ACADEMIC PRESS

Available online at www.sciencedirect.com

SCIENCE DIRECT

Journal of Sound and Vibration 262 (2003) 365–369

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Letter to the Editor

# Discussion on the local flexibility due to the crack in a cracked rotor system

J. Zou[a,*], J. Chen[a], J.C. Niu[b], Z.M. Geng[b]

[a] *The State Key Laboratory of Vibration, Shock & Noise, Shanghai Jiao Tong University, 1954 Huashan Road, Shanghai 200030, People's Republic of China*

[b] *School of Mechanical Engineering, Shandong University, Ji'nan 250061, People's Republic of China*

Received 2 January 2002; accepted 4 October 2002

## 1. Introduction

The dynamic analysis of a cracked rotor has been investigated since the 1970s, and now there are extensive researches on the vibrational behaviors of the cracked rotor and the use of response characteristics to detect crack [1–11]. With the energy principle of Paris in fracture mechanics, Dimarogonas and his colleagues derived a rough analytical estimation of the local flexibility, which is a function of the relative crack depth [3,7–9]. From the viewpoint of the theorem, the researches of Dimarogonas are of state-of-art; however, the local flexibility due to the crack in the cracked rotor system is calculated approximately. In the present study, the local flexibility of Dimarogonas is modified slightly, which is more suitable to the theoretical model.

## 2. Theoretical calculation of local flexibility

A transverse crack of depth $a$ is considered on a shaft of radius $R$ in rotor system, which is shown in Fig. 1. Here only the bending deformation is taken into consideration; axial forces and shear stresses are neglected. By the energy principle of Paris, the strain energy of a rectangular strip with a crack of constant depth $z$ is

$$\mathrm{d}U = \mathrm{d}\eta \int_0^z J(z)\,\mathrm{d}z, \tag{1}$$

where $\mathrm{d}\eta$ is the width of the rectangular strip, $J(z)$ is the strain energy density function:

$$J(z) = \frac{1-v^2}{E} K_I^2(z), \tag{2}$$

*Corresponding author.

*E-mail address:* jianzou@sjtu.edu.cn (J. Zou).



doi:10.1016/S0022-460X(02)01409-8

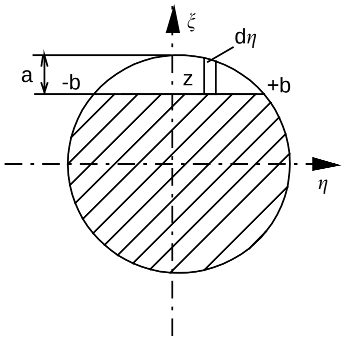


Fig. 1. Geometry of the cracked section of a shaft.

where $v$ is the Poisson ratio, $E$ is Young's modulus and $K_I$ is the stress intensity factor corresponding to bending moment $M$.

The solution of $K_I$ is

$$K_I = \frac{4M}{\pi R^4}\sqrt{R^2-\eta^2}\sqrt{\pi z}F_2\left(\frac{z}{h}\right), \tag{3}$$

where $h$ is the local height of the strip ($h = 2\sqrt{R^2-\eta^2}$),

$$F_2\left(\frac{z}{h}\right) = \sqrt{\frac{2h}{\pi z}\tan\frac{\pi z}{2h}}\frac{0.923+0.199(1-\ \sin(\pi z/2h))^4}{\cos(\pi z/2h)}.$$

For the crack of maximum depth $a$ and width $2b = 2\sqrt{R^2-(R-a)^2}$, the strain energy is obtained as follows:

$$\begin{aligned} U &= \frac{1-v^2}{E}\int_{-b}^{+b}\mathrm{d}\eta\int_0^z \frac{16M^2}{\pi^2R^8}(R^2-\eta^2)\pi z F_2^2\left(\frac{z}{h}\right)\mathrm{d}z \\ &= \frac{1-v^2}{E}\int_{-b}^{+b}\mathrm{d}\eta\int_0^{\sqrt{R^2-\eta^2}-(R-a)} \frac{16M^2}{\pi^2R^8}(R^2-\eta^2)\pi z F_2^2\left(\frac{z}{h}\right)\mathrm{d}z. \end{aligned}$$

The local flexibility due to the crack in the $\xi$-axis direction can be written as

$$c_\xi = \frac{\partial^2 U}{\partial M^2} = \frac{1-v^2}{E}\int_{-b}^{+b}\mathrm{d}\eta\int_0^{\sqrt{R^2-\eta^2}-(R-a)} \frac{32}{\pi^2R^8}(R^2-\eta^2)\pi z F_2^2\left(\frac{z}{h}\right)\mathrm{d}z. \tag{4}$$

In dimensionless form

$$\frac{R^3E}{1-v^2}c_\xi = \int_{-b/R}^{+b/R}\mathrm{d}\frac{\eta}{R}\int_0^{[\sqrt{R^2-\eta^2}-(R-a)]/R}\frac{32}{\pi^2}\left[1-\left(\frac{\eta}{R}\right)^2\right]\pi\frac{z}{R}F_2^2\left(\frac{z}{h}\right)\mathrm{d}\frac{z}{R}. \tag{5}$$

The expression on the right is a function of $a/R$. The numerical integration Simpson Method is adopted; the dimensionless local flexibility varying with the relative crack depth is shown in Fig. 2.

The dimensionless local flexibility of Dimarogonas in Refs. [7–9] is written as

$$\frac{R^3E}{1-v^2}c_\xi = \int_{-b/R}^{+b/R}\mathrm{d}\frac{\eta}{R}\int_0^{a/R}\frac{32}{\pi^2}\left[1-\left(\frac{\eta}{R}\right)^2\right]\pi\frac{z}{R}F_2^2\left(\frac{z}{h}\right)\mathrm{d}\frac{z}{R}. \tag{6}$$

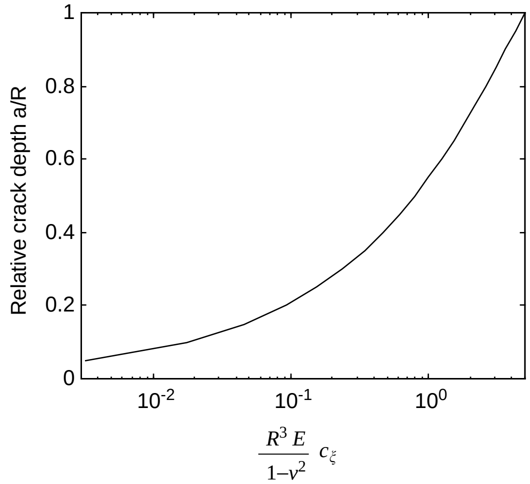


Fig. 2. Dimensionless local flexibility.

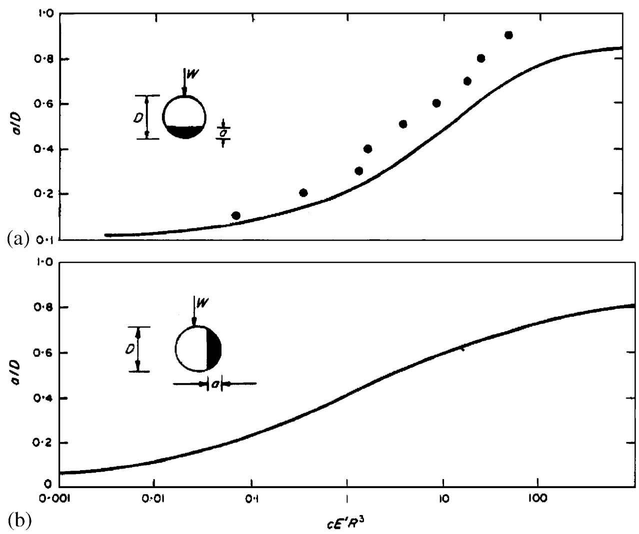


Fig. 3. Dimensionless flexibility of the cracked section: (a) load direction normal to crack edge; (b) load direction parallel to crack edge; • experimental results (from Ref. [7]).

The dimensionless local flexibility of Dimarogonas is shown in Fig. 3(a). The results shown in Fig. 2 are smaller than those shown in Fig. 3(a) for the difference in the integration boundary of the crack depth. The integration boundary of Eq. (6) is a constant crack depth $a/R$, but the integration boundary of Eq. (5) is a varying crack depth $[\sqrt{R^2-\eta^2}-(R-a)]/R$, which is smaller than $a/R$.

In comparison to Eq. (6), Eq. (5) is more suitable to the theoretical model shown in Fig. 1. The theoretical results correspond with the experimental results given by Grabowski (shown in Fig. 4)

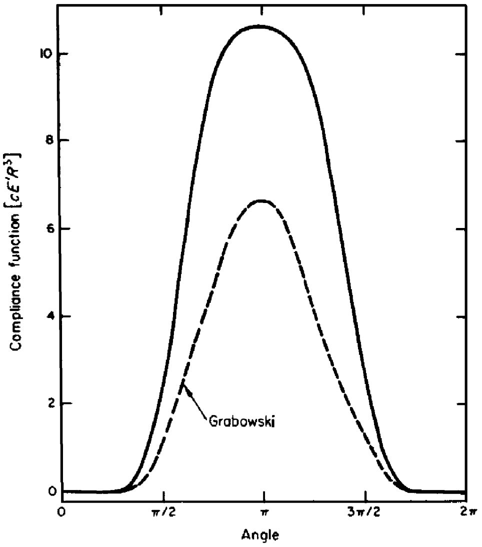


Fig. 4. Static deflection of a cracked shaft for changing edge orientation $a/R = 1$.

more effectively than those of Dimarogonas. For example, when the crack edge orientation is $\pi$ rad and the relative crack depth is $a/R = 1$, the dimensionless local flexibility of Dimarogonas is larger than 10, the dimensionless local flexibility in the present paper is 4.996, and the experimental result is about 6.3 [7–10]. The results are reasonable, which can be verified by the following equation:

$$k_\xi = \frac{k}{1 + c_\xi k l^2/8}, \tag{7}$$

where $k$ is the stiffness of the uncracked shaft and $l$ is the shaft length.

Substituting $c_\xi$ into Eq. (7), the conclusion is that the stiffness in the experiment of Grabowski will be smaller than the theoretical value in the paper, because the experimental results are obtained with a notched shaft rather than a cracked shaft.

The same modification can be made to the local flexibility in the $\eta$-axis direction of Dimarogonas in Refs. [7–9].

## Acknowledgements

This project is supported by the National Fundamental Foundation of Research and Development (approved No. G1998020321), by the National Natural Science Foundation and Institute of Engineering Physics of China (approved No. 10176014) and also by the State Key Laboratory of Vibration, Shock and Noise (approved No. VSN-2002-03). The authors are grateful for their financial support.

## References


[1] S. Prabhakar, A.S. Sekhar, A.R. Mohanty, Detection and monitoring of cracks in a rotor-bearing system using wavelet transforms, Mechanical Systems and Signal Processing 15 (2001) 447–450.

[2] J.B. Zheng, X.S. Gao, Y.C. Guo, Application of wavelet transform to bifurcation and chaos study, Applied Mathematics and Mechanics 19 (1998) 593–599.

[3] J. Wauer, On the dynamics of the cracked rotors: a literature survey, Applied Mechanics Review 43 (1990) 13–17.

[4] J. Wauer, Modelling and formulation of equations of motion for cracked rotating shafts, International Journal of Solids and Structures 26 (1990) 901–914.

[5] R. Gasch, Dynamic behavior of a simple rotor with cross-sectional crack, IMechE Conference Publication c178/76 (1976) 123–128.

[6] R. Gasch, A survey of the dynamic behavior of a simple rotating shaft with a transverse crack, Journal of Sound and Vibration 160 (1993) 313–332.

[7] A.D. Dimarogonas, C.A. Papadopoulos, Vibration of cracked shafts in bending, Journal of Sound and Vibration 91 (1983) 583–593.

[8] A.D. Dimarogonas, S.A. Paipetis, Analytical Methods in Rotor Dynamics, Applied Science Publishers, New York, 1983.

[9] C.A. Papadopoulos, A.D. Dimarogonas, Coupled longitudinal and bending vibrations of a rotating shaft with an open crack, Journal of Sound and Vibration 117 (1987) 81–93.

[10] B. Grabowski, The vibrational behavior of a turbine rotor containing a transverse crack, ASME Design Engineering Technology Conference, St. Louis, Paper No. 79-DET-67, 1979.

[11] J. Zou, J. Chen, Y.P. Pu, et al., On the wavelet time–frequency analysis algorithm in identification of a cracked rotor, Journal of Strain Analysis for Engineering Design 37 (2002) 239–246.