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Letter to the Editor

On the periodic solution for $\ddot{x} + x^{1/(2n+1)} = 0$

N. Shivakumar Swamy[a], H.R. Nataraja[b,*], K.S. Sai[c], Nageswara Rao[d]

[a] *Department of Computer Science and Engineering, Vivekananda Institute Of Technology, Gudimavu, Bangalore 560074, India*

[b] *Compressor Group, Gas Turbine Research Establishment, C.V. Raman Nagar, Bangalore 560093, India.*

[c] *Department of Mathematics, S.V.H. College of Engineering, Machilipatnam 521002, India.*

[d] *Structural Engineering Group, Vikram Sarabhai Space Centre, Trivandrum 695022, India*

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Mickens [1] has examined the equation of motion

$$\ddot{x} + x^{1/(2n+1)} = 0 \tag{1}$$

for a non-linear oscillator with the initial conditions

$$x(0) = x_0, \quad \dot{x}(0) = 0. \tag{2}$$

Here n is the positive integer and x_0 is the specified amplitude. Over dots denote differentiation with respect to time t. The approximate solution which satisfied the initial conditions (2) assumed in his analysis is

$$x(t) \cong A \cos \omega t. \tag{3}$$

Rewriting the equation of motion (1) in the form

$$[\ddot{x}]^{(2n+1)} + x = 0 \tag{4}$$

and using the method of harmonic balance [2], the amplitude, A, and the frequency of oscillations, ω, obtained by Mickens [1] are

$$A = x_0, \tag{5}$$

$$\omega = F_1(n) x_0^{-n/(2n+1)}, \tag{6}$$

*Corresponding author.

E-mail addresses: shivunes@yahoo.com (N.S. Swamy), hr_nataraj@yahoo.co.in (H.R. Nataraja), bnrao52@rediffmail.com (B.N. Rao).

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where

$$F_1(n) = \left\{ 2^{2n} \Big/ \begin{bmatrix} 2n+1 \\ n \end{bmatrix} \right\}^{1/(4n+2)}. \tag{7}$$

Ref. [3] indicates the possibility of obtaining exact solution for the non-linear second order differential equation (1) with the initial condition (2). The objective of the present study is to provide an approximate frequency–amplitude relation close to the exact by assuming a single-term solution or lower order harmonics (3) and following the Ritz procedure [3]. Following the procedure of Ref. [3] for "exact solution", the exact frequency–amplitude relationship obtained for the present problem is

$$\omega = F_2(n) x_0^{-n/(2n+1)}, \tag{8}$$

where

$$F_2(n) = \frac{n}{2}\sqrt{\frac{n}{(n+1)(2n+1)}} \Big/ \Gamma\left[\frac{2n+1}{2n+2}\right]. \tag{9}$$

Assuming the lower order harmonics or the single-term solution (3) and following the Ritz procedure [3], an approximate frequency–amplitude relation obtained for the present problem is

$$\omega = F_3(n) x_0^{-n/(2n+1)}, \tag{10}$$

where

$$F_3(n) = \left\{ \frac{1}{\sqrt{\pi}} \frac{1}{(n+1)} \Gamma\left[\frac{1}{4n+2}\right] \Big/ \Gamma\left[\frac{n+1}{2n+1}\right] \right\}^{1/2}. \tag{11}$$

In the present study, arguments of the gamma function are positive and they are less than unity. Expressions for the gamma function [4] which were found to be the accurate for small positive arguments (<1) were utilized in the present analysis.

Frequency–amplitude relations for the present problem given in Eqs. (6), (8) and (10) indicate that frequency of oscillations is inversely proportional to $x_0^{n/(2n+1)}$. Table 1 gives the comparison on the constant of proportionality in the frequency–amplitude relation (10) obtained using the single-term solution (3) and the Ritz procedure, is close to the exact relation (8). It is very interesting to note that the solution of the problem for large n will be close to that of antisymmetric constant force oscillation equation [5,6].

When $n \to \infty$, the exact frequency–amplitude relation (8) yields

$$\omega \to \frac{\pi}{2\sqrt{2x_0}}, \tag{12}$$

Table 1
Comparison on the constant of proportionality ($\omega x_0^{n/(2n+1)}$) in the frequency–amplitude relations

n	Present study			Mickens [1]	
	Exact solution Eq. (9)	Ritz procedure Eq. (11)	Relative error (%)	Harmonic balance Eq. (7)	Relative error (%)
1	1.070451	1.076845	−0.6	1.049115	2.0
2	1.086126	1.096092	−0.9	1.048122	3.5
3	1.093018	1.104867	−1.1	1.044052	4.5
4	1.096894	1.109890	−1.2	1.040169	5.2
5	1.099377	1.113145	−1.3	1.036840	5.7
10	1.104745	1.120276	−1.4	1.026280	7.1
25	1.108251	1.125009	−1.5	1.014945	8.4
50	1.109472	1.126671	−1.5	1.009188	9.0
100	1.110093	1.127519	−1.6	1.005458	9.4

which is nothing but the result of Ref. [6].

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