl_1^2 & 0 & 0 & 0 \\ 0 & cl_2^2 & 0 & 0 \\ . & & & \\ 0 & 0 & 0 & cl_n^2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ . \\ \dot{\theta}_n \end{Bmatrix}$$
$$+ \begin{bmatrix} k_r + 2k_t a^2 & -k_t a^2 & . & -k_t a^2 \\ -k_t a^2 & k_r + 2k_t a^2 & . & 0 \\ . & . & . & . \\ -k_t a^2 & 0 & . & k_r + 2k_t a^2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ . \\ \theta_n \end{Bmatrix} = \begin{Bmatrix} l_1 F_0 \sin \Omega t \\ l_2 F_0 \sin \Omega t \\ . \\ l_n F_0 \sin \Omega t \end{Bmatrix}. \tag{1}$$

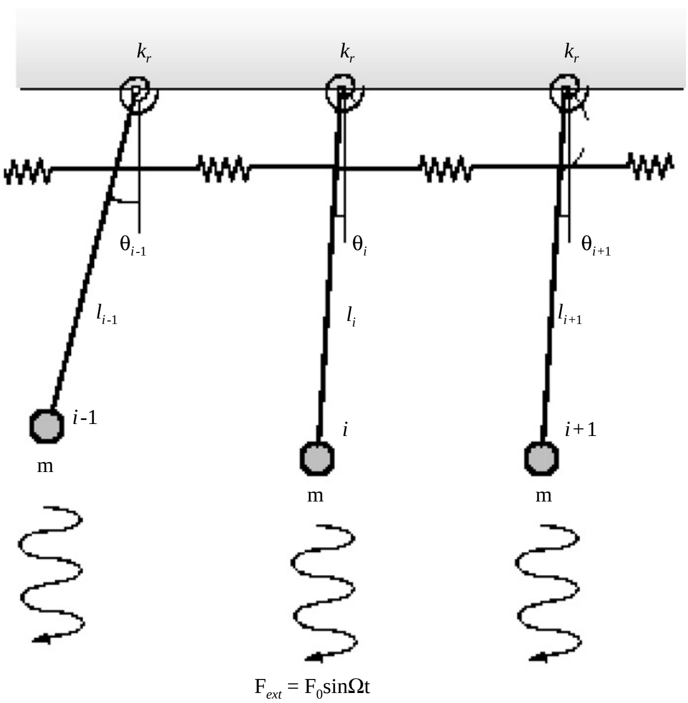

Fig. 1. Multiple coupled pendulum system.

To obtain more general and useful conclusions from the equations of motion, dimensionless parameters and a dimensionless variable are defined as follows:

$$\alpha_i \equiv \frac{l_i}{l}, \quad \beta \equiv \frac{k_t a^2}{k_r}, \quad \gamma \equiv \frac{Tc}{m}, \quad \omega \equiv T\Omega, \quad f \equiv \frac{T^2 F_0}{ml}, \quad \tau \equiv \frac{t}{T}, \tag{2}$$

where l represents the nominal length of the pendulums and

$$T = \sqrt{\frac{ml^2}{k_r}}. \tag{3}$$

Employing these dimensionless parameters and variable, Eq. (1) can be rewritten in a dimensionless form as follows:

$$\begin{bmatrix} \alpha_1^2 & 0 & . & 0 \\ 0 & \alpha_2^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \alpha_n^2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ . \\ \ddot{\theta}_n \end{Bmatrix} + \begin{bmatrix} \gamma\alpha_1^2 & 0 & 0 & 0 \\ 0 & \gamma\alpha_2^2 & 0 & 0 \\ . & & & \\ 0 & 0 & 0 & \gamma\alpha_n^2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ . \\ \dot{\theta}_n \end{Bmatrix}$$
$$+ \begin{bmatrix} 1+2\beta & -\beta & . & -\beta \\ -\beta & 1+2\beta & . & 0 \\ . & . & . & . \\ -\beta & 0 & . & 1+2\beta \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ . \\ \theta_n \end{Bmatrix} = \begin{Bmatrix} \alpha_1 f \sin \omega\tau \\ \alpha_2 f \sin \omega\tau \\ . \\ \alpha_n f \sin \omega\tau \end{Bmatrix}, \tag{4}$$

where a dot over a symbol now represents the differentiation of the symbol with respect to τ (instead of t). The steady state solutions of the above equations can be written as follows:

$$\theta_i = a_i \cos \omega\tau + b_i \sin \omega\tau \quad (i = 1, 2, \ldots, n). \tag{5}$$

Substituting Eq. (5) into Eq. (4), one obtains the following algebraic relations:

$$(-\omega^2\mathbf{M} + \mathbf{K})\mathbf{b} - \omega\mathbf{C}\mathbf{a} = \mathbf{f},$$

$$(-\omega^2\mathbf{M} + \mathbf{K})\mathbf{a} + \omega\mathbf{C}\mathbf{b} = 0, \tag{6}$$

where

$$\mathbf{M} = \begin{bmatrix} \alpha_1^2 & 0 & . & 0 \\ 0 & \alpha_2^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \alpha_n^2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \gamma\alpha_1^2 & 0 & 0 & 0 \\ 0 & \gamma\alpha_2^2 & 0 & 0 \\ . & . & . & . \\ 0 & 0 & 0 & \gamma\alpha_n^2 \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} 1+2\beta & -\beta & . & -\beta \\ -\beta & 1+2\beta & . & 0 \\ . & . & . & . \\ -\beta & 0 & . & 1+2\beta \end{bmatrix}, \quad \mathbf{f} = \begin{Bmatrix} \alpha_1 f \\ \alpha_2 f \\ . \\ \alpha_n f \end{Bmatrix}. \tag{7}$$

By solving Eq. (6), one can determine the coefficients a_i's and b_i's which constitute the column matrices **a** and **b**. By using the coefficients, one can obtain the dynamic magnification factors as follows:

$$\kappa_i = \sqrt{a_i^2 + b_i^2}/f. \tag{8}$$

In the next section, the vibration localization phenomena occurred in the mistuned pendulum system will be investigated by obtaining and comparing the dynamic magnification factors.

3. Numerical results and discussion

In this section, the effects of mistuning, coupling, and damping on the forced vibration responses are investigated. First, the simplest coupled pendulum system (double pendulum system) is employed for the investigation. In this system, the length of the first pendulum is taken as the nominal length. Therefore, there exists only one parameter α which is related to mistuning. Actually the mistuning of this system is the difference between α and 1.

Fig. 2 shows the frequency variations of dynamic magnification factors for the two pendulums. The dimensionless coupling parameter $\beta = 0.005$ and the damping parameter $\gamma = 0.01$ are used for the numerical results. These values represent a weakly coupled, lightly damped system. Fig. 2(a) shows the frequency variations of the two pendulums with very small mistuning ($\alpha = 1.001$). As one may expect, the two results show little difference. Now, as the mistuning increases up to 1% ($\alpha = 1.01$), the two results show significant difference as shown in Fig. 2(b). This indicates that even small non-uniformity among subcomponents of a cyclic structure may cause large difference in forced vibration response. When the mistuning increases more (for instance, $\alpha = 1.05$, as shown in Fig. 2(c)), the difference in maximal forced vibration response decreases. Thus, the mistuning up to certain level causes significant difference in maximal forced dynamic response. These results are in consistent with the results that were previously obtained by Ewins [1]. Fig. 3 exhibits the overall tendency clearly. The difference between the two maximal dynamic magnification factors becomes maximized when the mistuning is around 1% ($\alpha = 1.01$). As the mistuning increases more, the difference decreases. The coupling parameter β used to obtain these results is 0.005 and the damping parameter γ is 0.01. With a different coupling parameter, one may obtain the maximal dynamic magnification factor at different length ratio. Such results will be given later in Fig. 5.

There is one important thing that should be emphasized here. The maximal dynamic magnification factors should be compared even though they occur at slightly different frequencies. This is because the external force applied to a cyclic structure usually has a spread (if not wide) frequency range. Since the mistuning of a cyclic structure is usually not large, the frequencies of maximal dynamic magnification factors are not separated far. Thus, they are embraced by the frequency range of the external force applied to a cyclic structure. Comparing the two dynamic magnification factors at each identical frequency only leads to a trivial conclusion that the difference between the two dynamic magnification factors increases monotonically as α increases.

Fig. 4 shows the effect of damping on the forced vibration responses. The dimensionless parameters $\alpha = 1.01$ and $\beta = 0.005$ are used for the numerical results. Fig. 4(a) shows the

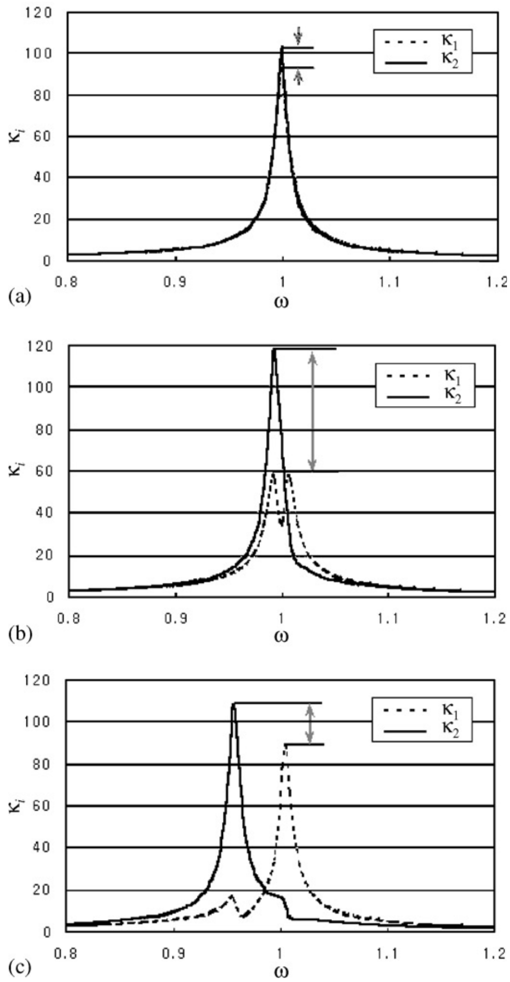

Fig. 2. Dynamic magnification factors of a weakly coupled, lightly damped system with three different length ratios. (a) $\alpha = 1.001$, $\beta = 0.005$, $\gamma = 0.01$; (b) $\alpha = 1.01$, $\beta = 0.005$, $\gamma = 0.01$; (c) $\alpha = 1.05$, $\beta = 0.005$, $\gamma = 0.01$.

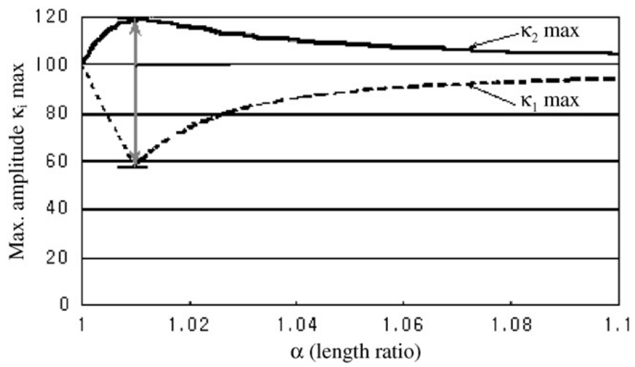

Fig. 3. Variations of dynamic magnification factors due to length ratio variation ($\gamma = 0.01$ for these results).

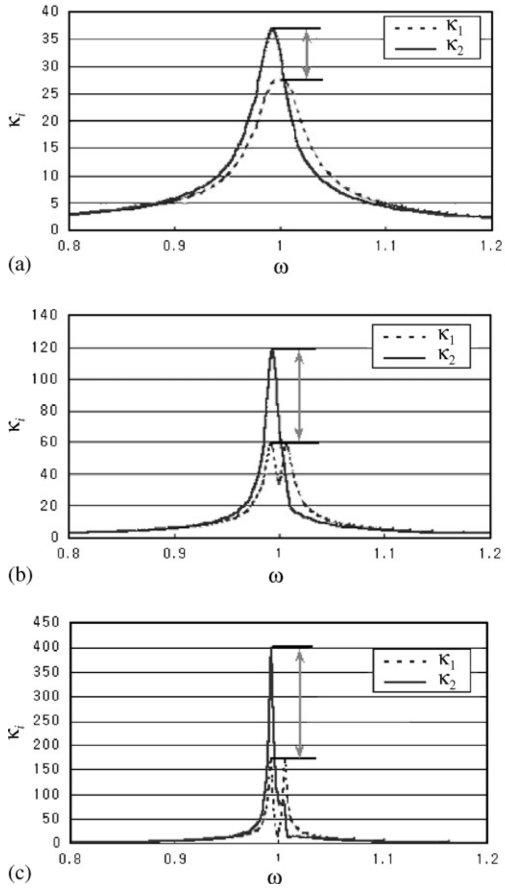

Fig. 4. Dynamic magnification factors of a weakly coupled system with three different damping parameters. (a) $\alpha = 1.01$, $\beta = 0.005$, $\gamma = 0.03$; (b) $\alpha = 1.01$, $\beta = 0.005$, $\gamma = 0.01$; (c) $\alpha = 1.01$, $\beta = 0.005$, $\gamma = 0.003$.

frequency responses of the pendulums under relatively strong damping ($\gamma = 0.03$). This indicates that strong damping results in the small difference in maximal forced vibration response of the mistuned pendulums. But as shown in Fig. 4(b) and (c), as the damping decreases, the difference in the maximal forced vibration response increases significantly.

Fig. 5 shows the combined effect of the mistuning, the coupling, and the damping. Darker area represents stronger vibration localization in this figure. It can be observed that certain conditions between the mistuning and the coupling can cause strong localization effect. In general, if the mistuning increases, the coupling should be also increased to obtain strong localization effect. Even if the mistuning and the coupling are very small, strong localization may occur if the damping remains small (see Fig. 5(a)). Now, as the damping increases, the vibration localization is significantly reduced. Fig. 5(c) also shows that maximum localization (under relatively strong damping) occurs when both of the mistuning and the coupling increase in an approximately proportional way.

Since a cyclic structure has many repeated subcomponents, the effect of the number of subcomponents on the vibration localization needs to be investigated. Therefore, the equations of

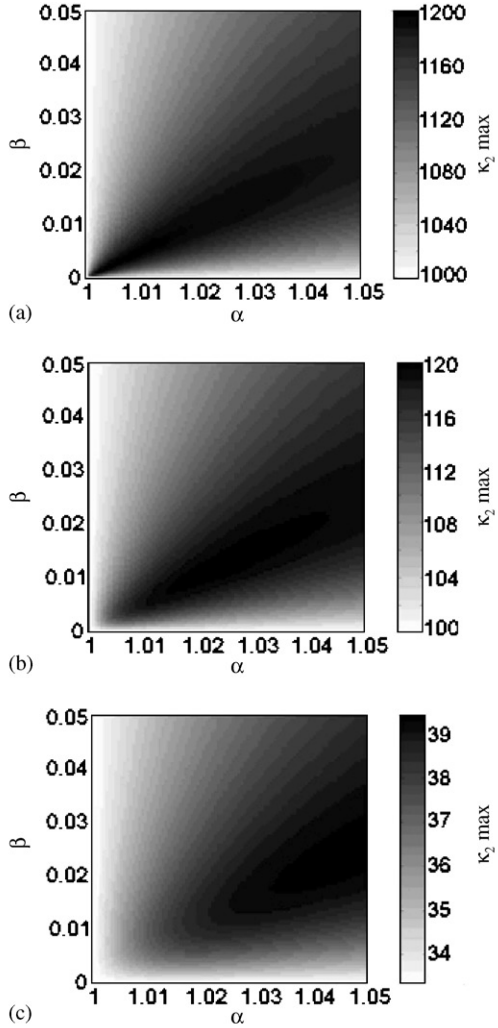

Fig. 5. Distribution of the maximal dynamic magnification factor in $\alpha - \beta$ plane with three different damping parameters. (a) $\gamma = 0.001$; (b) $\gamma = 0.01$; (c) $\gamma = 0.03$.

motion derived in the previous section are solved for the cases of having more than two pendulums. Solutions of dynamic magnification factors are first obtained at every grid point of parametric region. Then, the maximal and the minimal values and the corresponding grid points are searched. If one divides a parameter with 10 grids, adding one more parameter results in 10 times computation time. Since practical cyclic structures have large number of degrees of freedom (at least 80–100 degrees of freedom), they have large number of parameters. Thus, it requires prohibitively tremendous amount of computation time to find the maximal and the minimal responses and the corresponding grid points for such systems. Therefore, in this paper, the maximal number of degrees of freedom is restricted to five at most. The conclusions drawn from the small degrees of freedom system, however, can be applied to practical systems without loosing generality.

Table 1 shows the dimensionless parameter values at which maximal dynamic magnification factors appear. For these results, the dimensionless damping parameter γ and the dimensionless coupling parameter β are fixed to 0.01 and 0.005, respectively. The number of subcomponents of cyclic structure varies from two to five. As shown in these results, small non-uniformity in a cyclic structure may cause large difference in forced vibration response. For example, about 1% mistuning of five-coupled pendulums can cause 34.5% larger forced vibration response than those of the perfectly tuned assembly. Among the pendulums, the pendulum that has the largest length generates the strongest response. These results also show that the maximal forced vibration response increases as the number of subcomponents increases. This indicates that a cyclic structure having more subcomponents may be exposed to more serious vibration localization effect. However, the increasing rate seems to decrease as the number of subcomponents increases.

Different from the condition for Table 1, the dimensionless coupling parameter β is not fixed to obtain the results in Table 2 while the dimensionless damping parameter γ is fixed to 0.01. It can be observed that the maximal dynamic magnification factors (compared to those of Table 1)

Table 1
Length ratios that cause the maximum response at one of the pendulums under constant coupling and light damping condition ($\gamma = 0.01$)

	α_1	α_2	α_3	α_4	α_5
	κ_1	κ_2	κ_3	κ_4	κ_5
$n = 2$	1.000	1.012			
($\beta = 0.005$)	62.14	118.4			
$n = 3$	1.000	1.000	1.009		
($\beta = 0.005$)	69.24	69.24	128.9		
$n = 4$	1.000	1.003	1.011	1.003	
($\beta = 0.005$)	81.86	74.96	133.2	74.96	
$n = 5$	1.000	1.000	1.004	1.011	1.004
($\beta = 0.005$)	85.19	85.19	78.44	134.5	78.44

Table 2
Length ratios that cause the maximum response at one of the pendulums under variable coupling and light damping condition ($\gamma = 0.01$)

	α_1	α_2	α_3	α_4	α_5
	κ_1	κ_2	κ_3	κ_4	κ_5
$n = 2$	1.000	1.024			
($\beta = 0.012$)	52.45	119.5			
$n = 3$	1.000	1.000	1.026		
($\beta = 0.018$)	53.13	53.13	134.4		
$n = 4$	1.000	1.020	1.000	1.043	
($\beta = 0.029$)	55.11	75.93	55.11	146.1	
$n = 5$	1.000	1.019	1.019	1.000	1.045
($\beta = 0.032$)	59.20	71.03	71.03	59.20	155.7

increase more when the coupling parameter β increases. The coupling parameter to obtain the maximal dynamic magnification factors increases as the number of subcomponents increases. Now, for the system having five-coupled pendulums, about 56% larger forced vibration response can be obtained with 4.5% mistuning and the larger coupling parameter ($\beta = 0.032$).

Table 3 shows the dimensionless parameter values at which minimal dynamic magnification factors appear. As same as Table 1, the dimensionless damping parameter γ and the dimensionless coupling parameter β are fixed to 0.01 and 0.005, respectively. The number of subcomponents of cyclic structure varies from two to five. These results show that the minimal forced vibration response first increases and then decreases as the number of subcomponents increases. However, the decreasing rate seems to decrease as the number of subcomponents increases. There is an interesting result one can observe in the Table 3. Different from the maximal vibration results, the pendulum of the minimal vibration response does not always have the smallest length. When $n = 5$, the minimal vibration response occurs at the pendulum of α_4 instead of that of α_1.

Table 3
Length ratios that cause the minimum response at one of the pendulums under constant coupling and light damping condition ($\gamma = 0.01$)

	α_1	α_2	α_3	α_4	α_5
	κ_1	κ_2	κ_3	κ_4	κ_5
$n = 2$	1.000	1.010			
($\beta = 0.005$)	58.72	118.3			
$n = 3$	1.000	1.004	1.010		
($\beta = 0.005$)	59.32	74.17	127.6		
$n = 4$	1.000	1.005	1.000	1.012	
($\beta = 0.005$)	53.90	109.4	53.90	130.4	
$n = 5$	1.000	1.001	1.015	1.008	1.023
($\beta = 0.005$)	81.97	86.66	119.2	50.51	124.3

Table 4
Length ratios that cause the maximum response at one of the pendulums under constant coupling and larger damping condition ($\gamma = 0.02$)

	α_1	α_2	α_3	α_4	α_5
	κ_1	κ_2	κ_3	κ_4	κ_5
$n = 2$	1.000	1.015			
($\beta = 0.005$)	36.99	57.36			
$n = 3$	1.000	1.000	1.013		
($\beta = 0.005$)	41.83	41.83	59.77		
$n = 4$	1.000	1.008	1.020	1.008	
($\beta = 0.005$)	38.94	45.67	60.38	45.67	
$n = 5$	1.000	1.000	1.009	1.020	1.009
($\beta = 0.005$)	43.48	43.48	46.75	60.50	46.75

Comparing Tables 1 and 3, one can also figure out that the maximal and the minimal responses do not occur at the same system configuration having identical length ratios.

Lastly, Table 4 shows the dimensionless parameter values at which maximal dynamic magnification factors appear. For these results, however, the dimensionless damping parameter γ is fixed to 0.02. By comparing the results of this table with those of Table 1, one can easily see that how much damping affects the magnitudes of maximal dynamic magnification factors. As the damping parameter increases twice, the magnitudes decrease approximately half. It should be also noted that the conditions to obtain the maximal dynamic magnification factors change even though the largest pendulum still has the strongest vibration response.

4. Conclusions

In the present work, the conditions that cause strong vibration localization effects are investigated. A simplified model of cyclic structures (a multiple coupled pendulum system) is employed for this investigation. The parameters of mistuning, coupling, and damping are identified and their effects on the vibration localization effects are examined through numerical simulations. It is found that certain conditions relating mistuning, coupling, and damping can cause strong vibration localization phenomena. The pendulum having the largest length is likely to have maximal vibration response. However, the pendulum having the smallest length does not always have the minimal vibration response. It is also found that the maximal dynamic magnification factor increases as the number of subcomponents of a mistuned cyclic structure increases. The minimal dynamic magnification factor first increases and then decreases as the number of subcomponents of a mistuned cyclic structure increases. However, the increasing or the decreasing rate decreases as the number of subcomponents increases. As the number of subcomponents increases, the coupling parameter should be increased to obtain the maximal vibration response. These results provide overall insight into the vibration localization phenomena of mistuned cyclic structures and provide useful guidelines for safe and reliable designs of them.

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