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Efficient modal control strategies for active control of vibrations

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Abstract

Some efficient strategies for the active control of vibrations of a beam structure using piezoelectric materials are described. The control algorithms have been implemented for a cantilever beam model developed using finite element formulation. The vibration response of the beam to an impulse excitation has been calculated numerically for the uncontrolled and the controlled cases. The essence of the method proposed is that a feedback force in different modes be applied according to the vibration amplitude in the respective modes i.e., modes having lesser vibration may receive lesser feedback. This weighting may be done on the basis of either displacement or energy present in different modes. This method is compared with existing methods of modal space control, namely the independent modal space control (IMSC), and modified independent modal space control (MIMSC). The method is in fact an extension of the modified independent space control with the addition that it proposes to use the sum of weighted multiple modal forces for control. The proposed method results in a simpler feedback, which is easy to implement on a controller. The procedure is illustrated for vibration control of a cantilever beam. The analytical results show that the maximum feedback control voltage required in the proposed method is further reduced as compared to existing methods of IMSC and MIMSC for similar vibration control. The limitations of the proposed method are discussed.

1. Introduction

With increasing space activities, the use of lightweight and flexible structures is becoming important to reduce the high cost of lifting the mass into orbit. Because of the flexibility, the vibrations once introduced in the system grow to large amplitudes. The conventional form of

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external passive damping is not preferred as the addition of a damper adds to the overall system weight, which is undesirable. This has led to extensive research into active and passive control.

In active control, the effect of an unwanted disturbance is cancelled by deliberate addition of another disturbance, equal in magnitude but opposite in sign. Piezoelectrics (PZTs) bonded on the structure act as sensors to monitor and as actuators to control the response of a structure. The output of piezosensors is processed by a controller and fed to piezoactuators. Application of voltage to an actuator introduces the force on the base structure, which is proportional to displacement or velocity measured by sensors. This actuator force gets added to the stiffness force or damping force, thus changing the stiffness and damping characteristics of the system (structure). These changes, if properly adjusted, can reduce the amplitude of vibration.

Consider the case of a large-scale spacecraft structure which must be lightweight, hence it will become flexible. When the structure begins to vibrate, its vibration will continue for a long time at a lower natural frequency. In order that the position of the antenna and telescope is assured precisely, it is necessary to control the vibration. This case is similar to many practical systems where lower modes of vibration are having most of the energy and are more critical for control purposes. Hence, in such cases the effort to control all modes of the structure will be wastage of energy. In such cases, the system is transformed into modal space and its individual modes are controlled.

One such technique of controlling individual modes of vibration is the Pole Allocation method in which closed-loop poles of the system are altered to achieve a desired performance objective. In this category fall methods like the independent modal space control (IMSC) method [1], the modified independent space control (MIMSC) method [2], coupled control [3], etc.

2. Model of the system in modal space

The problem addressed is of controlling vibrations of a cantilever beam with a PZT patch actuator mounted at its surface as shown in Fig. 1. Electro-dynamic modelling of this system can be done using finite element formulation and is available in the literature [2,4–6]. The modelling uses Euler beam elements with two nodes per element and two degrees of freedom at each node. For the case of a piezo-patch-mounted element, the electrical strain energy is included along with

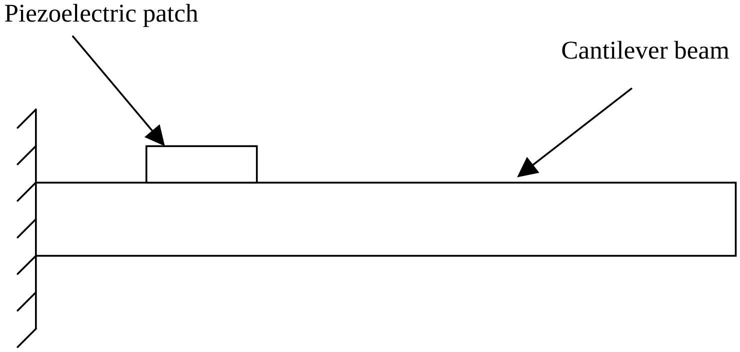

Fig. 1. A cantilever beam mounted with a PZT patch.

the mechanical strain energy and the electrical displacement terms are eliminated using the electro-mechanical constitutive equations of the piezoelectric material. The equation of motion of the system with no inherent damping present in it can be written in the form

$$[M]\{\ddot{d}(t)\} + [K]\{d(t)\} = [D]\{u(t)\}, \tag{1}$$

where $[M]$ and $[K]$ are, respectively, global mass and stiffness of the system including that of the PZT patch and $\{d\}$ is the displacement vector of the system. $[D]$ is an $n \times m$ dimensional location matrix of control force; this also includes electro-mechanical constants of the PZT patch. $\{u(t)\}$ is the $m \times 1$ vector of feedback control voltages applied to m actuators. This matrix equation can be reduced to a number of modal equations by using modal transformation [8]:

$$\{d(t)\} = [U]\{\eta(t)\}, \tag{2}$$

where $\{\eta(t)\}$ is $n \times 1$ vector of modal harmonic time functions and $[U]$ is the $n \times n$ ortho-normal modal matrix which relates modal co-ordinates to generalized co-ordinates $\{d(t)\}$. Substituting Eq. (2) in Eq. (1) and using the standard modal analysis procedure [7] , the de-coupled modal equations are obtained in the form

$$\ddot{\eta}_r + \lambda_r^2 \eta_r = q_r(t), \quad r = 1, 2, \ldots, n, \tag{3}$$

where λ_r^2 is the rth eigenvalue of the system and λ_r is equal to the natural frequency of the system. The subscript r is used to indicate quantities in the rth mode. The vector $q(t)$, defined by

$$q^{\mathrm{T}}(t) = [\, q_1(t), \quad q_2(t), \quad \ldots \quad q_n(t)] \tag{4}$$

is the control vector in modal space and is related to the physical control vector $u(t)$ by

$$\{q(t)\} = [U]^{\mathrm{T}}[D]\{u(t)\}. \tag{5}$$

When feedback is applied to the system the single degree-of-freedom equation (3) gets coupled through the modal control forces $q_r(t)$, since each $q_r(t)$ usually depends on all the modal co-ordinates.

3. Independent modal space control method

Mierovitch and Baruh [1] developed the IMSC method for controlling vibrations of a distributed mass body. In this method, feedback control parameters which are displacement and velocity gains are selected as modal gains.

The modal feedback force in each mode $q_r(t)$ is designed to depend on $\eta_r(t)$ and $\dot{\eta}_r(t)$ alone i.e.,

$$q_r(t) = -g_r \eta_r(t) - h_r \dot{\eta}_r(t). \tag{6}$$

This avoids re-coupling of modal equations through feedback. Thus, an independent controller can be designed for each of n modes.

The modal control forces $q(t)$ are determined using optimal control theory [7], which determines feedback control gains by minimizing a quadratic performance index J. In the IMSC method, the performance index considered is the sum of the potential energy $(\lambda_r^2 \eta_r^2)$ and kinetic energy $(\dot{\eta}_r^2)$ of the vibrating system as well as the required input control effort (q_r^2):

$$J = \int_0^\infty [(\lambda_r^2 \eta_r^2 + \dot{\eta}_r^2) + (R q_r^2)]\, dt. \tag{7}$$

The factor R is used to weight the importance of damping the system energy with respect to the required control effort.

Meirovitch and Baruh [1] developed closed-form solution to this problem and showed that the above performance index attains a minimum value when the feedback gains g_r and h_r are given by

$$g_r = \lambda_r - \sqrt{\lambda_r^2 + 1/R}, \quad h_r = -\sqrt{2\lambda_r(-\lambda_r + \sqrt{(\lambda_r^2 + 1/R)}) + 1/R}. \tag{8}$$

Using these feedback gains, the equation of motion of the system in any mode becomes

$$\ddot{\eta}_r(t) + h_r \dot{\eta}_r(t) + (\lambda_r^2 + g_r)\eta_r(t) = 0. \tag{9}$$

This equation can be solved to determine the control response of the system in any mode.

The physically applied voltage to the actuator can be calculated from the modal control force using relation (5). In Eq. (5), if the number of actuators is equal to the number of controlled modes then the inverse of the $[U]^T[D]$ can be calculated to directly determine the voltages to be applied to each actuator.

4. Modified independent modal space control method

While calculating feedback gains using the IMSC method, the previous vibration history is not considered. Hence, there is no priority given to controlling those modes which are excited to a greater extent by the disturbance force. Baz and Poh [2] modified the IMSC method to MIMSC to control different modes of vibrations of distributed structures separately depending upon the energy in each mode ($\lambda_r^2 \eta_r^2 + \dot{\eta}_r^2$). Energy present in different modes is checked at specific intervals of time and the mode with the highest energy is controlled. After some period of time when the mode being controlled subsides and the energy in some other mode becomes maximum, the control effort is directed towards that mode. Hence, there is shifting of control from one mode to another depending upon their energy contents. In this way, a lesser control effort may be applied to control structural vibrations.

5. Efficient modal control (EMC)

While controlling a vibrating system whose number of modes are excited, using modal feedback gains selected according to the IMSC algorithm, the physically applied voltages attain high values. This problem is largely overcome by the MIMSC method. This method uses the system information (energy present in different modes) of disturbed system to design a control system in which the mode having higher energy is controlled first. This way the feedback control voltage decreases. However, there is a need to weigh and compare the modal energies present in all modes at every instant of time. This puts computational effort on the digital controller and might lead to delays in feedback, especially when the frequencies to be controlled are high. Thus, feedback may not keep up with the disturbance. The modes are ranked according to energy present in them; the mode with maximum energy is controlled at that instant. After elapse of some time, the modal energies in all the modes become almost equal, the shifting of modes occurs very often and a controller has to exert an extra effort.

Rather than simultaneous control of different modes (IMSC) or sequential mode (MIMSC), the information about the uncontrolled system response can be utilized to tailor the control forces. The objective is to reduce the amplitude of vibration to some acceptable level with application of the least control force in a minimum period of time. It can be observed from the results of IMSC that optimal feedback gains are higher for higher modes of vibration but the amplitude of vibration is generally low in these modes.

If the criterion of an acceptable level of amplitude of vibration is set to be the goal, the time taken for achieving this amplitude depends upon the initial amplitude of vibration for the same value of damping. Hence, while controlling a number of modes simultaneously, to modes having lesser amplitude can be applied a lesser amount of active damping. Such a technique is proposed as an EMC strategy, in which modal feedback gains are weighted according to relative modal displacement or energy in that mode.

Here, it is important to note that the proposed method, just like any modal control method requires modal quantities to be measured. In case of free vibration, each mode vibrates at its own frequency, and in the overall response there is frequency-based coupling between modes. There, a frequency filter can be used to separate the modal displacements. Whereas, in forced vibrations the amplitude coupling is present, the response of the structure is estimated using an observer and the modal quantities are calculated using the mode shape functions. This requirement of a modal sensor in any modal control method is also present in the proposed method.

5.1. Weighting of the control force according to displacement in each mode

The fast Fourier transform (FFT) of uncontrolled system response is taken. The ratio of amplitudes in different modes is calculated with respect to the mode having maximum amplitude. The feedback gains in modes having less amplitude are reduced by their respective ratios. Thus, if the feedback control gains are to be applied for controlling ith, jth and kth modes, then the gain ratios are set by

$$\begin{aligned} &\mathit{Feedback\ in\ mode\ i} : \mathit{Feedback\ in\ mode\ j} : \mathit{Feedback\ in\ mode\ k} \\ &= 1 : \frac{\mathit{displacement}(j)}{\mathit{displacement}(i)} : \frac{\mathit{displacement}(k)}{\mathit{displacement}(i)}. \end{aligned} \tag{10}$$

5.2. Weighting of the control force according to energy in each mode and frequency weighting

An alternate way of weighting the modes is according to modal energy in each mode. Baz and Poh [2] have also used energy weighting to prioritize different modes. Secondly, a factor is provided for frequency weighting since a mode with higher frequency will execute more number of cycles in the same interval of time, hence it is needed to provide less damping in that mode. Thus, if feedback gains need to be applied to the ith, jth and kth modes, the criteria for controlling the

ith, jth and kth modes is

$$Feedback\ in\ mode\ i : Feedback\ in\ mode\ j : Feedbaack\ in\ mode\ k$$
$$= 1 : \frac{energy(j)}{energy(i)} \times \frac{frequency(i)}{frequency(j)} : \frac{energy(k)}{energy(i)} \times \frac{frequency(i)}{frequency(k)}. \quad (11)$$

6. Results and discussion

6.1. Properties of the system

Studies are conducted on a cantilever beam made of mild steel and mounted with a PZT patch on its surface. The physical and geometrical properties of the test beam and PZT patch are given in Table 1.

For the studies presented here, the PZT element is mounted about 20 mm away from the root of the cantilever beam. Using relation (8) with $R = 100$ the optimal feedback gains for the first few modes of the given cantilever beam are as given in Table 2.

One can apply all modal feedback gains to control all modes of the system or single modal feedback gain can be applied to control a dominating mode. To achieve simultaneous control of more than one mode, the modal feedback forces are calculated for each individual mode. These feedback forces are converted to physical forces at the same actuator. The physical actuator forces coming from different modes are added to give the resultant force required at the actuator location. This could be done because different modes of the system are linearly independent for a given structure. Each force has its own independent effect. Since the forces are vectorially added and these forces are directly proportional to the voltage to be applied to the piezocrystal, the

Table 1
Main geometrical and physical properties of test beam

Material	Length (mm)	Width (mm)	Thickness (mm)	PZT constant (d_{31})	Density (kg/m^3)	Young's modulus (N/m^2)
Steel beam	146.3	4.65	1.8	—	7800	2.1e11
PZT actuator	20.9	11.62	1.06	180e−12	7500	5e10

Table 2
Optimal feedback gains in different modes

Mode number	Displacement gain	Velocity gain
1	1.424e3	75.526
2	5.163e4	454.728
3	3.728e5	1.222e3
4	1.312e6	2.292e3
5	3.3778e6	3.678e3
6	7.465e6	5.468e3

voltages (coming from different mode control requirements) can be added to get the total voltage to be applied to the PZT crystal.

An impulse load is applied at the tip of the cantilever beam. This load excites more than one mode of the system. The uncontrolled response at the tip of the beam for this excitation is given in Fig. 2. The FFT of the uncontrolled response is as in Fig. 3. The control studies as per different algorithms are presented in the following sections.

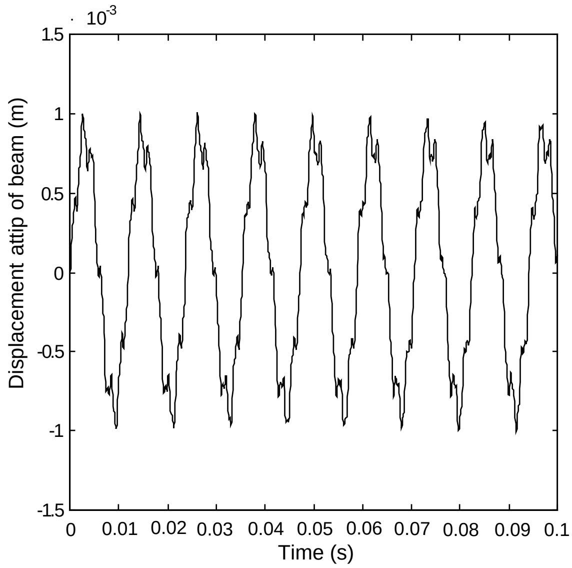

Fig. 2. Uncontrolled response of beam due to excitation of first three modes.

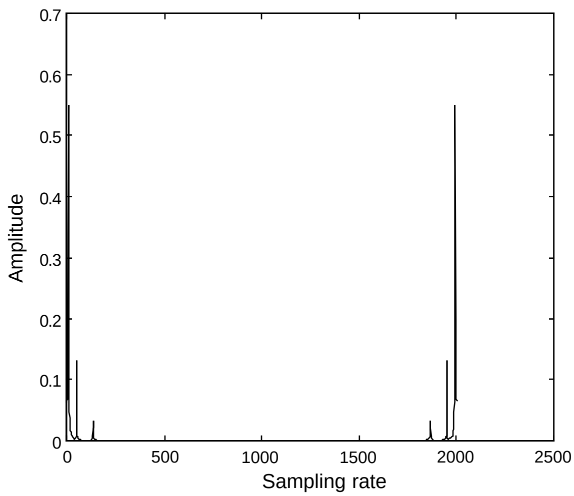

Fig. 3. FFT of the uncontrolled response.

6.2. Independent modal space control

The control of the first three modes of the system is achieved using IMSC. Optimal modal feedback gains are applied to the modal displacement and modal velocity in each mode. The physical feedback voltages calculated for each mode are added up and applied to the actuator. Fig. 4(a) shows the controlled response of the beam. With this configuration and weighting factor, the overall amplitude dies from the initial value of about 8.2×10^{-4} to 0.2×10^{-4} in about 0.1 s. Fig. 4(b) gives a plot of the feedback voltage required to be applied to the actuator crystal. The maximum control voltage requirement is about 1200 V.

6.3. EMC using displacement weighting

From the FFT of the response taken in Fig. 3, the displacement amplitudes in three modes were found to have a ratio of 1 : 0.2525 : 0.0727. Thus, optimal gains in the second and third modes are

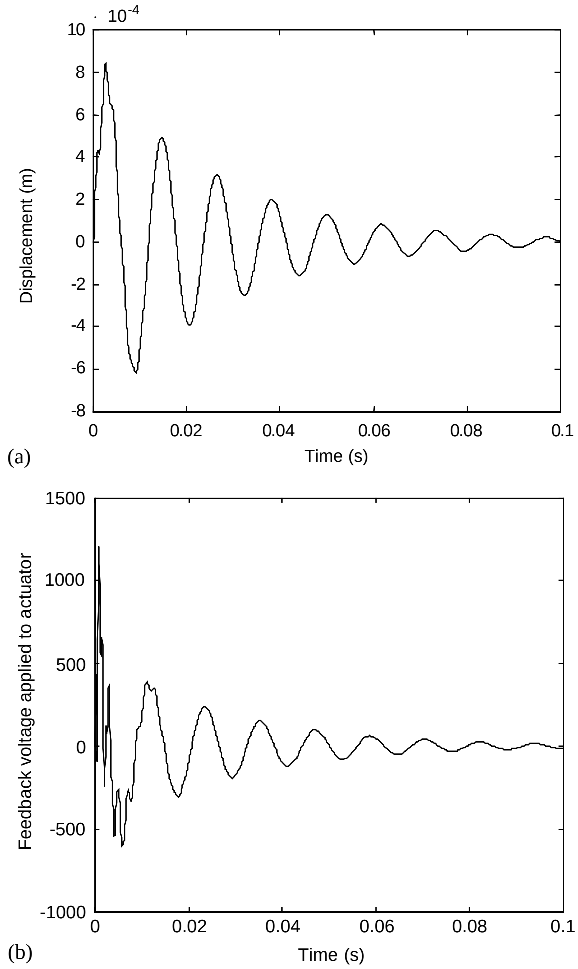

Fig. 4. (a) Controlled response at tip of beam due to feedback force applied according to IMSC. (b) Feedback voltage applied according to IMSC.

reduced by these ratios as per Eq. (10). Fig. 5(a) gives the response at the tip of the beam when these gains are applied. A comparison of Figs. 4(a) and 5(a) indicates that EMC gives almost the same settling time as IMSC. The difference lies in the control achieved on higher modes, which are now receiving reduced control effort. Comparing Fig. 5(b) of feedback voltages with Fig. 4(b) clearly shows that there is a reduction in maximum feedback voltage applied to the actuator in the EMC. The maximum control voltage required in IMSC is 1200 V as compared to only 650 V in the EMC.

6.4. EMC using energy weighting

The modal energies (sum of the potential energy ($\lambda_r^2\eta_r^2$) and kinetic energy ($\dot{\eta}_r^2$)) in the first three modes are calculated as 4.5608e–004, 4.2644e–004 and 3.4825e–004, respectively, and the first three natural frequencies are 78, 405 and 1089 Hz, respectively. Using energy/frequency as the

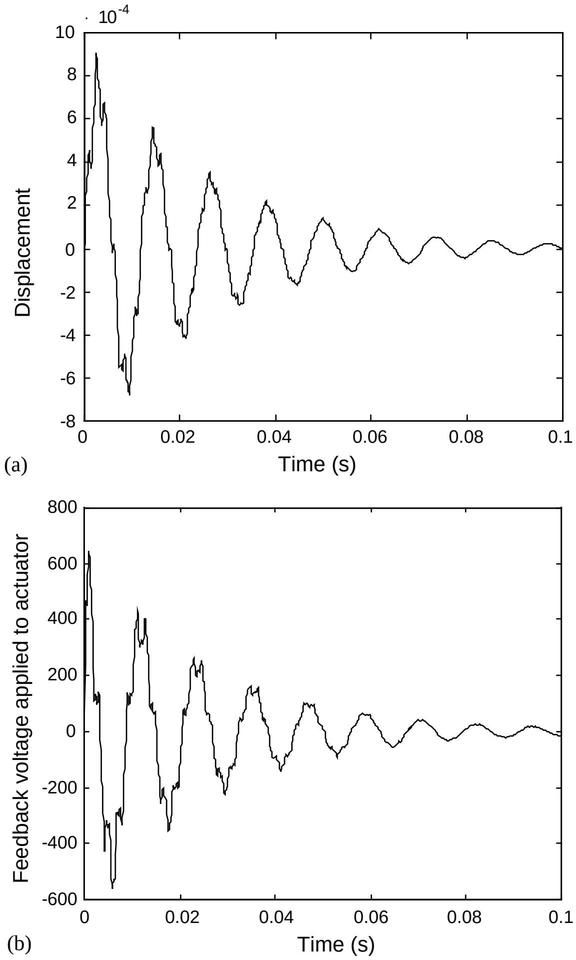

Fig. 5. (a) Controlled response at tip of beam due to feedback force applied according to EMC, with displacement weighting. (b) Feedback voltage applied to the actuator.

weighting parameter, the weighting of control force is

$$1 : \frac{4.2644e{-}4}{4.5608e{-}4} \times \frac{78}{405} : \frac{3.4825e{-}4}{4.5608e{-}4} \times \frac{78}{1089} = 1 : 0.1801 : 0.0547.$$

When the control force is applied by using these weighting factors on the modal contributions, the response at the tip of the beam for this case is as shown in Fig. 6(a). The corresponding feedback voltages are given in Fig. 6(b). Comparing the feedback voltages required (Fig. 6(b)) with that for IMSC (Fig. 4(b)) shows that there is a significant reduction in amplitude of the feedback voltage applied to the actuator whereas the control remains more or less the same. In comparison with Fig. 5(b), there is further reduction in the maximum feedback voltage required from 650 to 600 V. Also, a careful comparison of Figs. 6(a) and 5(a) reveals that in the

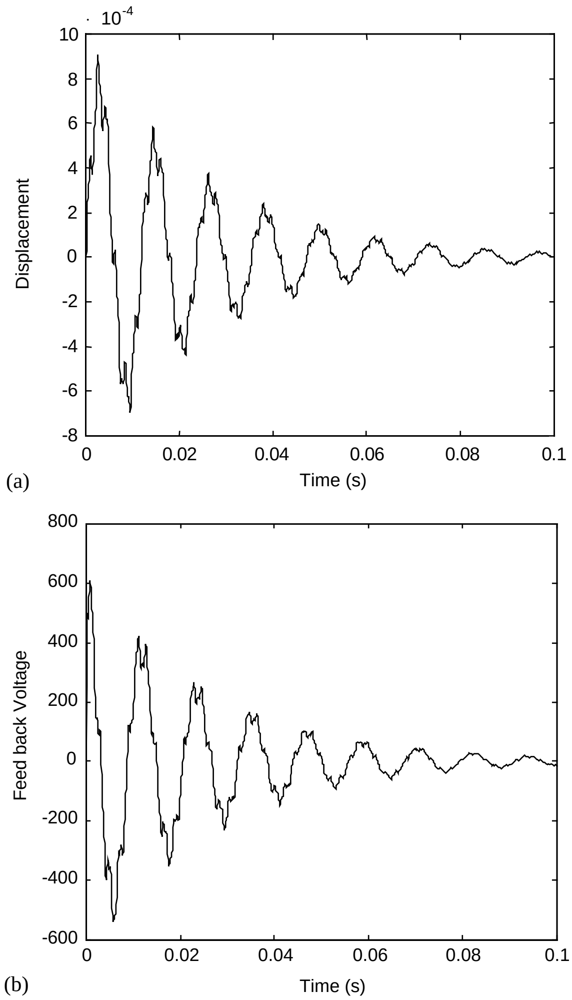

Fig. 6. (a) Controlled response at tip of beam due to feedback force applied according to EMC with energy weighting. (b) Feedback voltage applied according to EMC energy weighting.

energy-weighted approach, higher modes have been slower to decay and their decay is almost complete by the time fundamental mode diminishes out.

6.5. Comparison with MIMSC

The MIMSC method developed by Baz evaluates the modal energy present in each mode and control is applied to the mode having the highest energy. The method was applied for the same case, with energy weight being updated after every 0.1 ms. The controlled displacement at the tip of the beam is given in Fig. 7(a) when feedback gain according to MIMSC is applied to the beam. The feedback voltage applied to the actuator is given in Fig. 7(b).

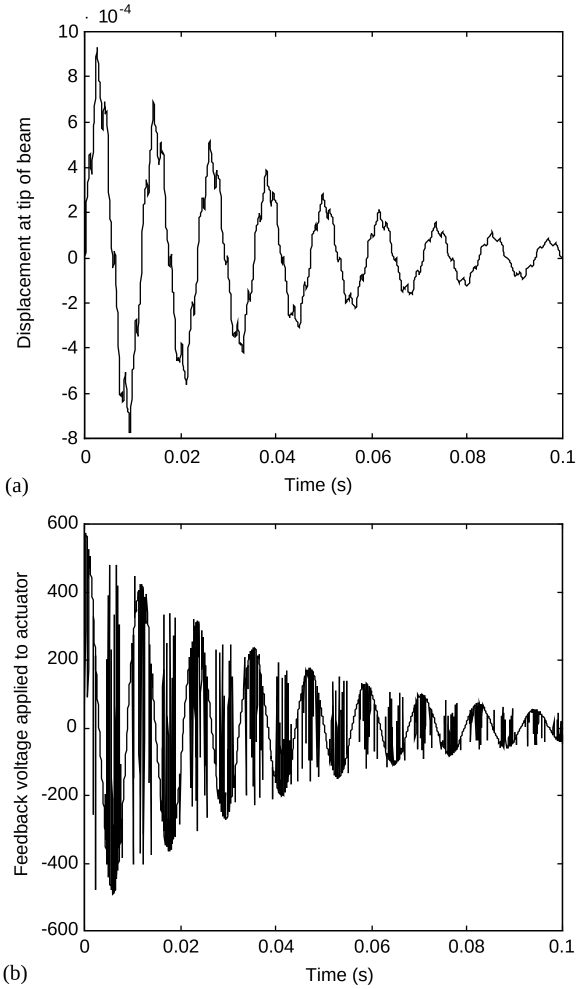

Fig. 7. (a) Controlled response at tip of beam due to feedback force applied according to MIMSC. (b) Feedback voltage applied according to MIMSC energy weighting.

Table 3
Comparison table for various modal control strategies.

Method	Initial displacement (mm)	Peak voltage (V)	Settling time (s)
Uncontrolled	1.0	—	—
IMSC	0.82	1200	0.1
MIMSC	0.92	590	0.11
EMC (displacement weighted)	0.9	620	0.1
EMC (energy weighted)	0.9	600	0.1

It can be seen that in this case the maximum feedback voltage is almost of the same order as for EMC. However the graph in Fig. 7(b) shows that at the time of mode switching, there is an abrupt change in feedback voltage. If energy is weighted at each instant and feedback is applied to the mode having the highest energy, after some time the modal energy in all modes becomes almost equal and it becomes difficult to shift control from one mode to another mode at every instant. The time interval between two weightings may be increased in order to decrease the shifting of control. Comparing the feedback in MIMSC with feedback voltage applied in EMC, it is clear that the maximum voltage applied in the two cases is approximately the same but in MIMSC the energy in each mode needs to be calculated and compared each fixed interval of time. In the case of EMC, the weighting may be done once for controlling a disturbance and the controller can supply fixed feedback gains after that. Hence the time for online computing is reduced. In that sense this method is similar to IMSC in which fixed gains are applied.

7. Conclusions

In IMSC, optimal feedback gains are found to be independent of the kind of applied force. When feedback is applied to control modes of vibration, feedback control voltages are found to be very high. EMC which compares the displacement or energy present in each mode to weight the feedback control force applied is proposed. Comparing these methods for controlling the first three modes of a cantilever beam shows that there is a large reduction in maximum amplitude of the feedback voltage applied to the actuator in case of EMC without much change in control effectiveness.

Comparison of this method with MIMSC shows that the maximum feedback voltage applied in two cases is similar but EMC is easier to implement practically since it uses fixed gains. There are no abrupt changes in feedback voltages and time for online computing is also reduced which is a critical consideration while controlling relatively high frequencies. A summary of comparison of the three modal control methods is presented in Table 3, for the example of cantilever beam analyzed in this work. However, there is one important limitation in the fixed gain technique that if the structural vibration nature changes due to changes in disturbances, then the gains no longer remain optimal. In that case, an updating procedure as proposed in MIMSC may be needed to increase control effectiveness.

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