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Letter to the Editor

Simplified formula for the vibration frequency of circular cylinders

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1. Introduction

The analysis of the dynamical behavior of a circular cylinder has attracted the attention of many researchers due to both its important practical application and relative simplicity [1–4]. It should be noted that a construction of the exact solution to the problem is possible but unfortunately it yields complicated transcendental equations, and serious difficulties occur while studying their roots (frequencies). Therefore, many various approximate techniques are applied yielding the solutions in an explicit form. However, the existing approximate approaches are only applicable for the defined wavelengths.

On the other hand, two limiting cases can be strictly distinguished while analyzing cylinder oscillations. The first one corresponds to long-wave oscillations well described by classical theory (with Raleigh's first correction). The second one governs disc oscillations. Therefore, a naturally motivated idea to match both limiting cases occurs. In Ref. [5] numerical matching has been illustrated. In this Letter, we apply a two-point Padé approximation method which yields the required approximate analytical solution.

2. Method

We introduce a parameter ε describing the ratio of the cylinder, radius R, over the oscillation wavelength l ($\varepsilon = R/l$). The first limiting case corresponds to long-wave oscillations corresponding to $\varepsilon \ll 1$. A zero order approximation ($\varepsilon \to 0$) is defined by the frequency of classical bar oscillations $\omega = 2\pi l^{-1}\rho^{1/2}E^{-1/2}$, where E is Young's modulus and ρ denotes material density. The next approximation (i.e., the next term in the sought series relating to ε) is defined by Rayleigh's

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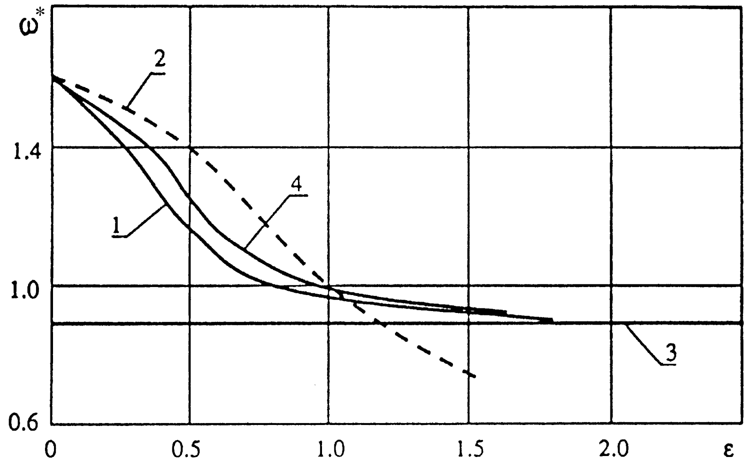

Fig. 1. Solutions obtained using different methods (see text).

formula [1,3], which reads as

$$\omega^* = (\omega/\sqrt{\rho/E}) - (2\pi/l)(1 - \pi^2 v^2 \varepsilon^2), \tag{1}$$

where v denotes the Poisson ratio.

The second limiting case corresponds to waves with very small length ($\varepsilon \gg 1$), which for the disc obtained from the cylinder serves as a zero order approximation to the analyzed oscillations. The frequency oscillations are defined via the following transcendental equation [3–5]:

$$\omega^* \lambda \varepsilon \mathrm{J}_0(\omega^* \lambda \varepsilon) - \mathrm{J}_1(\omega^* \lambda \varepsilon) = 0, \tag{2}$$

where $\lambda = 2\pi\sqrt{E/G - 1}$, G is the shear modulus of the cylinder material, and $\mathrm{J}_0, \mathrm{J}_1$ are the Bessel's functions.

The first root of Eq. (2) may be written as [5]:

$$\omega^* = 2.40483\lambda^{-1}\varepsilon^{-1}. \tag{3}$$

In Fig. 1 the exact solution (curve 1) found numerically [3], as well as solutions (2) and (3) are shown (curves 2 and 3, correspondingly, for $\omega = \omega^* \sqrt{E/G}, v = 0.29$).

It can be concluded that formula (2) approximates reasonably well the exact solution for $\varepsilon \to 0$, whereas formula (3) for $\varepsilon \to \infty$. The problem reduces to that of matching of both mentioned solutions and a construction in the average zone ($\varepsilon \sim 1$). To solve the stated problem we apply the two-point Padé approximants. We briefly describe its idea [6]. Let

$$f(\varepsilon) \sim \sum_{i=0}^{\infty} a_i \varepsilon^i \quad \text{for } \varepsilon \to 0, \tag{4}$$

$$f(\varepsilon) \sim \sum_{i=0}^{\infty} b_i \varepsilon^i \quad \text{for } \varepsilon \to \infty. \tag{5}$$

Then the two-point Padé approximant is governed by the expression

$$\varphi(\varepsilon) = \frac{\sum_{i=0}^{\infty} \alpha_i \varepsilon^i}{\sum_{i=0}^{\infty} \beta_i \varepsilon^i}. \tag{6}$$

The coefficients (α_i, β_i) are chosen in a way that the series of φ for $\varepsilon \to 0$ and $\varepsilon \to \infty$ coincide with series (4) and (5), correspondingly.

Matching of solutions (1) and (3) can be carried out using formula (6). The matching result is shown in Fig. 1 (curve 4). It is seen that a smooth approximation curve is obtained close to the exact solution.

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