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Topology group concept for truss topology optimization with frequency constraints

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Abstract

A practical methodology based on a topology group concept is presented for finding optimal topologies of trusses. The trusses are subjected to natural frequency, stress, displacement and Euler buckling constraints. Multiple loading conditions are considered, and a constant nodal mass is assumed for each existing node. The nodal cost as well as the member cost is incorporated in the cost function. Starting with a ground structure, a sequence of substructures with different node distribution, called topology group, is generated by using the binary number combinatorial algorithm. Before optimizing a certain topology, its meaningfulness should be examined. If a topology is meaningless, it is then excluded; otherwise, it is optimized as a sectional area optimization problem. In order to avoid a singular solution, the dimension of the structure for a given topology is kept unchanged in the optimization process by giving the member to be removed a tiny sectional area. A parabolic interpolation method is used to solve a non-linear constrained problem, which forms the part of the algorithm. The efficiency of the proposed method is demonstrated by two typical examples of truss.

1. Introduction

Considerable work has been done on structural optimum design since the 1960s. Most of this work is related to the optimization of the members' cross-sections, while less effort has been devoted to topology optimization. It is recognized, however, that topology optimization can greatly improve the design.

Some algorithms have been developed for the problem of finding optimal topologies of trusses with stress and displacement constraints under static loading conditions [1–10], and some review

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papers on this topic are also available [1,2]. The simplest approach to deal with topology optimization problems is referred to as the ground structure method [1]. A ground structure is formed by many connected members, including all the nodes on which the external loads are imposed and the supports. In the ground structure method, some unnecessary members are removed from a ground structure and a structural topology is believed to be determined by the remaining members. Then, the optimal topology is selected through the sectional area optimum. A two-level genetic algorithm-based search method [8] is an adaptation of the ground structure method. Cai and Cheng [9] studied the simulated annealing algorithm for topology optimization of truss. Zou et al. [10] developed the truss topology optimization program referred to as the biology hypothesis of Lamarck. But these methods are not suitable to the topology optimization with nodal mass. In addition, the natural frequencies are not considered as constraints in the methods.

Since stress constraints should not be satisfied in a removed member, the optimal topology might be an isolated or a singular solution, which can be found by varying the cross-sectional areas continuously and by deleting some members with a zero cross-sectional area. This is one of the most difficult troubles of structural topology optimization problems. Wang and Sun [11] developed a method to find an optimal topology by giving a tiny value to the member to be removed and thus to avoid singular solutions. However, it is almost impossible to apply this method to problems with natural frequency constraints. This is because when the nodes to be removed hypothetically remain by assigning tiny values to the members connected to the corresponding nodes, zero-value natural frequencies maybe led to by the process.

It is well known in engineering practices that the cost of the nodes connecting the members may sometimes be equivalent to or even greater than that of the members. A node should be kept if at least one member connected to the node has a positive cross-sectional area. Ohsaki [12] developed a genetic algorithm for topology optimization of trusses to consider the nodal cost as well as the member cost. In his method, the cost of a node is assumed to be constant where it exists. The proposed nodal cost concept is quite useful in engineering applications. However, the algorithm may require time-consuming computational effort if it is applied to large trusses and only static loading conditions are considered.

In order to avoid resonance or undesirable coupling effects between the structure and the external excitation, the natural frequencies of the structures have been considered as constraints or objectives in the optimization process since the late 1960s [13–15]. Very little work, however, has been done on topology optimization of trusses under natural frequency constraints. Nakamura and Ohsaki [16] presented an algorithm to deal with the optimum topology of plane trusses for a specified fundamental frequency based on the concept of an ordered set of optimal trusses. However, it is quite difficult to apply this method to problems with other constraints such as stress constraints, because it utilizes the conditions for global optimality.

In this paper, an algorithm based on the topology group concept is presented for topology optimization of trusses with natural frequency constraints as well as stress, displacement and Euler buckling constraints under multiple static loading conditions. The nodal cost and the member cost are incorporated in the objective function. Computer programs have been developed to perform this optimization and two numerical examples are presented to demonstrate the proposed method.

2. Topology optimization problem

The optimal design problem discussed in this paper can be stated as follows: for a given ground structure, find the node layout and the cross-sectional areas such that the cost of the structure is minimized.

Minimize

$$C(A) = \sum_{i=1}^{N_1}(c_i A_i L_i) + \sum_{k=1}^{N_2} b_k, \tag{1}$$

subject to

$$\underline{\sigma_i} \leqslant \sigma_{il} \leqslant \overline{\sigma_i}, \tag{2}$$

$$\underline{\delta_j} \leqslant \delta_{jl} \leqslant \overline{\delta_j}, \tag{3}$$

$$-\sigma_i \leqslant -\sigma_i^E, \tag{4}$$

$$\bar{f}_r \geqslant f_r \geqslant \underline{f_r} \quad (r = 1, 2, 3 \ldots R), \tag{5}$$

$$A_i \geqslant 0, \quad b_k \geqslant 0, \tag{6}$$

$$(i = 1, 2, \ldots, N_1; j = 1, 2, \ldots, n; k = 1, 2, \ldots, N_2; l = 1, 2, \ldots, m),$$

where N_1 and N_2 denote the number of the members and nodes of the structure, respectively, A_i, L_i and c_i are the sectional area, length and cost coefficient of the ith member, respectively, b_k denotes the cost of the kth node, n and m are the number of displacement constraints and loading conditions, respectively, σ_{il} is the stress of ith member under the lth loading condition, $\underline{\sigma_i}$ and $\overline{\sigma_i}$ are the corresponding compressive and tensile stress limits on σ_{il}, respectively, $\overline{\delta_{jl}}$ is the displacement of the jth degree of freedom under the lth loading condition, $\underline{\delta_j}$ and $\overline{\delta_j}$ are the corresponding lower and upper limits on δ_{jl}, respectively, σ_i^E is the stress of the ith member at which Euler buckling occurs. As a general simplification approach, the lower and upper bounds on stress and displacement are assumed to be the same for every loading condition. f_r denotes the rth eigenfrequency of the structure and $\underline{f_r}$ and $\bar{f}_r$ are the corresponding lower and upper bound, respectively. R is the number of natural frequency constraints. As stated in Eq. (1), the objective function is taken as the cost of the structure. In engineering practices, calculating the cost of the structure is very complicated. It depends on not only the volume of the material but also the manpower by which the structure is manufactured. For a simple presentation of the methods, the cost of the structure is assumed to be the mass it occupied and thus the coefficient c_i denotes the density of the ith member. The cost of the node is used in a specific structure, that is to say, the cost b_k of the kth node is defined as

$$b_k = b \quad \text{when } b_k \text{ exists}, \tag{7}$$

$$b_k = 0 \quad \text{when } b_k \text{ is removed}, \tag{8}$$

where b is the prescribed cost of a node.

The displacement and stress of the structure can be obtained through structural static analysis

$$[k]\{\delta\} = \{p\}, \tag{9}$$

$$\{\sigma\} = [S]\{\delta\}, \tag{10}$$

in which $[k]$ denotes the stiffness matrix of the structure based on finite element method (FEM), $\{p\}$ is the external force vector for a given loading condition, $[S]$ is the stress matrix defined according to the relationship between the stress and node displacement.

The Euler buckling stress can be formulated as

$$\sigma_i^E = \frac{-K_i A_i E}{L_i^2}, \tag{11}$$

in which K_i is a constant depending on the cross-sectional geometry of the ith element, E denotes Young's modulus of the material and it is assumed to be the same for every member.

The rth frequency of the structure can be calculated from the rth eigenvalue of the structure

$$f_r = \sqrt{\lambda_r}/2\pi, \tag{12}$$

in which λ_r is determined by solving the eigen-problem based on FEM

$$[k]\{\phi_r\} = \lambda_r[M]\{\phi_r\}, \tag{13}$$

where $[M]$ is the mass matrix of the structure, λ_r is the rth eigenvalue of the structure and $\{\phi_r\}$ is the corresponding eigenvector.

3. Topology optimization method

As mentioned in the introduction, most of the work on truss optimal topologies, up to now, has been focused on the problems subjected to stress and/or displacement under static loading conditions. It is well known that Euler buckling is very difficult to deal with in truss topology optimization. When natural frequencies are also considered in truss topology optimization, it becomes especially complicated to determine the optimum. In despite of this fact, from the view of engineering applications, the Euler buckling problem and fundamental frequency should be considered for a truss subjected to static and dynamic loads in order to guarantee its feasibility. In order to solve the above-mentioned problems and simplify the calculation, a practical truss optimization method is proposed. In this method, the imaginary bar and constraint deletion technique is introduced first; then a topology group method is proposed and the sectional area optimization for a certain topology is performed by using a improved one-dimensional search method.

3.1. Imaginary bar and constraint deletion

It is obvious from Eqs. (1)–(6) that the topology optimization problem may also be described as a parameter optimization model for cross-sectional areas. The only difference is that the cross-sectional areas of members and the cost of nodes can reach zero.

When a node is removed, the nodes and members of the remaining structure have to be re-numbered to construct a new FE model for analysis. However, this work is very cumbersome, as there are too many possible options for node and member removal. An alternative is to use a imaginary bar to replace the removed member, in other words, when a member's cross-sectional area is reaching zero, a tiny value is assigned to it to keep the mesh dimension of the FE model, i.e.,

$$A_i = \varepsilon \quad \text{when} \;\; A_i \leqslant \varepsilon, \tag{14}$$

where ε is prescribed a tiny positive value.

The feasibility of using an imaginary bar is easy to understand. In fact, as ε is a tiny value, the difference of the structural stiffness and mass matrices before and after removing the imaginary bar is very small and can be ignored.

It is a common practice in structural topology optimization that the structure is highly redundant. This results in an interesting phenomenon, where even if the sectional area of a member becomes zero, its corresponding expression of stress is still kept at a relatively large value. However, it can be explained easily. In fact, the stress of a member is determined by the relative displacement of the corresponding nodes to which the member is connected (see, Eq. (10)), while the relative displacement of the corresponding nodes are determined by all the members connected to them. Therefore, the stress constraints at a zero cross-sectional area can still be violated and the evaluating process of other members is affected. This is obviously unreasonable. Accordingly, singularity of the optimal solution under stress constraints as well as difficulties due to local buckling is extensively investigated in some papers [17–19]. Guo et al. [17] presented a second order smooth-extended technique and the so-called ε-relaxed method to find the solution of singular optima. A heuristic algorithm [18] for optimal design of trusses is also presented with account for stress and buckling constraints. Thus, in the paper we make use of a constraint deletion technique i.e., when a tiny cross-sectional area is reached, the corresponding stress and local stability constraints are ignored.

It is very efficient and feasible to overcome the difficulty of removing a member by using the above described imaginary bar method. But it is still very difficult to deal with node removal. When the mass of a node is considered, the node can never be removed by the imaginary bar method. In fact, when a node needs to be removed, all the sectional areas of the members connecting to the node should be equal to ε. While the mass of the node has a relatively large value, the truss will be led to a mechanism and the fundamental frequency will be led to zero. Even though we can assign a tiny value to the nodal mass for the node to be removed, the natural frequencies of the true structure may still be affected as the sectional areas of the members are also tiny. Therefore, further effort should be made towards node removal. A feasible method based on the concept of the proposed topology group will be discussed in detail in the following section.

3.2. Topology group method

The objective of topology optimization is to select a minimum cost truss from a given ground structure which contains a large set of candidate trusses. The ground structure is defined here as a truss with N predetermined members connecting M nodes. A candidate truss is obtained by removing possible members and nodes from the ground structure.

There are a large number of nodes and many more members in a practical ground structure. In the ground structure, some nodes can be removed, but others can never be removed; such as the supports, the nodes on which the external loads are imposed, and the nodes which if removed, a rigid body movement will be caused. As the ground structure for topology optimization is usually highly redundant, generally speaking, there are many possible nodal layouts and for a specific nodal layout there are still many candidate trusses. We define a specific possible configuration of the structure as a topology in which the support nodes, the nodes on which external load imposed, and the necessary nodes to keep the structure from rigid body movement and the members connected to those nodes in the ground structure should remain. It is obvious that as all possible nodal layouts are considered in topology group, the node-removal problem is overcome and it is unnecessary to remove any more nodes in a specific topology.

To illustrate this concept clearly, let us consider a well-known truss with six nodes and 10 members as shown in Fig. 1(a). For this ground structure, only node No. 1 could be removed, because nodes Nos. 5 and 6 are support nodes, Nos. 2 and 4 are nodes on which external loads are imposed, and node No. 3 is necessary to keep the structure from rigid body movement. Therefore, there are only two topologies for this ground structure as shown in Fig. 1(a) and (b), respectively.

Based on the concept of the topology group, a more general algorithm should be developed to work out all the possible topologies and to delete all meaningless topologies automatically. The algorithm is shown through a flow chart in Fig. 2. The optimization begins with the original structure that is specified as an initial ground structure. From the ground structure, a sequence of substructures with different node distributions, or a topology group, can be generated. In order to avoid the possibility of losing a good candidate truss and decrease the intervention of manual work, when we determine a possible topology, we need not care initially whether or not it is a mechanism, if it is not easily determined. Assume that there are NT possible topologies numbered in a sequence, where $NT = 2^{md}$ and $md = N_t - N_f$, in which N_t is the total node number of the ground structure and N_f is the number of the fixed nodes (such as support and loading nodes). After a specified topology is selected, consequently, the sectional area optimization process should be dealt with, for further weight reduction, and meanwhile we need to examine the topology as to whether it is a mechanism by checking or whether its stiffness matrix is singular. After optimizing all the NT possible topologies, the topology with a minimum cost is determined finally as the optimal topology.

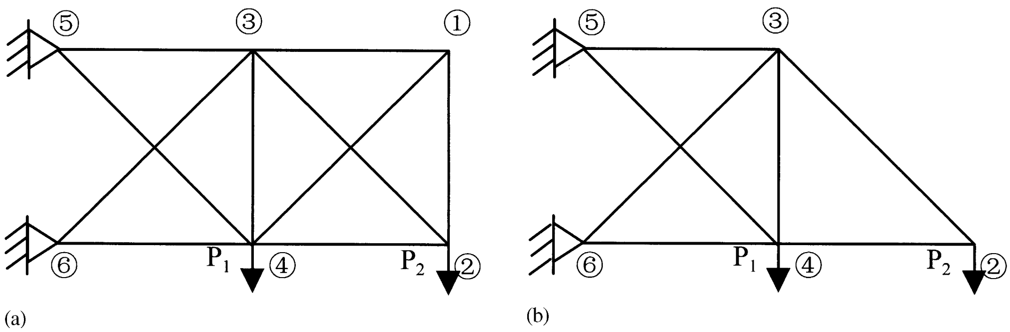


Fig. 1. (a) Ground structure and first topology for the 10-bar truss and (b) second topology for the 10-bar truss.

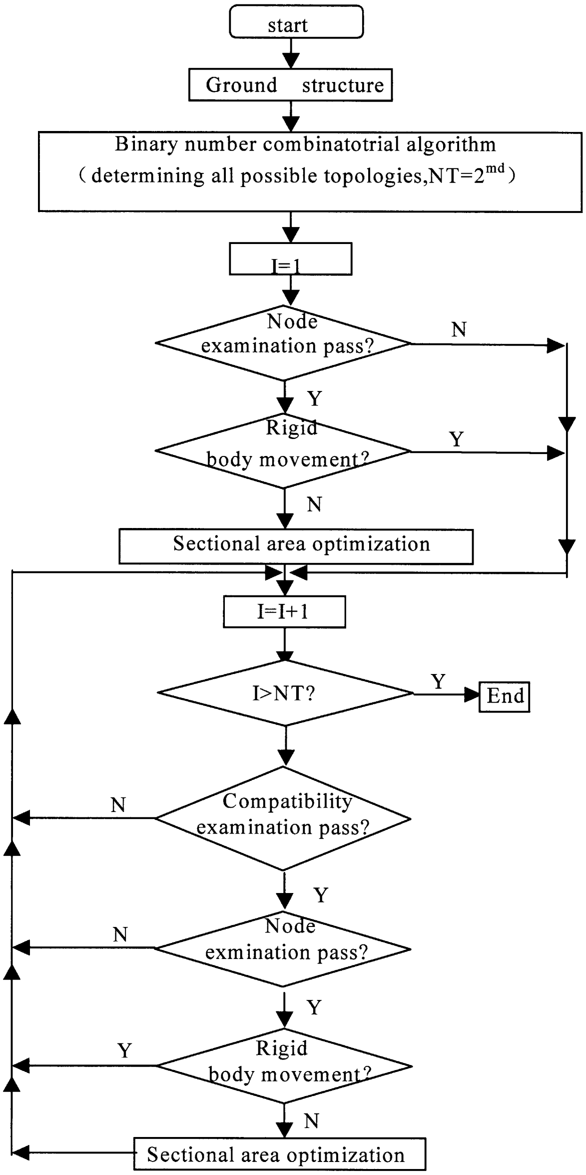


Fig. 2. Flow chart for the general algorithm.

In the general algorithm, a binary number combinatory algorithm is employed to determine all possible topologies. That is, when a node is removed, we note it as a binary number "1", if it remains, we note it as a binary number "0". For example, when there are three nodes, which can be possibly removed, then the possible topologies can be noted as (000), (001), (010), (100), (011),…, (111).

In the flow chart (Fig. 2), the "node examination" is performed by examining whether there is a node, which is only connected to two members. This is because when a node with no external loads to which is only connected to two members, which do not lie along with a straight line, the node must be meaningless, as those two members will not bear any stress. Therefore, when a topology has at least one such node, the topology can then be taken as meaningless and it is certainly unnecessary to do any further optimization analysis.

The "rigid body movement examination" is performed by examining whether the stiffness matrix is singular. Although all nodes of a certain topology are meaningful, the corresponding structure can still be a mechanism that results in rigid body movement. When a topology creates rigid body movement, it must not bear some types of external loads. Thus, it is meaningless and need not be analyzed any further too.

The "sectional area optimization" is performed by a one-dimensional search method. We perform the "compatibility examination" by examining whether the current topology needs to remove any nodes. For the above example, if node 2 cannot be removed, or the topology group (010) is meaningless, then topology group (011), (110) and (111) need not be examined for meaningfulness. Because when a node cannot be removed in a certain structure, it can never be removed in the substructures constructed from that certain one. Therefore, when a certain topology is determined meaningless, all the topologies constructed from the certain one need not to be examined or optimized.

3.3. Sectional area optimization

As a general rule, in solving non-linear programming problems, gradient and second-derivative method converge faster than the direct search methods. However, in practice, the derivative-type methods have two main barriers to their implementation. First, in structural optimization with dynamic constraints, it is laborious to provide analytical functions for the derivatives needed in a gradient algorithm. Second, the calculation of derivatives is time-consuming. Hence, a direct search optimization algorithm is introduced here. In order to use the unconstrained direct search technique, the original problem formulated in Eqs. (1)–(6) is transformed to an unconstrained one by using the penalty method that is

$$\text{Minimize} f(A, \lambda) = C(A) + \lambda \sum_{i=1}^{ncon} [\max(0, g_i(A))], \tag{15}$$

in which λ is a penalty multiplier, g_i denotes the constraints in Eq. (2)–(5) and $ncon$ denotes the total number of constraints stated. As constraints in sectional area and nodal cost can be easily satisfied in the calculation program, they are not considered in Eq. (15). The simplest, but feasible, type of search method, a one-dimensional search method, is used in this paper. The one-dimensional search algorithm is to determine the minimum point t_{min} of the element function

$$\varphi(t) = f(A^{(k)} + tS^{(k)}), \tag{16}$$

from the initial point $A^{(0)}$ (initial area values) in the searching direction $S^{(k)} = -\nabla f(A^{(k)})$, where f is objective function (e.g., the cost of the structure) and t is the interval of calculation. Actually, the search direction is defined using the gradient of the penalized objective function and the gradient is computed by the finite difference approach. Using the parabolic interpolation method to determine the optimal t_{min}, the corresponding optimum sectional areas are $A^{(k)} + t_{min}S^{(k)}$. When $A_i^{(k)} + t_{min}S_i^{(k)}$ is equal to zero, the corresponding member is then deleted in kth iterative loop.

However, as mentioned in the former sections, a member can hardly be deleted if it bears compressive stress and Euler buckling constraints are considered. In order to examine whether a member with compressive or tensile stress can truly be deleted or not, an attached step for this purpose should be considered in the one-dimensional search method, that is

$$A_i^{(k)} = \varepsilon \quad \text{if } f(A_i^{(k)} = \varepsilon) < f(A_i^{(k)} + t_{min} S_i^{(k)}), \tag{17}$$

where ε is prescribed a tiny positive value.

If a certain member with compressive or tensile stress can be removed, it will be removed by Eq. (17); otherwise, an optimal sectional area of the member will be determined by the one-dimensional search method.

4. Numercial example

The feasibility and effectiveness of the proposed method are illustrated by two examples. Let Euler buckling coefficient k_i in Eq. (11) be 4.0 which corresponds to one of a tubular member with a ratio of mean diameter to wall thickness of approximately 10.0. The elastic modulus and density of the structures are assumed to be 6.9×10^{10} Pa and 2740 kg/m^3, respectively. The corresponding allowable stress for each bar is assumed to be 172.43 MPa. The cost of each node is assumed to 5 kg of material in the examples.

4.1. 24-Bar truss

Fig. 3(a) shows the initial ground structure of the 24-bar truss structure. The initial sectional area is 10.0 cm^2 for each member. In this example, Stress, Euler buckling and the displacements of node 5 and 6 in the y direction, as well as the fundamental frequency constraint are imposed as the constraints. Two load conditions are considered as shown in Table 1. The displacement constraint demands that the amplitude of the corresponding displacement is not greater than 0.1 m. A fixed mass of 500 kg is lumped to node 3. The natural frequency constraint demands that the fundamental frequency is not less than 30.0 Hz.

There are four possible topologies as shown in Figs. 3(a)–(d). The optimal topology is determined as shown in Fig. 3(e), which is the output of topology group No. 4. The optimal sectional areas of the members are shown in Table 2. The characteristics before and after optimization are shown in Table 3. In addition, Euler buckling constraints are satisfied for each member.

4.2. 20-Bar truss

Fig. 4(a) shows the initial ground structure of the 20-bar truss structure. The initial sectional area is 50.0 cm^2 for each member. In this example, Stress, Euler buckling and the displacement of node 4 in the y direction as well as the fundamental frequency constraint are imposed as the constraints. Two load conditions are considered as shown in Table 4. The displacement constraint demands that the amplitude of the corresponding displacement is not greater than 0.01 m. The

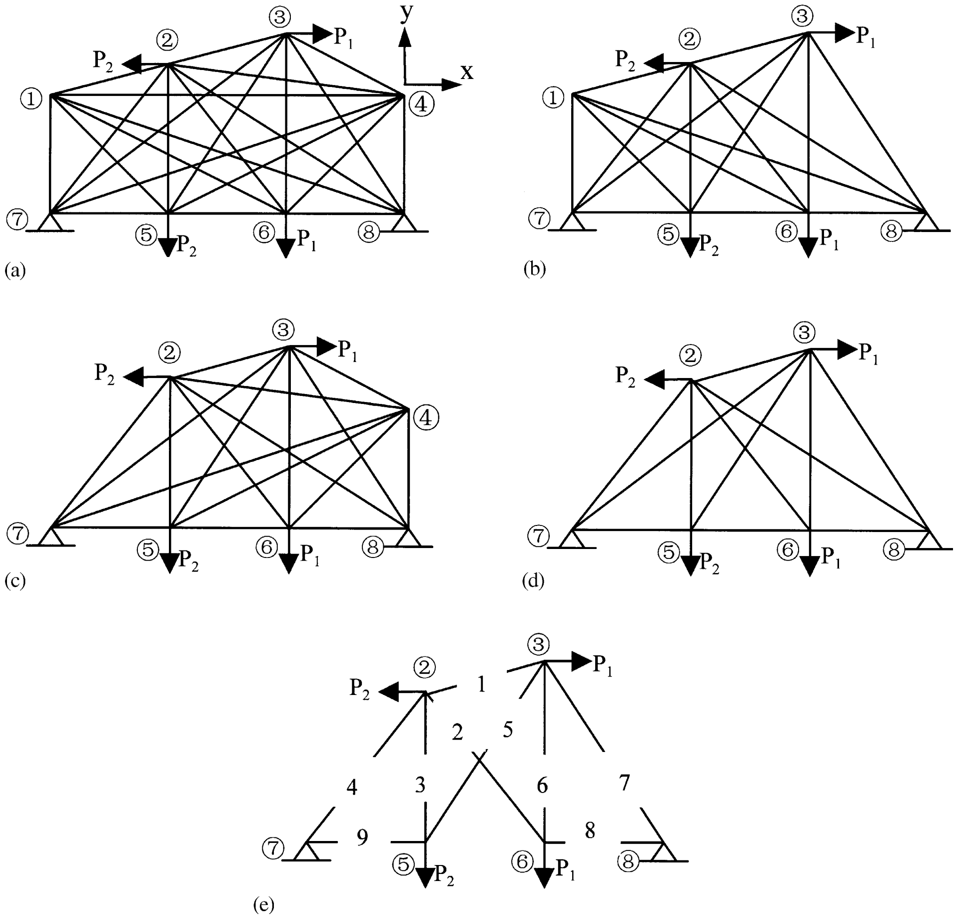


Fig. 3. (a) Ground structure and first topology for the 24-bar truss; (b) second topology for the 24-bar truss; (c) third topology for the 24-bar truss; (d) fourth topology for the 24-bar truss and (e) optimal topology for the 24-bar truss.

Table 1
Load condition for the 24-bar

	P_1 (N)	P_2 (N)
Load condition one	5.0×10^4	0.0
Load condition two	0.0	5.0×10^4

natural frequency constraint demands that the fundamental frequency and second frequency are not less than 60.0 and 100 Hz, respectively.

According to the binary number combinatorial algorithm, there are 64 possible topologies that are noted as (000000), (100000), (010000), (001000), (000100), (000010), (000001), (110000), (101000), …, (111111) in sequence. Node 5 cannot be removed, or else it results in rigid body movement. Based on the same reason, node 2 or 8 cannot be removed singly, i.e., topologies

Table 2
Optimal sectional area of the 24-bar

Bar no.	1	2	3	4	5	6	7	8	9
Area (cm^2)	36.5	15.0	9.51	11.0	17.6	13.8	16.0	14.5	11.02

Table 3
Characteristics of the 24-bar truss before and after optimization

	f_1 (Hz)	δ_{5y} (mm)	δ_{6y} (mm)	Cost (kg)
Before optimization	35.2	2.1	2.1	329.0
After optimization	30.0	3.2	3.0	167.0

(100000) and (000001) are meaningless. Thus, through the "nodal examination" and "rigid body movement examination", only 17 topologies are meaningful, which are (00000), (010000), (000100), (000010), (110000), (010100), (010010), (000110), (000011), (110100), (110010), (010110), (010011), (000111), (110110), (110011), (010111).Then through the "compatibility examination", there are only eight topologies that are optimized, which are noted as (000000), (010000), (000100), (000010), (010100), (010010), (000110), (010110) and shown in Figs 4(a)–(h). The optimal topology is determined as shown in Fig. 4(i), which is the output of topology No. 8. The optimal sectional areas of the members are shown in Table 5. The characteristics before and after optimization are shown in Table 6. In addition, Euler buckling constraints are satisfied for each member.

Because the nodal cost affects structural mass matrix, the optimal solution is different for different nodal cost. In order to clearly show how the nodal cost affects the optimal solution for specified constraints, optimal solutions are calculated for several different nodal masses. At the same time, total nodal masses are not included in optimal structural mass so as to show more obviously the relation between the optimal solution and the nodal cost. A very interesting linear relationship between total optimized bar weight and the different constant nodal cost are shown in Fig. 5. But, it should be further investigated whether the conclusion is a universal law for all the trusses in which the constant nodal cost is incorporated or just for the example of 20-bar truss only.

5. Conclusion

An optimization method that is called "a topology group" has been developed for finding the optimal topology of trusses subjected to constraints on natural frequencies, stress, displacement and Euler buckling under static loading conditions All the candidate node locations are fixed but a certain node may be deleted. Starting with the ground structure, all possible node layouts can be determined by using the binary number combinatorial algorithm. A possible node layout should at least include the support nodes and the nodes on which the external excitations are imposed. A sequence of substructures with different node layouts, called a topology group, can then be

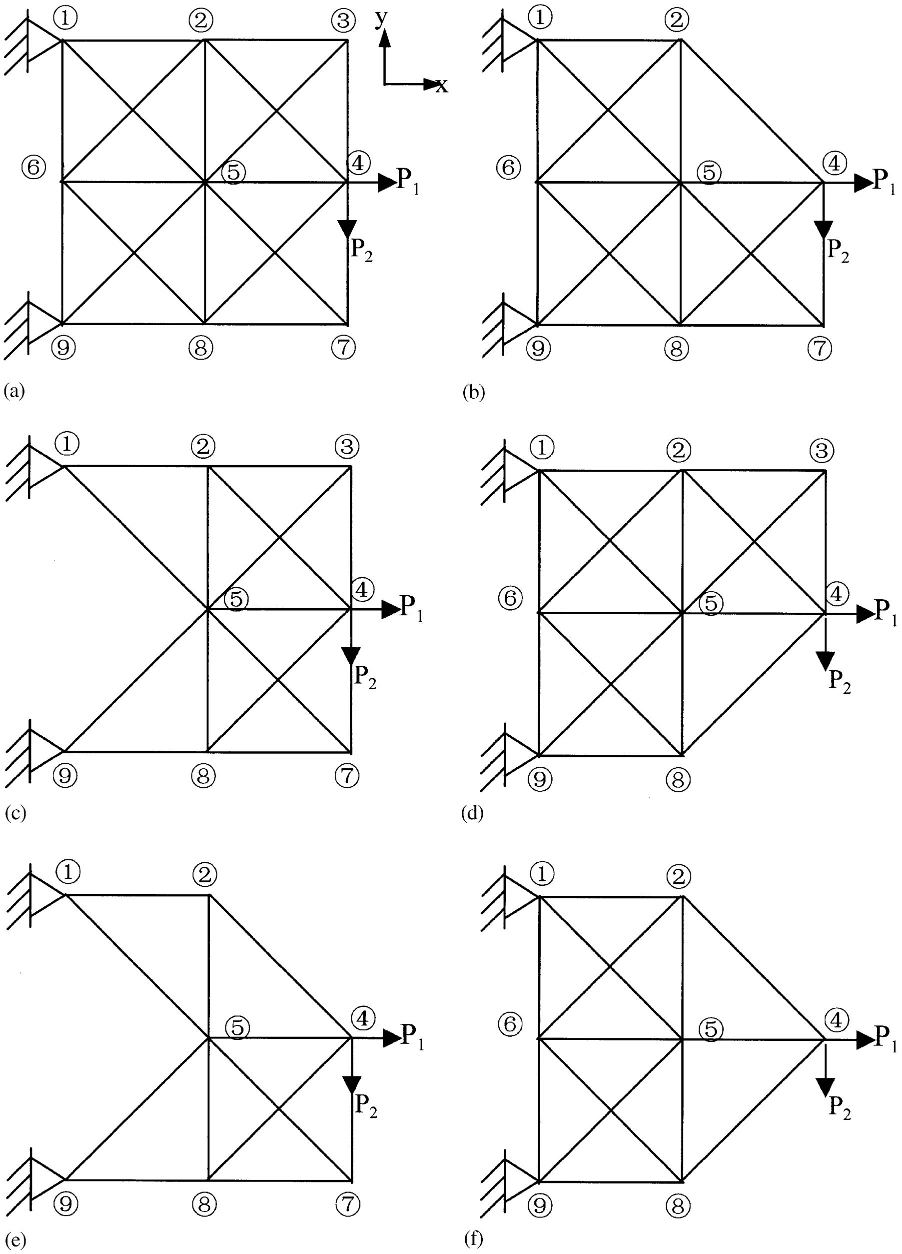
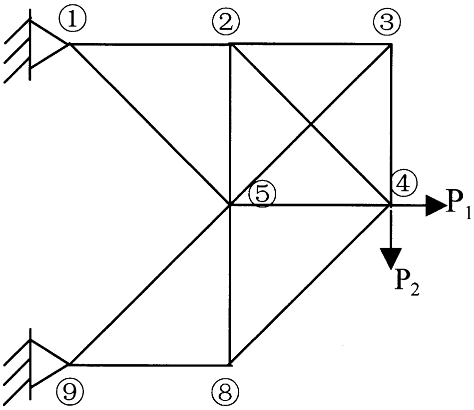
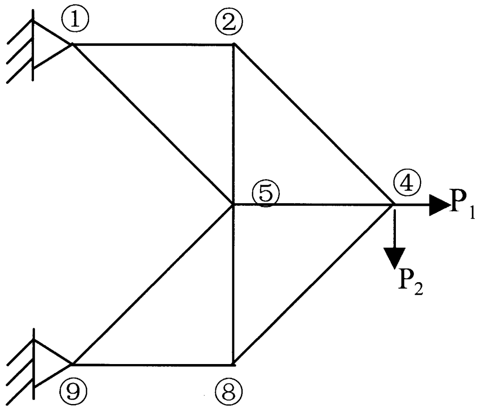


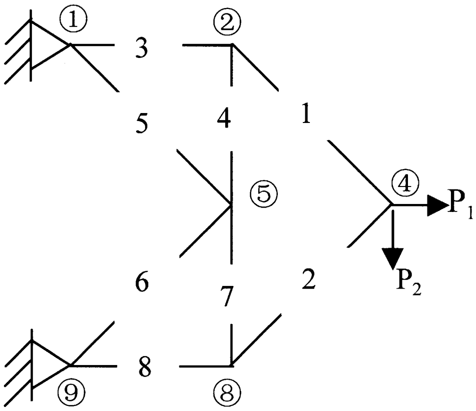
Fig. 4. (a) Ground structure and first topology for the 20-bar truss; (b) second topology for the 20-bar truss; (c) third topology for the 20-bar truss; (d) fourth topology for the 20-bar truss; (e) fifth topology for the 20-bar truss; (f) sixth topology for the 20-bar truss; (g) seventh topology for the 20-bar truss; (h) eighth topology for the 20-bar truss, and (i) optimal topology for the 20-bar truss.



(g)



(h)



(i)

Fig. 4 (*continued*).

Table 4
Load condition for the 20-bar truss

	P_1 (N)	P_2 (N)
Load condition one	5.0×10^5	0.0
Load condition two	0.0	5.0×10^5

Table 5
Optimal sectional area of the 20-bar truss

Bar no.	1	2	3	4	5	6	7	8
Area (cm^2)	71.97	3.76	53.38	53.38	42.12	53.44	3.04	3.02

Table 6
Characteristics of the 20-bar truss before and after optimization

	f_1 (Hz)	f_2 (Hz)	δ_{4y} (mm)	Cost (kg)
Before optimization	57.2	97.20	7.7	688.3
After optimization	60.0	130.4	10	225.9

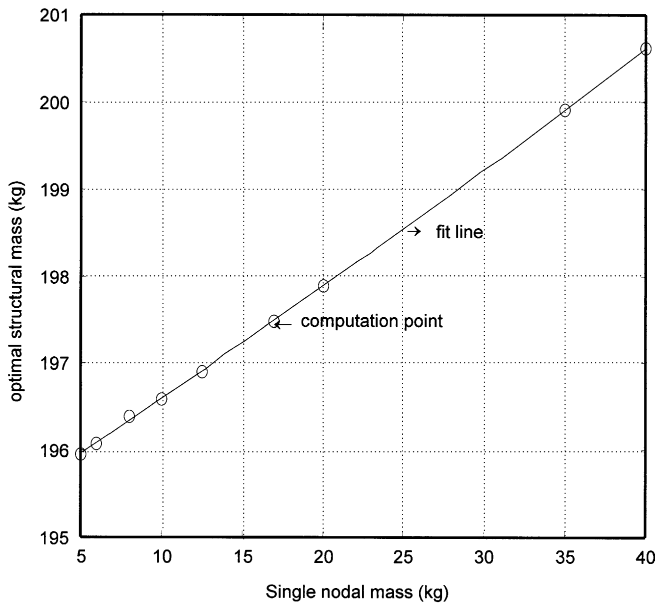


Fig. 5. Aspect of optimal solution w-r-t nodal cost.

generated. Before optimizing a topology, the meaningfulness of the corresponding topology should be examined as some topologies may include rigid body movement or have meaningless nodes. If a topology is meaningful, using the improved one-dimensional search method then optimizes it. The key technique in this paper to deal with Euler buckling constraints is that when a member is bearing compressive stress, a attached step is added to examine whether the corresponding member should be deleted or not. The imaginary bar method is also a feasible technique to maintain the computational dimension for a given topology. In order to optimize only those meaningful topologies, an efficient method is proposed in this paper to ignore the obviously meaningless topologies completely.

Acknowledgements

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