Available online at www.sciencedirect.com
SCIENCE DIRECT

Journal of Sound and Vibration 262 (2003) 497–507

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Analysis and design of pod silencers

M.L. Munjal*

Facility for Research in Technical Acoustics, Department of Mechanical Engineering, Indian Institute of Science, Bangalore 560 012, India

Received 1 February 2002; accepted 24 October 2002

Abstract

Parallel baffle mufflers or split silencers are used extensively in heating, ventilation and air conditioning systems for increased attenuation of noise within a short or given length. Acoustic analysis of rectangular parallel baffle mufflers runs on the same lines as that of a rectangular duct lined on two sides. This simplification would not hold for circular configurations. Often, a cylindrical pod is inserted into a circular lined duct to increase its attenuation (or transmission loss), thereby making the flow passage annular and providing an additional absorptive layer on the inner side of this annular passage. This configuration, called a pod silencer, is analyzed here for the four-pole parameters as well as transmission loss, making use of the bulk reaction model.

The effect of thin protective film or a highly perforated metallic plate is duly incorporated by means of a grazing-flow impedance. Use of appropriate boundary conditions leads to a set of linear homogeneous equations which in turn lead to a transcendental frequency equation in the unknown complex axial wave number. This is solved by means of the Newton–Raphson method, and the axial wave number is then used in the expressions for transmission loss as well as the transfer matrix parameters. Finally, results of a parametric study are reported to help the designer in optimization of a pod silencer configuration within a given overall size for minimal cost.

1. Introduction

Acoustically lined ducts are used extensively in heating, ventilation and air conditioning (HVAC) systems. Acoustic performance of a lined duct is generally measured in terms of transmission loss, TL, which is approximately governed by the expression [1]

$$TL \approx TL_h l/h, \quad h \equiv S/P, \tag{1}$$

*Tel.: +91-80-309-2303; fax: +91-80-360-0648.
E-mail address: munjal@mecheng.iisc.ernet.in (M.L. Munjal).

0022-460X/03/$ - see front matter
doi:10.1016/S0022-460X(03)00108-1

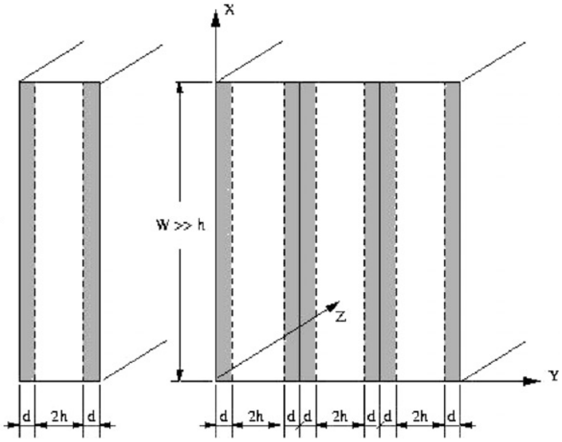

Fig. 1. A parallel-baffle muffler and an equivalent lined duct.

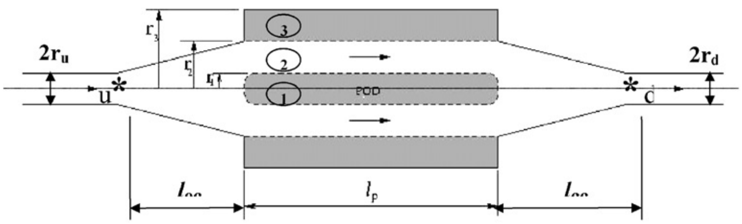

Fig. 2. A pod silencer.

where l is axial length of the lined duct, S is area of cross-section of the flow passage, P is the lined perimeter of the flow passage, and TL_h is transmission loss of the duct with length equal to h; it depends upon thickness and flow resistivity of the lining, and the excitation frequency.

As can be observed from Eq. (1), one sure way to obtain increased TL from a given length is to reduce h, which can be done by means of a parallel-baffle muffler or a splitter silencer, as shown in Fig. 1. Such mufflers or silencers are generally rectangular (or square) in cross-section for ease of fabrication. In circular ducts, similar effect (increase of TL through reduction in h) is often obtained by inserting a pod in the centre, as shown in Fig. 2. In fact, majority of the commercially available pre-fabricated silencers are pod silencers. These are investigated here analytically for the lowest order mode or plane wave propagation.

Analysis of rectangular lined ducts and parallel-baffler mufflers has been done by Ver [2] and Mechel [3] among several others [1] for the bulk reacting as well as locally reacting linings. The effect of protective perforated plate or thin impervious layer has been analyzed by Munjal and Thawani for both types of linings for a circular as well as rectangular lined duct [4]. Mechel [3] has presented analytical models for the least attenuated mode as well as for the (0,0) mode that can be called plane wave. In most applications, however, a major part of the acoustical energy is associated with the lowest order mode or the plane wave. Therefore, overall attenuation or transmission loss is nearly equal to that of the plane wave component.

Most of the systems where lined ducts, parallel-baffler mufflers or pod silencers are used have a moving medium, with the Mach number being 0.15 or less. That is, the flow is incompressible. The convective effect of mean flow on the silencer performance is negligible. However, it alters the grazing flow impedance of the perforated protective plate/layer [5] which may affect attenuation considerably. This is indeed fortuitous inasmuch as the convective effect of mean flow would unnecessarily complicate the analysis while the grazing flow effect can readily be incorporated into the expression for impedance of the perforate, as has indeed been done here under for the pod silencers (Fig. 2) making use of the bulk reaction model.

The following analysis is aimed at derivation of transfer matrix of the pod of length l_p, which can be combined (pre- and post-multiplied) with transfer matrices of the upstream and downstream elements to obtain the overall transfer matrix and thence the transmission loss. Finally, a parametric study has been done to arrive at appropriate design guidelines.

2. The analysis

Fig. 2 shows a typical pod silencer, where r_1, r_2 and r_3 represent radii of the absorptive pod, the annular flow path, and the outer layer of absorptive material, respectively.

Let k_w and Y_w denote respectively the frequency-dependent complex wave number and characteristic impedance of the absorptive material in the wall lining [1,3] (region 3) and pod (region 1), as shown in Fig. 2. Let k_0 and Y_0 denote the wave number and characteristic impedance of the annual airgap medium (region 2). Thus,

$$k_w = \omega/c_w, \quad Y_w = \rho_w c_w, \quad k_0 = \omega/c_0, \quad Y_0 = \rho_0 c_0, \tag{2}$$

k_w and Y_w are evaluated from the value of the flow resistivity of the material.

Use of the compatibility equation

$$k_r^2 + k_z^2 = k^2 \tag{3}$$

for the three regions yields

$$k_{r1} = (k_w^2 - k_z^2)^{1/2} = k_{r3}, \quad k_{r2} = (k_0^2 - k_z^2)^{1/2}, \quad k_0 = \omega/c_0, \tag{4}$$

where k_{ri} (i=1,2 or 3) is the radial component of the wave number in the ith region, k_z is the common axial component of the wave number, as per the bulk reaction model. Evaluation of k_z is the main purpose of the analysis that follows.

The lowest order (0,0) mode or the plane wave solutions of the wave equations in cylindrical co-ordinates in the three regions for forward progressive wave may be written as [1]

$$p_1(r,z,t) = C_1 \mathrm{J}_0(k_{r1} r)\ \mathrm{e}^{-\mathrm{j}k_z z}\mathrm{e}^{\mathrm{j}\omega t}, \tag{5}$$

$$p_2(r,z,t) = \{C_2 \mathrm{J}_0(k_{r2} r) + C_3 \mathrm{N}_0(k_{r2} r)\}\mathrm{e}^{-\mathrm{j}k_z z}\mathrm{e}^{\mathrm{j}\omega t}, \tag{6}$$

$$p_3(r,z,t) = \{C_4 \mathrm{J}_0(k_{r3} r) + C_5 \mathrm{N}_0(k_{r3} r)\}\mathrm{e}^{-\mathrm{j}k_z z}\mathrm{e}^{\mathrm{j}\omega t}, \tag{7}$$

where J and N denote Bessel functions of the first and second order respectively. In the absence of any protective layer, the relevant boundary conditions are represented by equation of acoustic pressure p and radial particle velocity u_r at the interfaces. Thus,

$$u_{r1}(r_1) = u_{r2}(r_1), \qquad p_1(r_1) = p_2(r_1), \tag{8, 9}$$

$$u_{r2}(r_2) = u_{r3}(r_2), \qquad p_2(r_2) = p_3(r_2), \tag{10, 11}$$

and the assumption of rigid shell wall gives

$$u_{r3}(r_3) = 0. \tag{12}$$

In these equation, $u_{r1}(r_1)$ and $p_1(r_1)$ denote radial velocity and acoustic pressure in region 1 at radius r_1, and so on.

With highly perforated thin plates or impervious layers, for plane waves in stationary medium, the inter-face boundary conditions for pressure would become [4]

$$p_1(r_1) - p_2(r_1) = u_{r1}(r_1)Z_p, \quad Z_p = \rho_0 a_0 \zeta, \tag{9a}$$

$$p_2(r_2) - p_3(r_2) = u_{r2}(r_2)Z_p, \tag{11a}$$

where ζ is the non-dimensional or normalized grazing flow impedance of the perforate [5]. It is a function of the plate thickness t, hole diameter d_h, porosity σ, and the mean flow Mach number M along the perforated tube or duct. For the case where it is the same gaseous medium on both sides of the perforated plate, ζ is given by the expression

$$\zeta = [0.006 + \mathrm{j}k_0(t + 0.75d_h)]/\sigma \tag{13}$$

for a stationary medium [6] and

$$\zeta = [0.0073(1 + 72.23M) + \mathrm{j}2.2245 \times 10^{-5}(1 + 51t)(1 + 204d_h)f]/\sigma \tag{14}$$

for a grazing flow [5]. But, for the present case where on one side of the perforate is absorptive material, the perforate impedance is given by the following expression for stationary medium [7,8]:

$$\zeta = [0.006 + \mathrm{j}k_0\{\cdot t + 0.75d_hF\}, \quad F = \frac{1}{2}\left\{1 + \frac{\rho_w c_w k_w}{\rho_o c_o k_o}\right\}. \tag{15}$$

Unfortunately, for grazing flow, there is no simple expression for this latter case where there is absorptive material on one side of the perforate [7]. Therefore, Eqs. (12) and (13) have been combined heuristically to obtain the expression

$$\zeta = [0.0073(1 + 72.23M) + \mathrm{j}2.2245 \times 10^{-5}(1 + 51t)(1 + 240d_hF)f]/\sigma, \tag{16}$$

where the correction factor F is given by Eq. (15).

This expression has been used here for linings of all radii or all thicknesses as well as for rectangular ducts. However, it has been observed that use of the correction factor F makes little difference in the computed values of transmission loss. In other words, Eq. (14) is as good as Eq. (16) for practical purposes.

All other boundary conditions, Eqs. (8), (10) and (12), remain unchanged.

Making use of the momentum equation in the radial direction,

$$\rho\frac{\partial u_r}{\partial \mathrm{t}} = -\frac{\partial p}{\partial r}, \tag{17}$$

we get

$$u_r = \frac{\mathrm{j}}{\omega\rho}\frac{\partial p_r}{\partial r} = \frac{\mathrm{j}}{kY}\frac{\partial p}{\partial r}, \quad k = \frac{\omega}{c}, Y = \rho c. \tag{18}$$

Also noting that, with ($'$) denoting derivative of a Bessel function with respect to its argument,

$$\mathrm{J}_0'(x) = -\mathrm{J}_1(x) \quad \text{and} \quad \mathrm{N}_0'(x) = -\mathrm{N}_1(x). \tag{19}$$

Substituting Eqs. (5)–(7), (18) and (19) into the five boundary conditions (8)–(12) yields

$$\begin{bmatrix} \frac{-k_{r1}\mathrm{J}_1(k_{r1}r_1)}{k_w Y_w} & \frac{-k_{r2}\mathrm{J}_1(k_{r2}r_1)}{k_0 Y_0} & \frac{k_{r2}\mathrm{N}_1(k_{r2}r_1)}{k_0 Y_0} & 0 & 0 \\ \mathrm{J}_0(k_{r1}r_1) & -\mathrm{J}_0(k_{r1}r_1) & -N_0(k_{r2}r_1) & 0 & 0 \\ 0 & \frac{-k_{r2}\mathrm{J}_1(k_{r2}r_2)}{k_0 Y_0} & \frac{-k_{r2}\mathrm{N}_1(k_{r2}r_2)}{k_0 Y_0} & \frac{-k_{r3}\mathrm{J}_1(k_{r3}r_2)}{k_w Y_w} & \frac{-k_{r3}\mathrm{N}_1(k_{r3}r_2)}{k_w Y_w} \\ 0 & \mathrm{J}_0(k_{r2}r_2) & \mathrm{N}_0(k_{r2}r_2) & -\mathrm{J}_0(k_{r3}r_2) & -\mathrm{N}_0(k_{r3}r_2) \\ 0 & 0 & 0 & \frac{k_{r3\;3}\mathrm{J}_1(k_{r3}r_3)}{k_w Y_w} & \frac{k_{r3}\mathrm{N}_1(k_{r3}r_3)}{k_w Y_w} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

or

$$[A]\{C\} = \{0\}. \tag{20}$$

In the presence of protective plates/layers, the first, third and fifth rows of the matrix equation (20) would remain the same. Altered coefficients of the coefficient matrix will be as follows:

$$A_{21} = \mathrm{J}_0(k_{r1}r_1) + \frac{k_{r1}Z_p}{k_w Y_w}\mathrm{J}_1(k_{r1}r_1), \tag{21}$$

$$A_{42} = \mathrm{J}_0(k_{r2}r_2) + \frac{k_{r2}Z_p\mathrm{J}_1(k_{r1}r_1)}{k_0 Y_0}, \tag{22}$$

$$A_{43} = \mathrm{N}_0(k_{r2}r_2) + \frac{k_{r2}Z_p\mathrm{N}_1(k_{r2}r_2)}{k_0 Y_0}, \tag{23}$$

where Z_p is the grazing flow impedance of the perforate given by Eq. (16).

Thus, only three of the 25 coefficients get altered. Of course, the additions/alterations tend to zero as Z_p tends to zero; that is, when there are no protective foils or thin perforated plates.

For this set of five equations to be compatible, determinant of the coefficient matrix $[A]$ may be zero. This provides a transcendental frequency equation for k_z, the axial wave number, the only unknown in view of Eq. (2). This equation may be solved numerically by means of the Newton–Raphson method. The choice of the starting value of k_z for this iteration method may be done as for a circular lined duct [1].

Finally, the required transfer matrix for the pod silencer section of length l_p (Fig. 2) is given by [1]

$$\begin{bmatrix} \cos(k_z l_p) & \mathrm{j}Y\sin(k_z l_p) \\ \frac{\mathrm{j}}{Y}\sin(k_z l_p) & \cos(k_z l_p) \end{bmatrix}, \tag{24}$$

where

$$Y = Y_0 k_0 / k_z, \quad Y_0 = \rho_0 c_0 / S_2, \quad S_2 = \pi(r_2^2 - r_1^2). \tag{25}$$

Radii r_1 and r_2 are as indicated in Fig. 2. The required overall transfer matrix for the full pod silencer, between the upstream point u and downstream point d, may be obtained by pre- and post-multiplying the pod element matrix of Eq. (24) as follows:

$$\begin{bmatrix} expansion \\ cone \end{bmatrix} \begin{bmatrix} sudden \\ expansion \end{bmatrix} \begin{bmatrix} pod \\ element \end{bmatrix} \begin{bmatrix} sudden \\ contraction \end{bmatrix} \begin{bmatrix} contraction \\ cone \end{bmatrix}. \tag{26}$$

Finally, the transmission loss TL may be obtained from the four-pole parameters of the resultant overall transfer matrix [1].

3. Results and discussion

TL of the full pod silencer is compared with that of the absorptive section alone (pod section of length l_p) in Fig. 3 over the frequency range of up to 5 kHz. It may be noted that the former is 0–8 dB (on the average, about 4 dB) higher than the latter due to the effect of area changes on either side of the pod section (see Fig. 3). The dotted curve representing the absorptive section alone is much smoother however, and is selected for the parametric study hereunder.

In this study, the flow resistivity of the absorptive material is kept constant at 15 000 Pa s/m^2.

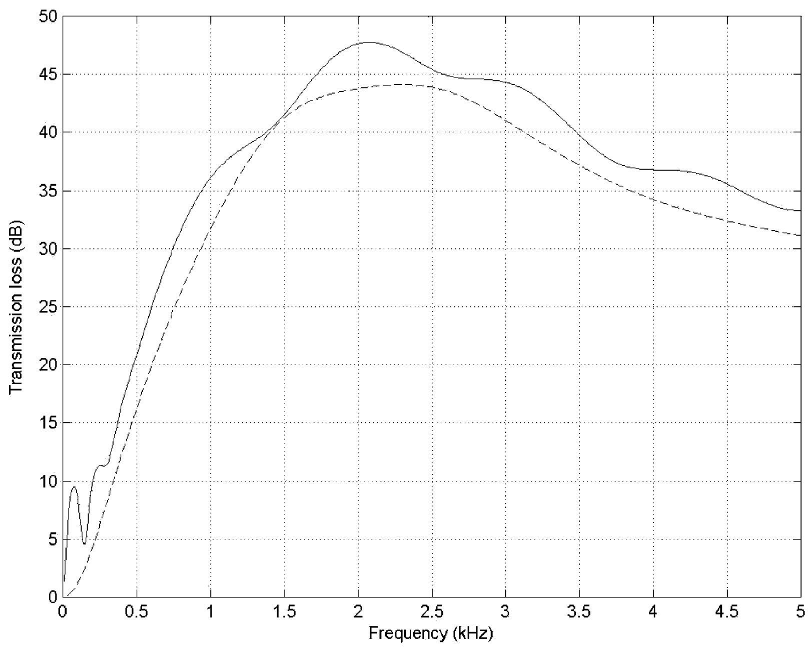

Fig. 3. Comparison of TL of the (full) pod silencer with that of the absorptive section alone: ——, full pod silencer; - - -, absorptive section alone.

Values of the parameters kept constant during the parametric study are as follows (see Fig. 2): $r_u = r_d = 5\,cm$, $r_3 = 15\,cm$, $l_p = 50\,cm$, $l_{ec} = l_{cc} = 15\,cm$, $M = 0.05$, $\rho_0 = 1.18\,kg/m^3$, $c_0 = 346\,m/s$.

The following two parameters are varied in order to investigate the role of the pod vis-à-vis a simple lined circular duct of the same outer radius, r_3. pod radius, $r_1 = 2.5$, $\underline{5.0}$, 7.5 cm; annular radius $r_2 = 7.5$, $\underline{10.0}$, 12.5 cm; where the underlined values indicate the default values of the respective parameter, when the other is being varied.

Figs. 3 to 7 have been drawn up to a frequency of 5000 Hz. This is much beyond the plane-wave cut-off frequency. For a maximum value of r_2 as 12.5 cm and minimum value of r_1 as 2.5 cm, the maximum value of the equivalent diameter of the flow passage amounts to 24.5 cm. Considering axisymmetric waves, the first higher order mode would get cut on (start propagating, if produced) at 1722 Hz. Therefore, beyond this cut-off frequency, values of TL shown in Figs. 3–7 should be understood as describing the behaviour of the lowest order (0,0) mode or plane wave component only.

The effect of variation in the pod radius r_1, with r_2 being fixed at 10 cm, is shown in Fig. 4. Comparison of Figs. 1 and 2 indicates that an increase in the pod radius is equivalent to an increase in d and decrease in h in Fig. 1. The former would widen the TL curve while a decrease in h would result in an increased value of l/h in Eq. (1) and thence raise the peak value of TL. Both these effects are seen in Fig. 4.

The effect of variation in the annular radius r_2, with r_1 being kept constant at 5 cm (see Fig. 2), is shown in Fig. 5. A combination of the same two effects is manifested here too, except that at lower frequencies, the effect of the r_2 variation (in Fig. 5) is more pronounced than the effect of

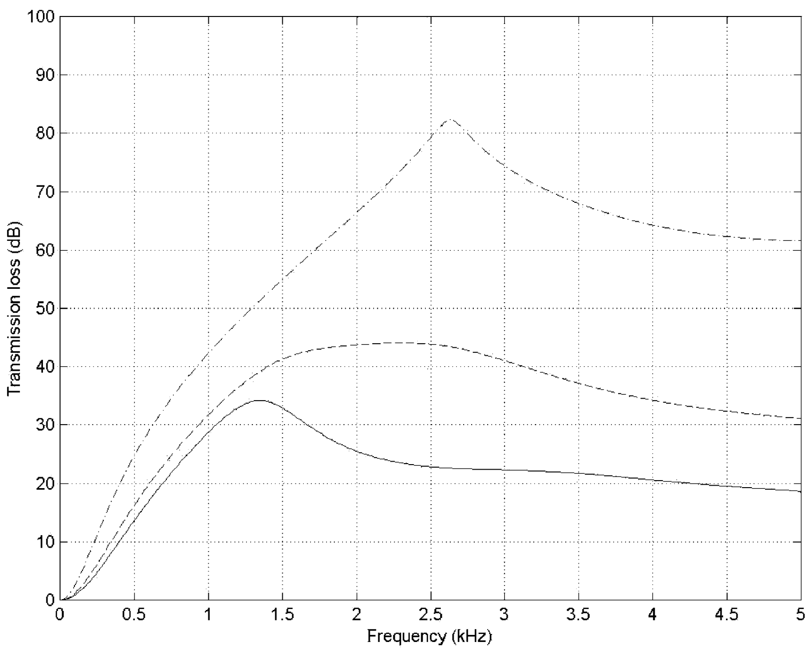

Fig. 4. Effect of the pod silencer on TL of the absorptive section of the silencer: ——, $r_1 = 25\,mm$; - - -, $r_1 = 50\,mm$; –·–, $r_1 = 75\,mm$

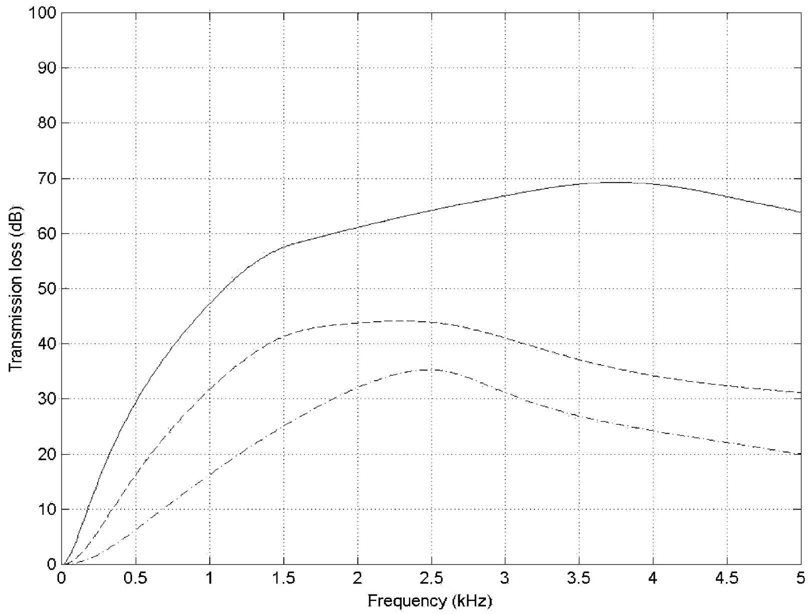

Fig. 5. Effect of radius r_2 on TL of the absorptive section of the silencer: ——, $r_2 = 75$ mm; - - -, $r_2 = 100$ mm; –·–, $r_2 = 125$ mm.

the r_1 variation (in Fig. 4). Incidentally, variation in r_2 involves much more variation in the absorptive material and flow area (and thence the mean pressure drop) than a variation of r_1 would involve.

Incidentally, for an annular flow passage

$$h \equiv \frac{S}{P} = \frac{\pi(r_2^2 - r_1^2)}{2\pi(r_2 + r_1)} = \frac{r_2 - r_1}{2}. \tag{27}$$

Thus, values of h for the three curves in Fig. 4 are equal to those of the corresponding curves of Fig. 5. This explains why the effect of variation in r_1 (Fig. 4) is qualitatively similar, though opposite, to the effect of variation in r_2 (Fig. 5).

In order to investigate the desirability of introducing an absorptive pod vis-à-vis increasing the lining thickness of an ordinary (simply connected) lined duct [4], Fig. 6 shows the effect of a variation of r_2 (with shell radius r_3 fixed) without any pod. Comparing the curves of Fig. 6 with those of Figs. 4 and 5, it may be observed that it is more cost-effective to introduce an absorptive pod than to increase the thickness of the lining of a simple lined duct of the same outer shell radius r_3. In fact, this is consistent with the design principle underlying parallel-baffle mufflers where it is acoustically better to increase the number of baffles than increasing the lining thickness, except at very low frequencies.

Higher value of axial TL in Figs. 3–6 at higher frequencies do not translate into higher values of net TL because the breakout noise (transverse radiation from the shell) increases as the frequency increases. This is shown in Fig. 7 where axial, transverse and net values of TL are computed and compared for the default configuration making use of theory of Ref. [9]. As TL_{net} generally follows the lower of the TL_a and TL_{tp} curves, it merges with the lower of the two at all frequencies

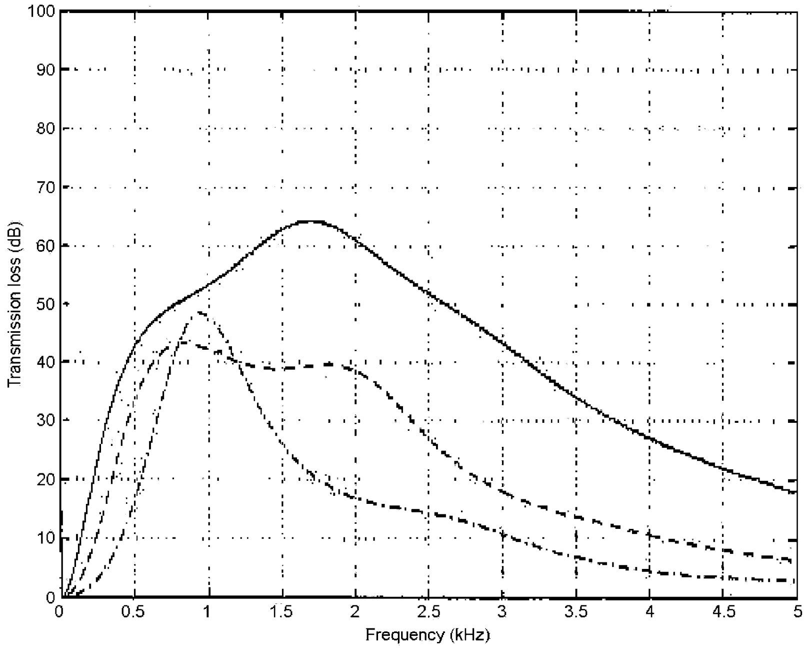

Fig. 6. Effect of radius r_2 on TL of the absorptive section of the silencer without pod ($r_1 = 0$): ——, $r_2 = 50$ mm; - - -, $r_2 = 75$ mm; –·–, $r_2 = 100$ mm.

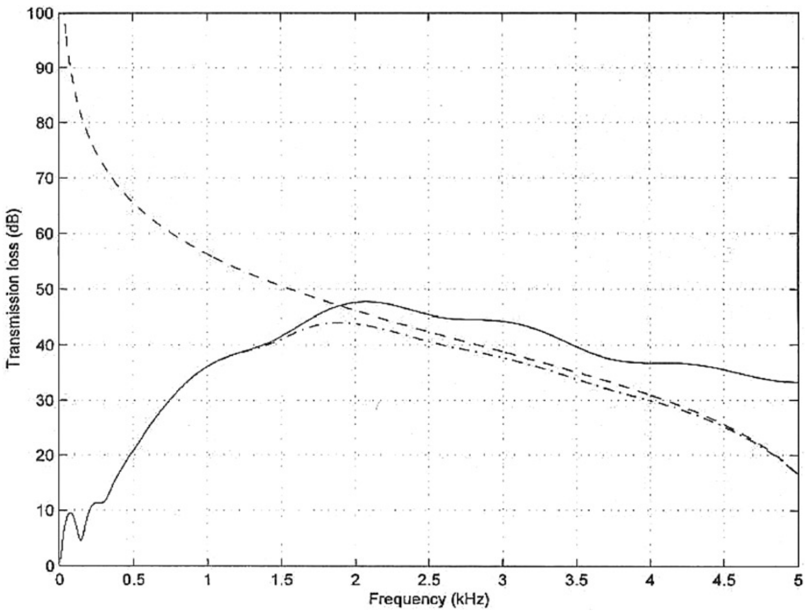

Fig. 7. Comparison of the axial, transverse and net values of TL: ——, TL_a; - - -, TL_{tp}; –·–, TL_{net}.

except around the intersection where it stands 3 dB lower than either. Fig. 7 clearly brings out the need for increasing transverse TL (TL_{tp}) by double wrapping or acoustic lagging of ducts [10] designed for large value of net TL, particularly at higher frequencies.

Incidentally, it may be observed from Eqs. (3) to (13) that the analytical procedure outlined here can easily be extended to multi-pass parallel-baffle mufflers of circular cross-section.

4. Conclusions

In this paper, plane-wave analysis of a pod silencer is presented. Equations have been formulated in such a way that these can be extended readily to the analysis of a multi-pass parallel-baffle muffler of circular cross-section. Parametric studies have revealed that the acoustic effect of the parameter $h \equiv S/P = (r_2 - r_1)/2$ is qualitatively similar to that in a rectangular lined duct or a multi-pass parallel-baffle muffler of rectangular cross-section. Of course, there are considerable quantitative differences because of the Bessel functions replacing trigonometric or circular functions. It is shown that it is more cost-effective to introduce an absorptive pod than to increase the radial thickness of the absorptive layer in a circular lined duct. Of course, one should make sure that the net area of cross-section of the flow passage remains more or less unchanged, otherwise pressure drop across the silencer, and thence flow rate and the fan power requirements would change. It is important to note however that the pod being in the centre would decrease the flow passage area less than the corresponding increase in the lining thickness, $r_3 - r_2$ Finally, the need for strengthening the shell (by double-wrapping or acoustic lagging) has been established for reducing the break-out noise in order to increase the net value of transmission loss

Acknowledgements

It is a pleasure to acknowledge funding of the research work by Department of Science and Technology of the Government of India through their Facility for Research in Technical Acoustics (FRITA).

References

[1] M.L. Munjal, Acoustics of Ducts and Mufflers, Wiley-Interscience, New York, 1987.
[2] A.G. Galaitsis, I.L. Ver, Passive silencers and lined ducts, in: L.L. Beranek, I.L. Ver (Eds.), Noise and Vibration Control Engineering, Wiley-Interscience, New York, 1992 (Chapter 10).
[3] F.P. Mechel, Extension of low-frequencies of the formulae of Delany and Bazley for absorbing materials, Acustica 35 (1976) 210–213 (in German).
[4] M.L. Munjal, P.T. Thawani, Effect of protective layer on the performance of absorptive ducts, Noise Control Engineering Journal 45 (1997) 14–18.
[5] K.N. Rao, M.L. Munjal, Experimental evaluation of impedance of perforates with grazing flow, Journal of Sound and Vibration 108 (1986) 283–295.
[6] J.W. Sullivan, A method of modeling perforated tube muffler components. II: applications, Journal of the Acoustical Society of America 66 (1979) 779–788.

[7] R. Kirby, A. Cummings, The impedance of perforated plates subjected to grazing gas flow and backed by porous media, Journal of Sound and Vibration 217 (1998) 619–636.
[8] A. Selamet, I.J. Lee, Z. Ji, N.T. Huff, Acoustic attenuation performance of perforated concentric absorbing silencers, SAE Noise and Vibration Conference and Exposition, April 30–May 3, Traverse City, MI, SAE Paper No. 2001-01-1435, 2001.
[9] M.L. Munjal, P.T. Thawani, Acoustic performance of hoses–a parametric study, Noise Control Engineering Journal 44 (1996) 274–280.
[10] M.L. Munjal, Acoustic analysis and parametric studies of lagged pipes, Noise Control Engineering Journal 45 (1996) 113–118.