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Multiple chatter frequencies in milling processes

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Abstract

Analytical and experimental identifications of the chatter frequencies in milling processes are presented. In the case of milling, there are several frequency sets arising from the vibration signals, as opposed to the single well-defined chatter frequency of the unstable turning process. Frequency diagrams are constructed analytically and attached to the stability charts of mechanical models of high-speed milling. The corresponding quasiperiodic solutions of the governing time-periodic delay-differential equations are also identified with some milling experiments in the case of highly intermittent cutting.

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1. Introduction

The history of machine tool chatter goes back to almost 100 years, when Taylor [1] described machine tool chatter as the most obscure and delicate of all problems facing the machinist. After the extensive work of Tlustý et al. [2] and Tobias [3], the so-called regenerative effect has become the most commonly accepted explanation for machine tool chatter [4–7]. This effect is related to the cutting force variation due to the wavy work-piece surface cut during the previous revolution. The corresponding mathematical models are delay-differential equations (DDE). Stability properties can be predicted through the investigation of these DDEs [8,9]. The identification of the resulting vibrations can effectively be supported by frequency analysis of the chatter signal [10–12]. The stability charts published in the specialist literature are almost always accompanied by frequency diagrams that represent the chatter frequencies at the loss of stability. The reason for this custom is that these frequencies can be identified precisely experimentally and so this is a direct way to verify theoretical models and predictions.

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For the simplest model of turning, the governing equation of motion is an autonomous DDE with a corresponding infinite dimensional state space. This fact implies the existence of an infinite number of characteristic roots, most of them having negative real parts referring to damped components of the vibration signals. There may be some finite number of characteristic roots that have positive real parts. Each of those roots which is pure imaginary corresponds to a single well-defined vibration frequency. For turning, these critical chatter frequencies are usually 0–15% above the well-separated lowest (or single) natural frequency of the machine tool structure [8]. The study of non-linear phenomena in the cutting process showed that these chatter frequencies are related to unstable periodic motions about stable stationary cutting, i.e., a so-called subcritical Hopf bifurcation occurs, as it was proved experimentally by Shi and Tobias [13] and analytically by Stépán and Kalmár-Nagy [14].

The model of the milling process is more complex. The tooth pass excitation effect results in a parametric excitation of the system, and the governing equation of motion is a time periodic DDE. These systems can be investigated by the extended Floquet theory of DDEs [15–17]. A time periodic DDE also has an infinite dimensional state space, but characteristic multipliers are defined instead of characteristic roots. Most of the infinite number of characteristic multipliers are located within the open unit disc of the complex plane referring to damped oscillation components, and only a finite number of multipliers can have a magnitude greater than 1. The critical multipliers are located on the unit circle and each of them refers to an infinite series of vibration frequencies.

Several analytical methods were developed to determine the stability properties of the milling process [18–23]. Numerical simulation may also serve to provide a satisfactory result for this purpose [24,25]. The analytical investigations lead to the realization of new bifurcation phenomena. In addition to Hopf bifurcation, period doubling bifurcation is also a typical way of stability loss in milling processes, as it was shown analytically by Davies et al. [21], Insperger and Stépán [26], Corpus and Endres [27], Bayly et al. [22] and experimentally by Davies et al. [21], Bayly et al. [22]. The non-linear analysis of Stépán and Szalai [28] showed that this period doubling bifurcation is subcritical.

In spite of all these research efforts, the identification of the critical chatter frequencies at the loss of stability is not a trivial task either experimentally or theoretically. The power spectra of the signals show several peaks of complicated structure. Some of them refer to the tooth pass excitation effect, others refer to the regenerative effect and the natural frequency of the tool also appears. In the subsequent Sections, a clear picture is given about these frequencies arising in chatter during the milling process.

2. Mechanical model

The mechanical model of the milling process can be seen in Fig. 1. The mass m of the tool, the damping coefficient c and the spring stiffness k can be determined via the modal analysis of the machine tool. x is the displacement of the centre of the tool relative to the workpiece. The structure is assumed to be flexible in the x direction only. This reduces the model to single degree of freedom (s.d.o.f.). This s.d.o.f. model is appropriate, for example, when a thin-walled

workpiece is the most flexible part of the structure. In this case, the workpiece is likely to be asymmetric and compliant in one direction while rigid in the orthogonal direction.

Assume the prescribed feed motion is uniform with a constant speed v of the tool. According to Newton’s law, the equation of motion reads

$$m\ddot{x}(t) = -F_x + k(vt - x(t)) + c(v - \dot{x}(t)). \tag{1}$$

To determine the cutting force F_x , further analysis of the cutting process is needed. The tangential component of the cutting force acting on an active tooth (number j) can be approximated by

$$F_{jt} = Kw(f \sin \varphi_j)^{x_F}, \tag{2}$$

where K is the cutting coefficient, w is the depth of cut, f is the feed per tooth and φ_j denotes the angular position of the tool. The exponent x_F is a small constant. $x_F = 0.8$ is a typical value for this parameter [29]. The normal component of the cutting force is usually estimated as [29]

$$F_{jn} = 0.3F_{jt}. \tag{3}$$

Recently, Halley [30] also verified this formula experimentally. The x component of the cutting force depends on the angular position of the tool as it can be seen in Fig. 2:

$$F_{jx} = g_j(t)(F_{jt} \cos \varphi_j + F_{jn} \sin \varphi_j). \tag{4}$$

Here, $g_j(t)$ is a screen function [31]; it is equal to 1, if the j th tooth is active and 0 if it is not.

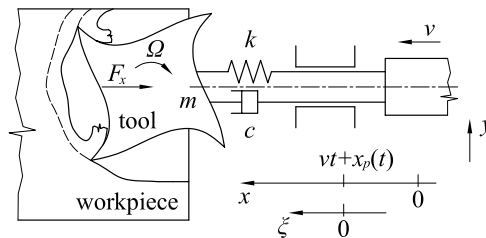


Fig. 1. Regenerative mechanical model for milling.

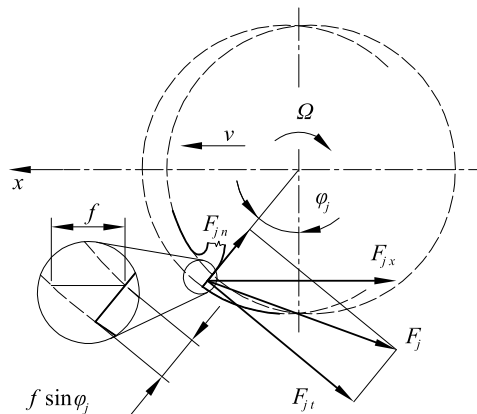


Fig. 2. Cutting force components.

Let the spindle speed of the tool be denoted by Ω (r.p.m.), so the tooth pass period is $\tau = 60/(z\Omega)$ (s), where z is the number of the teeth. The feed is equal to the difference of the present and the delayed position of the tool, i.e., $f = x(t) - x(t - \tau)$. The angular position of each tooth depends on the time as follows: $\varphi_j = \Omega t + j\vartheta$, where $\vartheta = 2\pi/z$. Consequently, the x component of the cutting force acting on the tool is given by the sum of F_{jx} (see Eq. (4)) for all j . Introducing the τ -periodic function $q(t)$, the excitation force in Eq. (1) reads

$$F_x = wq(t)(x(t) - x(t - \tau))^{x_F}, \quad (5)$$

where

$$q(t) = K \left(\sum_{j=1}^z g_j(t) \sin^{x_F}(\Omega t + j\vartheta) (\cos(\Omega t + j\vartheta) + 0.3 \sin(\Omega t + j\vartheta)) \right). \quad (6)$$

Thus, the equation of motion is the following non-autonomous non-linear delay differential equation

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -wq(t)(x(t) - x(t - \tau))^{x_F} + kv t + cv. \quad (7)$$

Note that the time period of $q(t)$ is equal to the time delay τ .

3. Linearization about the unperturbed motion

Assume the tool motion in the form

$$x(t) = vt + x_p(t) + \zeta(t), \quad (8)$$

where vt is the linear feed motion, $x_p(t) = x_p(t + \tau)$ is a τ -periodic motion that can also be considered as the unperturbed, or ideal tool motion when no self-excited vibrations arise, and $\zeta(t)$ is the perturbation (see Fig. 1). Substitute Eq. (8) into Eq. (7) to get

$$m\ddot{x}_p(t) + c\dot{x}_p(t) + kx_p(t) + m\ddot{\zeta}(t) + c\dot{\zeta}(t) + k\zeta(t) = -wq(t)(vt + \zeta(t) - \zeta(t - \tau))^{x_F}. \quad (9)$$

In the ideal case, $\zeta(t) \equiv 0$ and the tool moves according to $x(t) = vt + x_p(t)$. This case gives an ordinary differential equation for x_p as

$$m\ddot{x}_p(t) + c\dot{x}_p(t) + kx_p(t) = -w(v\tau)^{x_F} q(t). \quad (10)$$

Since this is a linear system with τ -periodic excitation, it has a τ periodic solution, namely, the particular one. This proves the existence of the τ -periodic function $x_p(t)$ and verifies Eq. (8). Furthermore, it can be seen that $x_p(t)$ has the same harmonics as the excitation $q(t)$. In general, this means that all the higher harmonics of the basic frequency $2\pi/\tau$ appear in $x_p(t)$.

For linear stability analysis, the variational system of Eq. (7) is determined about the combined linear and periodic motion $vt + x_p(t)$. Expand the non-linear term in Eq. (9) into Taylor series with respect to ζ and neglect the higher order terms to get

$$\begin{aligned} m\ddot{x}_p(t) + c\dot{x}_p(t) + kx_p(t) + m\ddot{\zeta}(t) + c\dot{\zeta}(t) + k\zeta(t) \\ = -w(v\tau)^{x_F} q(t) - w x_F (v\tau)^{x_F-1} q(t) (\zeta(t) - \zeta(t - \tau)). \end{aligned} \quad (11)$$

Using Eqs. (10) and (11), a linear time periodic DDE is obtained for ξ as

$$m\ddot{\xi}(t) + c\dot{\xi}(t) + k\xi(t) = -wh(t)(\xi(t) - \xi(t - \tau)), \quad (12)$$

where $h(t) = x_F(v\tau)^{x_F-1}q(t)$ is the specific force variation.

Eq. (12) is considered as a standard linear DDE model of the milling process.

4. Frequencies during chatter

Chatter arises if the linear Eq. (12) loses stability or there is resonance in Eq. (10). Resonance can easily be described by the ratio of the natural frequency of the machine tool structure and the exciting frequency. This resonant case is not considered here, and only the self-excited chatter related to the loss of stability considered in Eq. (12) is investigated.

The stability properties of Eq. (12) are determined by the infinite number of characteristic multipliers, as explained in the Section 1, by the extended Floquet theory of DDEs. If $\mu = e^{\lambda\tau}$ is a characteristic multiplier of Eq. (12), then there exists a solution in the form

$$\xi(t) = p(t)e^{\lambda t} + \bar{p}(t)e^{\bar{\lambda}t}, \quad (13)$$

where $p(t) = p(t + \tau)$ is τ -periodic function, λ is the so-called characteristic exponent and bar denotes complex conjugates. Eq. (12) is asymptotically stable, if and only if, all the characteristic multipliers are in modulus less than one; in other words, if all characteristic exponents have negative real part.

The stability analysis can be based on the determination of the relevant characteristic multiplier. There are several approximation methods to carry out this calculation [26,22,32]. The vibration frequencies corresponding to the relevant characteristic multiplier can be determined in the following way.

If Eq. (12) is at the border of stability, then there is at least one characteristic multiplier (either one real, or one complex pair) with a modulus of one. All of the other infinite number of characteristic multipliers have moduli less than one, so they are not important for chatter frequency analysis.

The critical characteristic multipliers can be located in three ways:

1. They are a complex pair located on the unit circle ($|\mu| = 1$ and $|\bar{\mu}| = 1$). This case is topologically equivalent to the Hopf bifurcation of autonomous systems and called a secondary Hopf or Naimark–Sacker bifurcation.
2. $\mu = 1$. The associated bifurcation is topologically equivalent to the saddle-node bifurcation of autonomous systems and is called a period one bifurcation.
3. $\mu = -1$. There is no topologically equivalent type of bifurcation for autonomous systems. This case is called period two, period doubling, or flip bifurcation.

It can easily be seen that the $\mu = 1$ case cannot arise in Eq. (12) [21,23].

For a given $|\mu| = 1$, $\lambda = i\omega$ is pure imaginary, where $\omega = (\ln \mu)/\tau$. Essentially, the chatter frequencies are assigned by ω . Since the complex exponential function is periodic, the logarithmic function is not unique in the plane of complex numbers. This raises the possibility of multiple chatter frequencies. To give a clear view of the resulting frequencies, Eq. (13) must be analyzed.

For the secondary Hopf case, the characteristic exponents are also a complex pair. It is possible to substitute $\lambda = i\omega$ into Eq. (13), expand $p(t)$ into Fourier series and use trigonometrical transformations. Then Eq. (13) can be written in the form

$$\xi(t) = \sum_{n=-\infty}^{\infty} (C_n e^{i(\omega+n2\pi/\tau)t} + \bar{C}_n e^{i(-\omega+n2\pi/\tau)t}), \quad (14)$$

where C_n 's are complex coefficients. This shows that the frequencies arising in the signal $\xi(t)$ are

$$f_H = \left\{ \pm\omega + n\frac{2\pi}{\tau} \right\} [\text{rad/s}] = \left\{ \pm\frac{\omega}{2\pi} + n\frac{z\Omega}{60} \right\} [\text{Hz}], \quad n = \dots, -1, 0, 1, \dots, \quad (15)$$

where τ is given in seconds, Ω in r.p.m. The index of f_H refers to the secondary Hopf bifurcation. There are an infinite number of frequencies with amplitudes corresponding to the coefficients C_n 's. This is in accordance with the periodic property of the complex exponential function mentioned before. Of course, only the positive values of f_H have physical meaning.

For the period doubling case ($\mu = -1$), the characteristic exponent is $\lambda = (\ln(-1))/\tau$ and the frequencies can be written in the simple form of

$$f_{PD} = \left\{ \frac{\pi}{\tau} + n\frac{2\pi}{\tau} \right\} [\text{rad/s}] = \left\{ \frac{z\Omega}{30} + n\frac{z\Omega}{60} \right\} [\text{Hz}], \quad n = \dots, -1, 0, 1, \dots, \quad (16)$$

where the index of f_{PD} refers to the period doubling bifurcation.

Either the frequency set f_H or f_{PD} shows up during chatter. If Eq. (12) is stable, then these frequencies do not arise.

5. Other non-chatter vibration frequencies during milling

The frequencies excited during the milling operation are related to all components of $x(t)$ defined by Eq. (8). The term vt is the linear feed motion, and it does not contain any periodicity, but the periodic motion $x_p(t)$ contains the following frequencies:

$$f_{TPE} = \left\{ \frac{nz\Omega}{60} \right\} [\text{Hz}], \quad n = 1, 2, \dots, \quad (17)$$

as it was shown by Eq. (10). The index of f_{TPE} refers to the tooth pass excitation effect.

Since the damping of machine tools is small, the transient phenomena decay slowly. This results in another peak in the spectrum at the well-separated lowest damped natural frequency $f_d = \omega_n \sqrt{1 - \zeta^2}/(2\pi)$ of the machine tool structure. Here, $\omega_n = \sqrt{k/m}$ is the angular natural frequency and $\zeta = c/(2m\omega_n)$ is the relative damping factor.

The frequencies f_{TPE} and f_d are present in the vibration signal both for stable and unstable cutting.

6. Experimental verification

Milling tests were performed with an experimental flexure designed to mimic the s.d.o.f. system described above. A monolithic, uni-directional flexure was machined from aluminium and

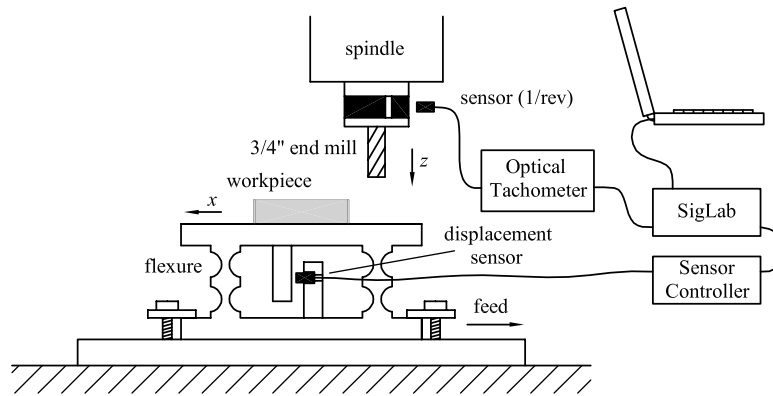


Fig. 3. Scheme of the experiment.

instrumented with a single non-contact, eddy current displacement transducer as shown in Fig. 3. Aluminium (7075-T6) test samples of width 1/4 in (6.35 mm) were mounted on the flexure and centrally milled by a 3/4-inch (19.05 mm) diameter carbide end mill with a single flute (the second flute was ground off to remove any effects due to asymmetry or runout). Feed was held constant: $v\tau = 0.004$ in = 0.1016 mm.

The measured stiffness of the flexure to deflections in the x direction was $k = 2.2 \times 10^6$ N/m. In comparison, the values of stiffness in the orthogonal y and z directions were more than 20 times greater than that in the x direction. The natural frequency was experimentally determined to be $f_n = 146.8$ Hz, and the relative damping factor was $\zeta = 0.0038$, which corresponds to very light damping. Consequently, the damped natural frequency of the flexure was $f_d \approx f_n = 146.8$ Hz.

The displacement transducer output was anti-alias filtered with 500 Hz cutoff and sampled (16-bit precision, 12 800 samples/s) with SigLab 20-22A data acquisition hardware connected to a Toshiba Tecra 520 laptop computer. A periodic 1/rev pulse was obtained with the use of a laser tachometer to sense a black–white transition on the rotating tool holder (see Fig. 3).

Displacement data were recorded for 15 s. The spectral analysis was performed using Hanning window. No averaging procedures were used. The calibration of the displacement sensor was 1.303×10^{-4} m/V.

Theoretical stability charts and the chatter frequencies were determined through investigation of the characteristic multipliers calculated by the semi-discretization method [32]. The infinite dimensional system (12) was approximated by a 22-dimensional discrete system, which resulted in errors of less than 2% for the stability boundaries in the presented parameter domain. The execution time of one computation with fixed parameters was 0.023 s with a 400 MHz Pentium II processor. For the construction of the chart in Fig. 4, an 800×200 grid was taken in the parameter plane of spindle speed and depth of cut, and the relevant characteristic multipliers were determined for each parameter points. In this way, the computation time of the stability chart was about 1 h.

For the calculations, the following experimentally identified parameters were used: $m = 2.586$ kg, $k = 2.2 \times 10^6$ N/m, $c = 18.13$ N s/m. Based on the experimental results of Halley [30], the cutting coefficient was chosen to the reasonable value $K = 1.9 \times 10^8$ N/m $^{1+x_F}$, with $x_F = 0.8$. The value of x_F is also confirmed in the book of Tlustý [29].

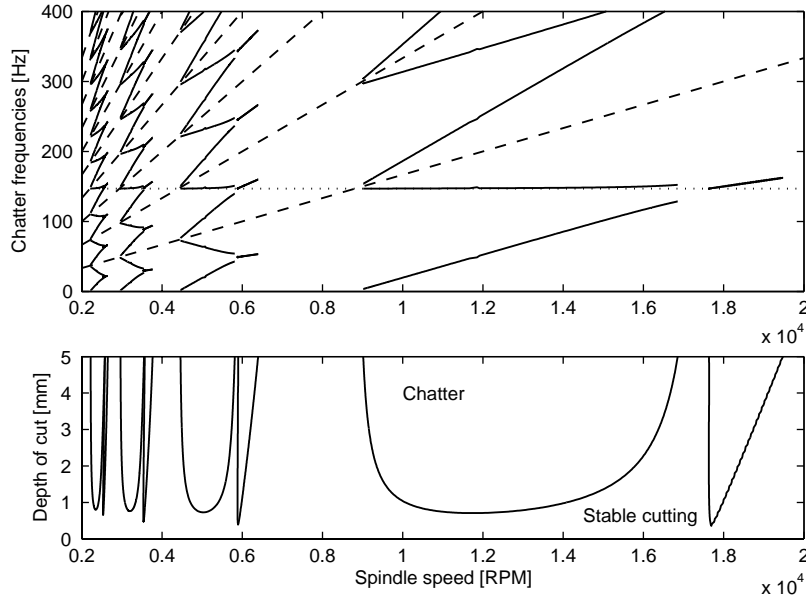


Fig. 4. Theoretical stability chart and chatter frequencies.

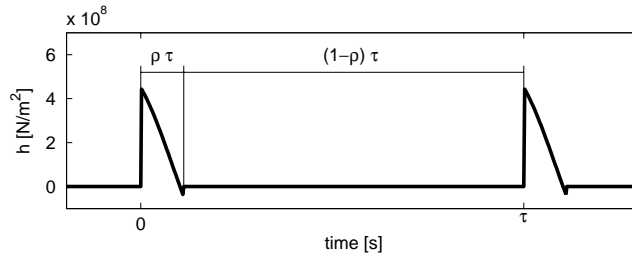


Fig. 5. The specific force variation $h(t)$.

The relative position of the tool and the workpiece defines the specific force variation $h(t)$ from Eq. (12). The ratio of time spent cutting to not cutting is $\rho = 0.1082$, as it can be seen in Fig. 5.

The theoretical stability chart and the corresponding chatter frequencies can be seen in Fig. 4. Solid lines denote the chatter frequencies f_H and f_{PD} . Dashed lines refer to the frequencies f_{TPE} caused by the tooth pass excitation effect, and a dotted line denotes the damped natural frequency f_d of the flexure.

Milling tests were carried out over a specified range of speeds and axial depths of cut. The results are presented in Fig. 6, where \circ denotes stable cutting, and \times denotes unstable operations. The experimental data correlate with the theoretical predictions gained from this simple s.d.o.f. mechanical model. The specified parameter points A, B and C relate to constant depth of cut $w = 2$ mm and three different spindle speeds $\Omega = 3300, 3500,$ and 3590 r.p.m., respectively. Point A is in an unstable parameter domain of Hopf type, point B is in a stable

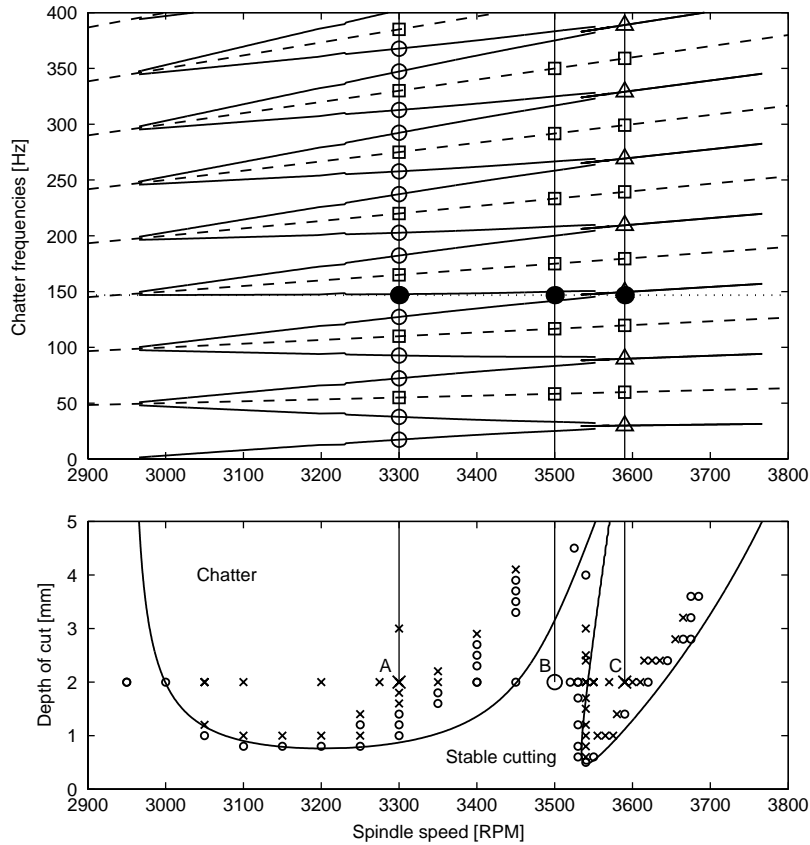


Fig. 6. Theoretical stability boundaries with the corresponding vibration frequencies (\circ — f_H , \triangle — f_{PD} , \square — f_{TPE} , \bullet — f_d) and experimental results (\circ —stable cutting, \times —chatter).

domain, and point C is in an unstable domain of period doubling type. The vertical lines raised from the corresponding parameter points of the chart intersect the frequency diagram above the chart and assign the frequency sets belonging to the corresponding vibration signal of the machine tool. The symbols \circ , \triangle , \square and \bullet refer to the four different classes of frequency sets f_H , f_{PD} , f_{TPE} and f_d , respectively. The same symbols also appear in Fig. 8. The three power spectra are calculated from the three vibration signals presented in Fig. 7 in three different forms: time history, sampled time history, and Poincaré (or stroboscopic) map. In the power spectra of Fig. 8, the dashed lines denote the theoretical tooth pass excitation frequency and its higher harmonics. The symbols mentioned above help to identify all the various frequency sets.

For parameter point A, the theory shows that the relevant characteristic multiplier is a complex pair. The experiment confirms the theoretical expectation: the most dominant peaks in the power spectrum show up at the frequencies f_H , f_{TPE} and f_d .

Cutting defined by parameter point B is stable, so only frequencies f_{TPE} and f_d are expected. This is also confirmed by the measurement result.

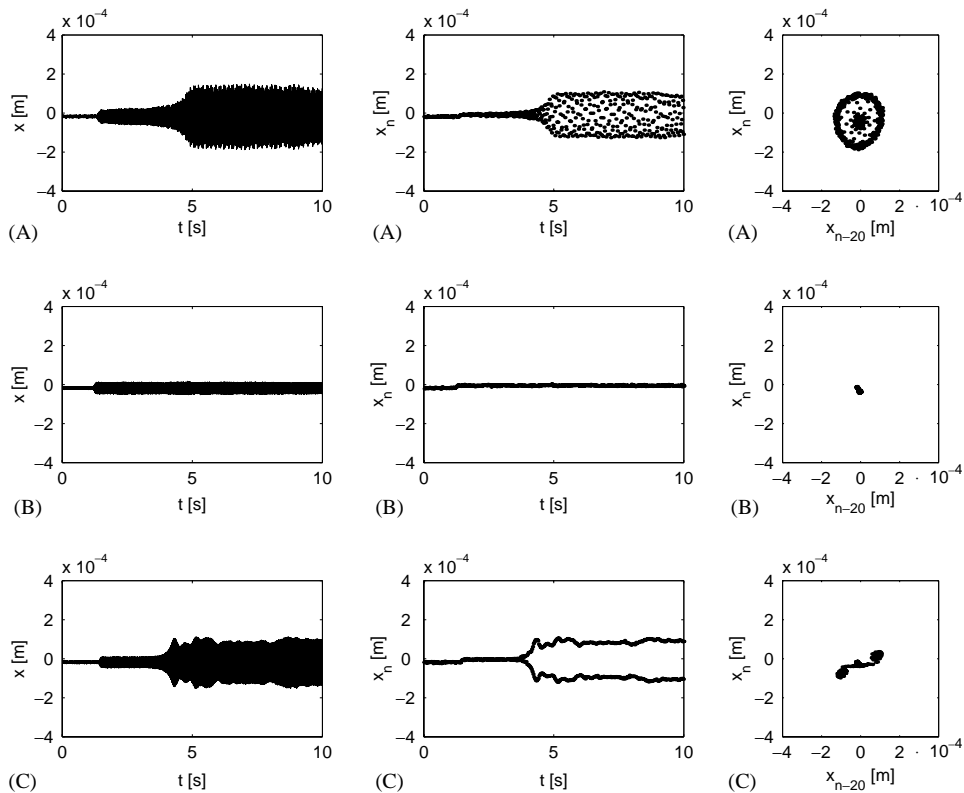


Fig. 7. Continuous time histories, 1/rev sampled signals, and Poincaré sections for parameter points A, B and C.

Parameter point C defines an unstable, period doubling cutting process. In this case, the most dominant peaks in the power spectrum are at the frequencies f_{PD} , f_{TPE} and f_d , and clearly, it is also confirmed by the experiments.

The transition between the secondary Hopf and the period doubling case can be followed in the chatter frequency plots of Figs. 4 and 6. For a secondary Hopf type chatter, there are two f_H -frequencies in the neighbourhood of each f_{TPE} -frequency, one below, and one above. As the spindle speed is increased, the f_H -frequencies move away from the f_{TPE} -frequencies until they meet the f_H -frequencies belonging to the neighbourhood of the other nearby f_{TPE} -frequencies, and they meet right at the middle of two f_{TPE} -frequencies. Above this spindle speed, the bifurcation is period doubling: that is, the f_{PD} -frequencies are just in midway between two nearby f_{TPE} -frequencies.

7. Conclusions

The dynamics of the milling process are intricate due to the infinite dimensional phase space caused by the regenerative effect, and to the parametric excitation caused by the time-varying

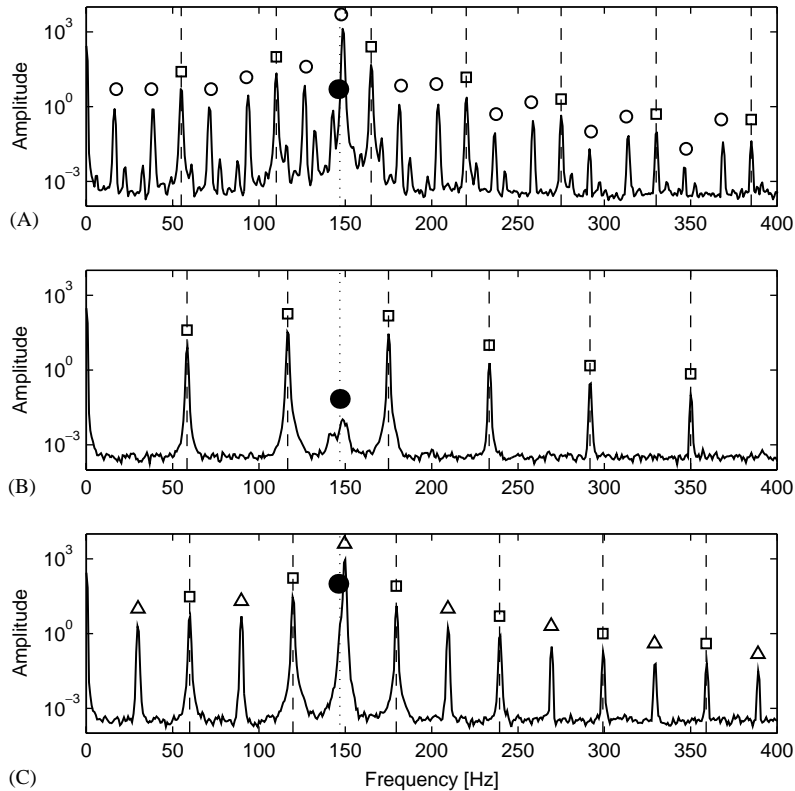


Fig. 8. Power spectra for parameter points A, B and C (\circ — f_H , \triangle — f_{PD} , \square — f_{TPE} , \bullet — f_d).

number of active teeth. Frequency analysis is also not trivial: several sets of peaks appear in the power spectra of the vibration signals.

The analytical investigation of the governing time-periodic DDEs identifies four types of frequency sets. The tooth pass excitation frequency together with its higher harmonics (f_{TPE}), and also the damped natural frequency (f_d) of the tool arise for both stable and unstable milling processes. In the unstable case, additional frequency sets occur: either Hopf type (f_H) or period doubling type (f_{PD}) frequencies. As opposed to the chatter frequencies of turning, some of the milling vibration frequencies may be smaller than the natural frequency of the tool. The results were confirmed by power spectra gained via Fourier transformation of the experimental data.

In case of high-speed milling, the period doubling type loss of stability is a recently identified mechanism of chatter in cutting processes. The clear picture of the structure of frequency components helps to distinguish between the different types of oscillations in intermittent cutting.

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