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An overview of MIMO-FRF excitation/averaging/processing techniques

Allyn W. Phillips*, Randall J. Allemang

Structural Dynamics Research Laboratory (SDRL), University of Cincinnati, P.O. Box 210072 Cincinnati, OH 45221-0072, USA

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Abstract

The use of characterized excitation and choice of averaging techniques are fundamental to the estimation of multiple input, multiple output (MIMO) frequency response function (FRF) data. The characteristics of the excitation and averaging selected greatly influence the quality of the resulting MIMO-FRF measurements. Presented is an overview of the basic excitation methods, such as random, periodic random, pseudorandom, and burst random (random transient) as well as more advanced excitation methods, such as burst-cyclic random, slow random, MOOZ random, and periodic chirps. The application of these excitation and averaging methods is discussed relative to lightly or heavily damped systems, systems with small non-linearities, FRF models, and peak to RMS (crest factor) as well as signal-to-noise ratio (SNR) issues. Experimental examples are given to demonstrate the important issues.

1. Introduction

Single- and multiple-input estimation of frequency response functions (FRFs) via shaker excitation has become the mainstay of most mechanical structure measurements, particularly in the automotive and aircraft industries. While there are appropriate occasions for the use of deterministic excitation signals (sinusoids), the majority of these measurements are made using broadband (random) excitation signals. These signals work well for moderate to heavily damped mechanical structures which exhibit linear characteristics. When the mechanical structures are very lightly damped, care must be taken to minimize the leakage error so that accurate frequency

*Corresponding author. Tel.: +1-513-556-2720; fax: +1-513-556-3390.
E-mail address: allyn.phillips@uc.edu (A.W. Phillips).

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response function (FRF) data can be estimated in the vicinity of the modal frequencies of the system. Frequently, when random excitation methods are compared to deterministic methods (sinusoids), the comparisons are questionable since proper procedures for eliminating the leakage error have not been followed.

Historically, a number of random excitation signals have been utilized, together with appropriate digital signal processing techniques [1–5], to obtain accurate FRF data. The most common random signal that is used in this situation is the pure random signal together with a Hanning window. This signal is normally generated by the data acquisition system utilizing built-in random signal generator(s) or via external random signal generator(s). While this approach does not eliminate the source of leakage and the effect of applying the Hanning window must be considered, this approach is normally considered as a baseline random excitation method for estimating FRF measurements since this method is available with almost any data acquisition system.

Other forms of random signals (pseudorandom, periodic random, burst random, etc.) utilize more control or frequency shaping of the excitation signal(s) and generally require digital-to-analog (DAC) converter(s). For this reason, some of these alternate methods are infrequently available and therefore not used. This is unfortunate since these methods often yield a superior FRF measurement in less total testing time.

When FRFs are measured on lightly damped systems, great care must be taken to eliminate the leakage error. Regardless of the type of excitation signal hardware involved (random signal generator or DAC), there are random excitation methods that can nearly eliminate the leakage error. In some cases, one approach will be superior on the basis of minimizing the total test time but on the basis of accurate, leakage-free FRFs, one of the methods will always work if test time can be sacrificed. Note that these alternate forms of random excitation focus on eliminating the source of leakage by customizing the random signal to match the requirements of fast Fourier transform (FFT) that is used in converting from the time to frequency domain. The FFT requires that the time domain signal must either be totally observed in the observation period (T) or be periodic in the observation period (T). For leakage free FRF measurements, all of the input and output signals must match one of these two requirements. Burst random excitation is an attempt to match the first requirement; pseudo and periodic random excitations are attempts to match the second requirement.

2. Traditional perspective

The concepts behind commonly used random excitation methods come from a number of sources, including technical papers and vendor documentation. These concepts are briefly reviewed in the following sections.

2.1. Random excitation methods

Inputs, which can be used to excite a system in order to determine frequency response functions (FRFs), belong to one of the two classifications, random or deterministic [6–8]. Random signals are widely utilized for general single- and multiple-input shaker testing when evaluating structures that are essentially linear. Signals of this form can only be defined by their statistical properties over some time period. Any subset of the total time period is unique and no explicit mathematical relationship can be formulated to describe the signal. Random signals can be further classified as stationary or non-stationary. Stationary random signals are a special case where the statistical properties of the random signals do not vary with respect to translations with time. Finally, stationary random signals can be classified as ergodic or non-ergodic. A stationary random signal is ergodic when a time average on any particular subset of the signal is the same for any arbitrary subset of the random signal. All random signals, which are commonly used as input signals, fall into the category of ergodic, stationary random signals. Deterministic signals can be characterized directly by mathematical formula and the characteristic of the excitation signal can be computed for any instance in time. While this is true for the theoretical signal sent to the exciter, it is only approximately true for the actual excitation signal due to the amplifier/shaker/structure interaction that is a function of the impedances of these electro-mechanical systems. Deterministic signals can, nevertheless, be controlled more precisely and are frequently utilized in the characterization of non-linear systems for this reason. The random classification of excitation signals is the only signal type discussed in this paper.

The choice of input to be used to excite a system in order to determine frequency response functions depends upon the characteristics of the system, the characteristics of the modal parameter estimation, and the expected utilization of the data. The characterization of the system is primarily concerned with the linearity of the system. As long as the system is linear, all input forms should give the same expected value. Naturally, though, all real systems have some degree of non-linearity. Deterministic input signals result in frequency response functions that are dependent upon the signal level and type. A set of frequency response functions for different signal levels can be used to document the non-linear characteristics of the system. Random input signals, in the presence of non-linearities, result in a frequency response function that represents the best linear representation of the non-linear characteristics for a given RMS level of random signal input. For systems with small non-linearities, use of a random input will not differ greatly from the use of a deterministic input.

The characterization of the modal parameter estimation is primarily concerned with the type of mathematical model being used to represent the frequency response function. Generally, the model is a linear summation based upon the modal parameters of the system. Unless the mathematical representation of all non-linearities is known, the parameter estimation process cannot properly weight the frequency response function data to include non-linear effects. For this reason, random input signals are most commonly used to obtain the best linear estimate of the frequency response function when a parameter estimation process using a linear model is to be utilized.

The expected utilization of the data is concerned with the degree of detailed information required by any post-processing task. For experimental modal analysis, this can range from implicit modal vectors needed for trouble-shooting to explicit modal vectors used in an orthogonality check. As more detail is required, input signals, both random and deterministic, will need to match the system characteristics and parameter estimation characteristics more closely. In all possible uses of frequency response function data, the conflicting requirements of the need for accuracy, equipment availability, testing time, and testing cost will normally reduce the possible choices of input signal.

With respect to the reduction of the random and bias errors of the frequency response function, random or deterministic signals can be utilized most effectively if the signals are periodic with respect to the sample period or totally observable with respect to the sample period. If either of these criteria are satisfied, regardless of signal type, the predominant bias error, specifically leakage, will be minimized. If these criteria are not satisfied, the leakage error may become significant. In either case, the random error will be a function of the signal-to-noise ratio and the amount of averaging.

Many signals are appropriate for use in experimental modal analysis. Some of the most commonly used random signals, used with single- and multiple-input shaker testing are described in the following sections.

2.1.1. Pure random

The pure random signal is an ergodic, stationary random signal which has a Gaussian probability distribution. In general, the frequency content of the signal contains energy at all frequencies (not just integer multiples of the FFT frequency increment ($\Delta f = 1/T$)). This characteristic is shown in Fig. 1. This is undesirable since the frequency information between the FFT frequencies is the cause of the leakage error. The pure random signal may be filtered (F_{min} to F_{max}) to include only information in a frequency band of interest. The measured input spectrum of the pure random signal, as with all random signals, will be altered by any impedance mismatch between the system and the exciter. The number of RMS spectral averages used in the pure random excitation approach is a function of the reduction of the random error and the need to have a significant number of averages to be certain that all frequencies have been adequately excited.

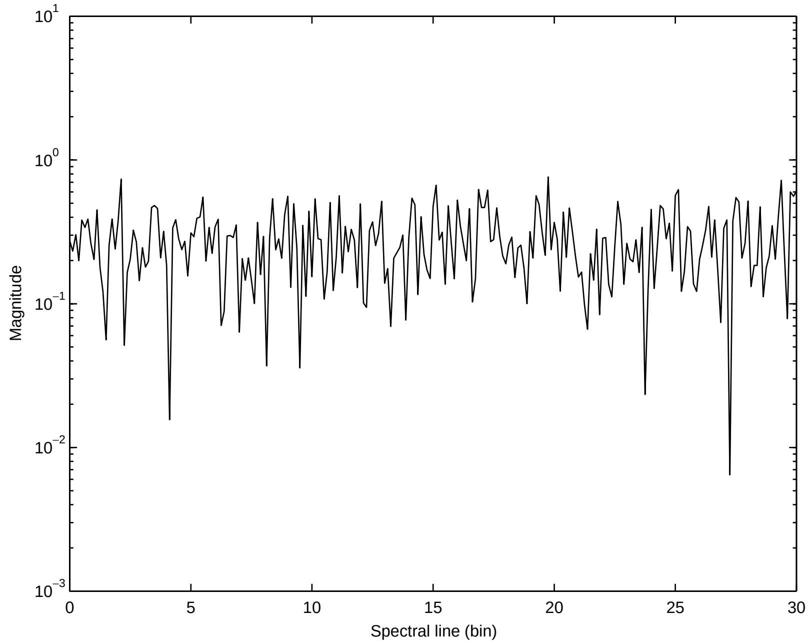

Fig. 1. Signal energy content—pure random.

2.1.2. Pseudorandom

The pseudorandom signal is an ergodic, stationary random signal consisting of energy content only at integer multiples of the FFT frequency increment (Δf). The frequency spectrum of this signal is shaped to have constant amplitude with random phase. This characteristic is shown in Fig. 2. If sufficient delay time is allowed in the measurement procedure for any transient response to the initiation of the signal to decay (number of delay blocks), the resultant input and output histories are periodic with respect to the sample period. The number of RMS spectral averages used in the pseudorandom excitation approach is a function of the reduction of the random error. In a noise-free environment, only one average (per input) may be necessary (provided the inputs are not perfectly correlated at any frequency).

2.1.3. Periodic random

The periodic random signal is an ergodic, stationary random signal consisting only of integer multiples of the FFT frequency increment. The frequency spectrum of this signal has random amplitude and random phase distribution. This characteristic is shown in Fig. 3. For each average, input signal(s) are created with random amplitude and random phase. The system is excited with these input(s) in a repetitive cycle until the transient response to the change in excitation signal decays (number of delay blocks). The input and response histories should then be periodic with respect to the observation time (T) and are recorded as one RMS spectral average in the total process. With each new average, a new history, random with respect to previous input signals, is generated so that the resulting measurement will be completely randomized. The number of RMS spectral averages used in the periodic random excitation approach is a function

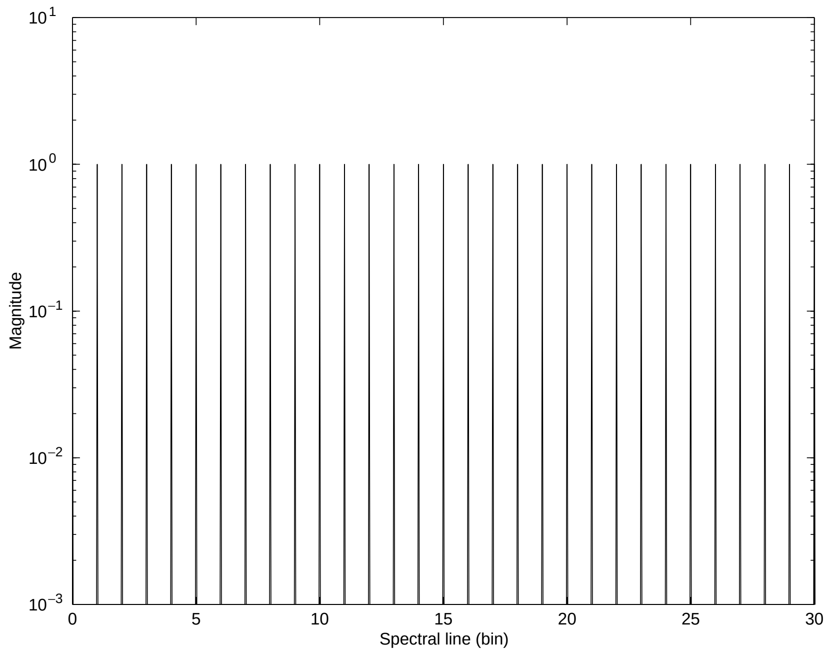

Fig. 2. Signal energy content—pseudorandom.

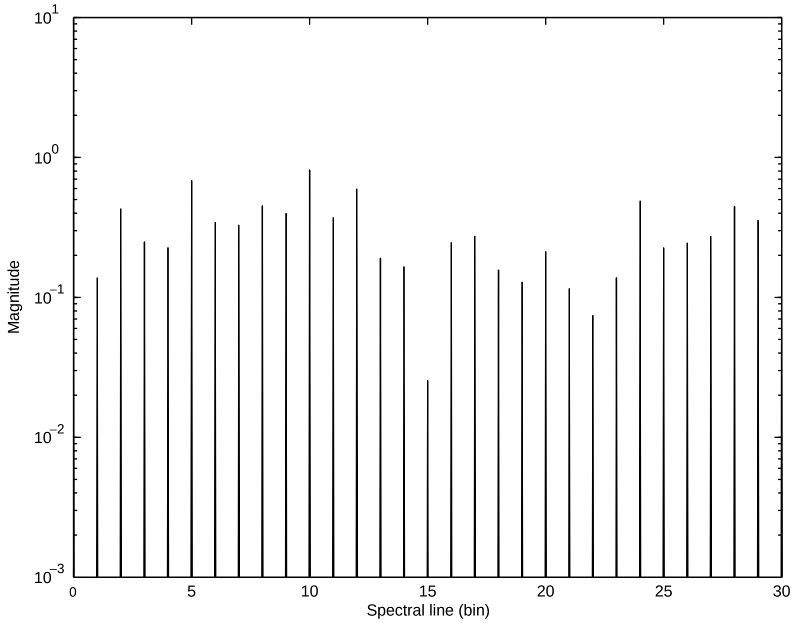

Fig. 3. Signal energy content—periodic random.

of the reduction of the random error and the need to have a significant number of averages to be certain that all frequencies have been adequately excited.

2.1.4. Burst random (random transient)

The burst random signal is neither a completely transient deterministic signal nor a completely ergodic, stationary random signal but contains properties of both signal types. The frequency spectrum of this signal has random amplitude and random phase distribution and contains energy throughout the frequency spectrum. This characteristic is shown in Fig. 4. The difference between this signal and the random signal is that the random transient history is truncated to zero after some percentage of the observation time (T). Normally, an acceptable percentage is 50–80%. The measurement procedure duplicates the random procedure but without the need to utilize a window to reduce the leakage problem. The burst length (0–100%) is chosen so that the response history decays to zero within the observation time (T). For light-to-moderate damped systems, the response history will decay to zero very quickly due to the damping provided by the exciter/amplifier system trying to maintain the input at zero (voltage feedback amplifier in the excitation system). This damping, provided by the exciter/amplifier system, is often overlooked in the analysis of the characteristics of this signal type. This exciter damping input, although not part of the generated signal, is measured and it includes the variation of the input during the decay of the response history. Therefore, since the input and response histories are totally observable within the sample period, the system damping that will be computed from the measured FRF data is unaffected by the exciter system. For very lightly damped systems, the burst length may have to be shortened below 20%. This may yield an unacceptable signal-to-noise ratio (SNR). The number of RMS spectral averages used in the burst random excitation approach is a function of the

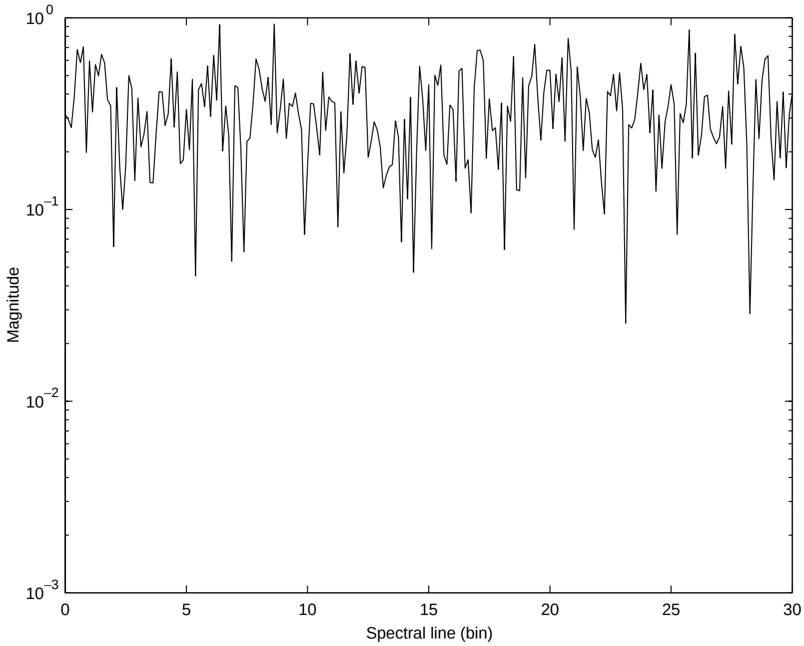

Fig. 4. Signal energy content—burst random.

reduction of the random error and the need to have a significant number of averages to be certain that all frequencies have been adequately excited.

2.1.5. Slow random

The slow random signal is an ergodic, stationary random signal consisting only of integer multiples of the FFT frequency increment. This signal behaves just like the pseudorandom signal but without the frequency shaping of the amplitude. The slow random signal is generated by cyclic averaging a random signal in order to produce digitally comb-filtered excitation signal(s) with the proper characteristics. Note, frequency shaping is covered in Section 3.1.2.

2.1.6. MOOZ random

The MOOZ random signal is an ergodic, stationary random signal consisting only of integer multiples of the FFT frequency increment frequency band limited to the frequency band of a zoom fast Fourier transform (FFT) (F_{min} to F_{max}). The MOOZ (zoom spelled backwards) random signal requires synchronization between the data acquisition and the digital-to-analog converter (DAC). The MOOZ random signal is essentially a slow random excitation signal adjusted to accommodate the frequencies of a zoom FFT.

2.1.7. Periodic chirp

The periodic chirp is a deterministic signal where a sinusoid is rapidly swept from F_{min} to F_{max} within a single observation period (T). This signal is then repeated in a periodic fashion. While this signal is not random in characteristic, it is often included in discussions of random excitation since it has similar properties as pseudorandom.

2.2. RMS spectral averaging concept

When frequency response function(s) are estimated using any of the current methods (H_1, H_2, H_v, H_s, etc.), a number of averages are normally utilized. These averages are performed in the frequency domain and are thus referred to as spectral averages. Since the functions that are averaged are the auto- and cross-power spectra, the averaging that takes place is a least-squares averaging procedure that is often referred to as an RMS averaging procedure. The purpose of RMS spectral averages is to eliminate the noise that is random with respect to the averaging procedure in order to reduce the variance on the resulting FRF estimate. This type of averaging does not reduce the effects of bias errors like the leakage error.

2.3. Averaging methods—triggering issues

Generally, averaging is utilized primarily as a method to reduce the error in the estimate of the frequency response function(s). This error can be broadly considered as noise on either the input and/or the output and can be considered to the sum of random and bias components. Random errors can be effectively minimized through the common approach to averaging, RMS spectral averaging. However, bias errors cannot generally be effectively minimized through this form of averaging alone.

The triggering criteria (choice) for signal averaging for measurement situations that include random and bias errors is critical if both types of error are to be minimized. With this in mind, the signal averaging useful to frequency response function measurements can be divided into three classifications:

- Asynchronous (free-run)
- Synchronous (event triggered)
- Cyclic (contiguous)

These three classifications refer to the trigger and sampling relationships between sample functions. In all three cases, RMS spectral averaging will be used to minimize the random portion of the error. Only in the last case, specifically cyclic averaging, does the triggering method also minimize the significant bias error caused by the discrete Fourier transform (DFT) of a truncated time domain signal. This error is commonly known as the *leakage error*. Normally, cyclic averaging will be applied in the time domain, but since the Fourier transform is a linear function, there is no theoretical difference between the use of time histories or linear spectra. (Practically, though, there are numerical precision considerations.)

2.3.1. Asynchronous signal averaging (free-run)

The classification of asynchronous signal averaging refers to the case where no known relationship exists between individual sample functions. The FRF is estimated solely on the basis of the intrinsic uniqueness of the frequency response function. In this case, the power spectra (least-squares) approach to the estimate of frequency response must be used since no other way of preserving phase and improving the estimate is available. In this situation, the trigger to initiate digitization (sampling and quantization) takes place in a random fashion dependent only upon the equipment availability. The triggering is said to be in a free-run mode.

2.3.2. Synchronous signal averaging (event triggered)

The synchronous classification of signal averaging adds the additional constraint that each sample function must be initiated with respect to a specific trigger condition (often the magnitude and slope of the excitation). This means that the frequency response function can be formed as a summation of ratios of $X(\omega)$ divided by $F(\omega)$ since phase is preserved. Even so, the power spectra (least-squares) approach is still the preferred FRF estimation method due to the reduction of variance and the usefulness of the ordinary coherence function. The ability to synchronize the initiation of digitization allows for use of non-stationary or deterministic inputs with a resulting increased signal-to-noise ratio and reduced leakage. Both of these improvements in the frequency response function estimate are due to more of the input and output being observable in the limited time window.

The synchronization takes place as a function of a trigger signal occurring in the input (internally) or in some event related to the input (externally). An example of an internal trigger would be the case where an impulsive input is used to estimate the frequency response. All sample functions would be initiated when the input reached a certain amplitude and slope. A similar example of an external trigger would be the case where the impulsive excitation to a speaker is used to trigger the estimate of frequency response between two microphones in the sound field. Again, all sample functions would be initiated when the trigger signal reached a certain amplitude and scope.

2.3.3. Cyclic signal averaging (contiguous)

Cyclic signal averaging is often used with excitation characteristics in order to better match the time domain input and output signals to the requirements of the FFT prior to the application of the FFT. The cyclic classification of signal averaging involves the added constraint that the digitization is coherent between sample functions. This means that the exact time between each sample function is used to enhance the signal averaging process. Rather than trying to keep track of elapsed time between sample functions, the normal procedure is to allow no time to elapse between successive sample functions. This process can be described as a comb digital filter in the frequency domain with the teeth of the comb at frequency increments dependent upon the periodic nature of the sampling with respect to the event measured. The result is an attenuation of the spectrum between the teeth not possible with other forms of averaging [9–11].

This form of signal averaging is very useful for filtering periodic components from a noisy signal since the teeth of the filter are positioned at harmonics of the frequency of the sampling reference signal. This is of particular importance in applications where it is desirable to extract signals connected with various rotating members. This same form of signal averaging is particularly useful for reducing leakage during frequency response measurements and also has been used for evoked response measurements in biomedical studies.

A very common application of cyclic signal averaging is in the area of analysis of rotating structures. In such an application, the peaks of the comb filter are positioned to match the fundamental and harmonic frequencies of a particular rotating shaft or component (note that for this case sampling is synchronous with angular position and not time). This is particularly powerful, since in one measurement it is possible to enhance all of the possible frequencies generated by the rotating member from a given data signal. With a zoom Fourier transform type

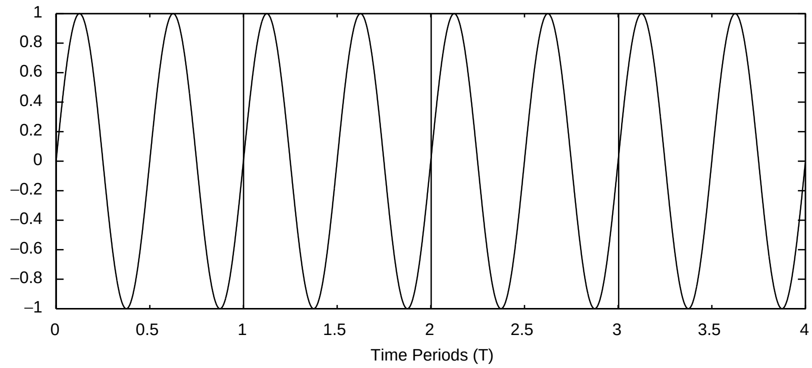

Fig. 5. Contiguous time records (periodic signal).

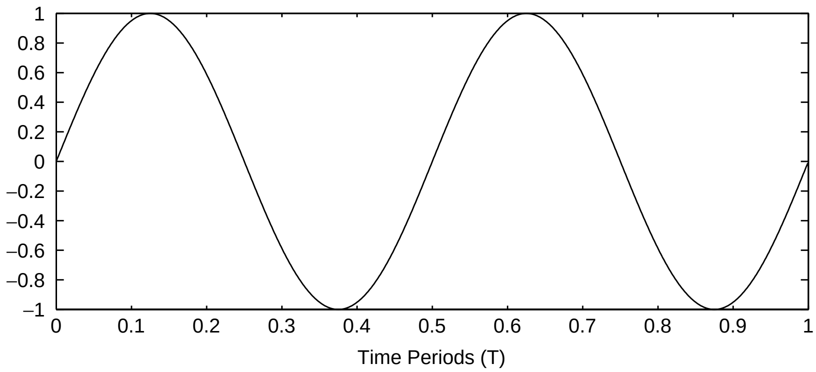

Fig. 6. Averaged time records (periodic signal).

of approach, one shaft frequency at a time can be examined depending upon the zoom power necessary to extract the shaft frequencies from the surrounding noise.

The application of cyclic averaging to the estimation of frequency response functions can be easily observed by noting the effects of cyclic averaging on a single frequency sinusoid. Figs. 5 and 6 represent the cyclic averaging of a sinusoid that is periodic with respect to the observation time period T. Figs. 7 and 8 represent the cyclic averaging of a sinusoid that is aperiodic with respect to the observation time period T. By comparing Fig. 6 with Fig. 8, the attenuation of the non-periodic signal can be clearly observed.

2.3.3.1. Theory of cyclic averaging. In the application of cyclic averaging to frequency response function estimates, the corresponding fundamental and harmonic frequencies that are enhanced are the frequencies that occur at the integer multiples of Δf. In this case, the spectra between each Δf is reduced with an associated reduction of the bias error called *leakage*.

The first observation to be noted is the relationship between the Fourier transform of a history and the Fourier transform of a time-shifted history. In the averaging case, each history will be of some finite time length T which is the observation period of the data. Note that this time period of

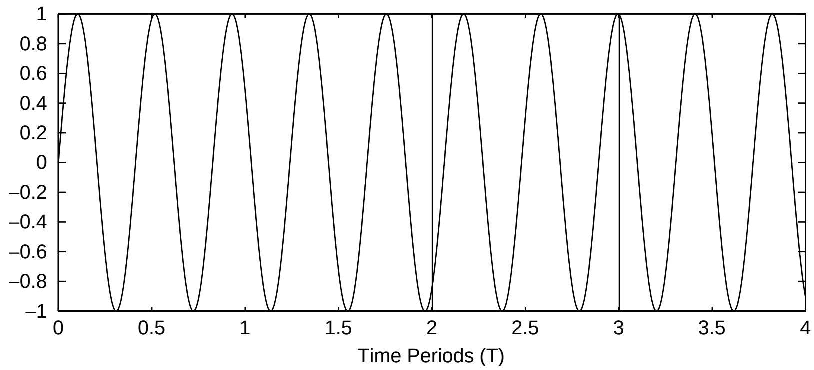

Fig. 7. Contiguous time records (non-periodic signal).

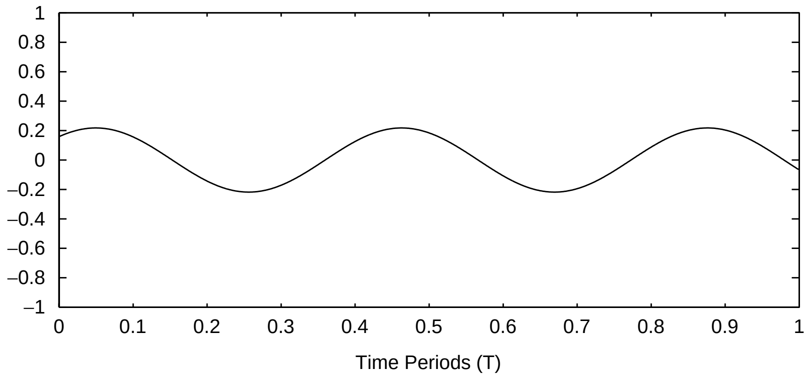

Fig. 8. Averaged time records (non-periodic signal).

observation T determines the fundamental frequency resolution Δf of the spectra via the Rayleigh criteria ($\Delta f = 1/T$).

The Fourier transform of a history is given by

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}t. \tag{1}$$

Using the time shift theorem of the Fourier transform, the Fourier transform of the same history that has been shifted in time by an amount t_0 is

$$X(\omega)\mathrm{e}^{-\mathrm{j}\omega t_0} = \int_{-\infty}^{+\infty} x(t+t_0)\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}t. \tag{2}$$

For the case of a discrete Fourier transform, each frequency in the spectra is assumed to be an integer multiple of the fundamental frequency $\Delta f = 1/T$. Making this substitution in Eq. (2) ($\omega = n2\pi/T$ with n as an integer) yields

$$X(\omega)\mathrm{e}^{-\mathrm{j}n\frac{2\pi}{T}t_0} = \int_{-\infty}^{+\infty} x(t+t_0)\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}t. \tag{3}$$

Note that in Eq. (3), the correction for the cases where $t_0 = NT$ with N is an integer will be a unit magnitude with zero phase. Therefore, if each history that is cyclic averaged occurs at a time shift, with respect to the initial average, that is an integer multiple of the observation period T, then the correction due to the time shift does not effect the frequency domain characteristics of the averaged result. All further discussion will assume that the time shift t_0 will be an integer multiple of the basic observation period T.

The signal averaging algorithm for histories averaged with a boxcar or uniform window is

$$\bar{x}(t) = \frac{1}{N_c} \sum_{i=0}^{N_c-1} x_i(t), \tag{4}$$

where $x_i(t)$ = time history, average i, N_c = number of cyclic averages, $\bar{x}(t)$ = cyclic-averaged time history.

For the case where $x(t)$ is continuous over the time period N_cT, the complex Fourier coefficients of the cyclic-averaged time history become

$$C_k = \frac{1}{T} \int_0^T \bar{x}(t) \mathrm{e}^{-\mathrm{j}\omega t} \, \mathrm{d}t, \quad C_k = \frac{1}{T} \int_0^T \frac{1}{N_c} \sum_{i=0}^{N_c-1} x_i(t) \mathrm{e}^{-\mathrm{j}\omega t} \, \mathrm{d}t. \tag{5,6}$$

Finally,

$$C_k = \frac{1}{N_cT} \int_0^T \sum_{i=0}^{N_c-1} x_i(t) \mathrm{e}^{-\mathrm{j}\omega t} \, \mathrm{d}t. \tag{7}$$

Since $x(t)$ is a continuous function, the sum of the integrals can be replaced with an integral evaluated from 0 to N_cT over the original function $x(t)$. Therefore,

$$C_k = \frac{1}{N_cT} \int_0^{N_cT} x(t) \mathrm{e}^{-\mathrm{j}\omega t} \, \mathrm{d}t. \tag{8}$$

The above equation indicates that the Fourier coefficients of the cyclic-averaged history (which are spaced at $\Delta f = 1/T$) are the same Fourier coefficients from the original history (which are spaced at $\Delta f = 1/N_cT$).

3. Current perspective

The current perspective on excitation methods is not limited to these historical excitation methods. A number of new excitation methods are now possible that, based on constantly evolving hardware and software, do not directly fit these historical definitions of excitation methodology. In order to discuss these methods, several terminology issues must be clarified.

3.1. Terminology

A number of terminology issues have not been rigorously defined when excitation methods have been described historically. The following terminology is important to the explanation of different excitation methods together with the associated digital signal processing requirements.

3.1.1. Signal type

Signal type refers to the basic form of the signal, such as random, impact, sinusoidal, or chirp.

3.1.2. Frequency shaping

Frequency shaping refers to any frequency domain constraint or characteristic that is applied to the specific signal type. With respect to random excitation, a common frequency shaping is pseudorandom. Other frequency shaping is commonly applied to sinusoids and chirps via the rate at which the change of frequency and/or amplitude occurs. Additionally, there are also frequency-dependent amplitude shaping approaches that may be used to improve dynamic range/numerical conditioning characteristics of the measurements [12]. Impact excitation is commonly frequency shaped by controlling the tip characteristic of the hammer.

3.1.3. Delay blocks

The number of contiguous blocks of excitation that take place without the associated input and output data being acquired are referred to as the delay blocks (N_d). This is normally associated with an excitation technique that is periodic in nature. The delay blocks are needed in order to give the transient response to any start or change in the periodic excitation to decay out of the response signal(s) so that both the input(s) and output(s) are periodic with respect to any observation period (T). It is this requirement that makes swept sinusoidal excitation methods (analog swept or digitally stepped) so time consuming, particularly on lightly damped systems. Each delay block is equal in length to the observation period (T) and the number of delay blocks is normally chosen as an integer. The number of delay blocks does not have to be an integer for all excitation methods but, for the purposes of this paper and in common usage, is normally chosen as an integer. The delay blocks are not recorded and are not used in the estimation of the FRFs.

3.1.4. Capture blocks

The number of capture blocks refers to the number of contiguous blocks of time data (excitation (input) and response (output)) that are recorded or captured for each average (N_c). Each group of contiguous capture blocks is used as the time domain data contributing to one RMS spectral average that contributes to the estimate of the FRF measurements.

3.1.5. Window function

The window function refers to the digital signal processing, time domain weighting function that is applied to the capture blocks. The application of the window function to the capture blocks is on the basis of the group of contiguous capture blocks not on each capture block individually.

3.1.6. Average (ensemble)

The average or ensemble refers to the total collection of contiguous time blocks that contribute to each RMS spectral average. The total time of each average is equal to the sum of the number of delay blocks (N_d) plus the number of capture blocks (N_c) times the observation period (T) which is the same for all delay and capture blocks. The number of averages (N_{avg}) refers to the number of these contiguous collections of time blocks and is, therefore, the same as the number of RMS spectral averages.

3.1.7. Periodic

If the excitation signal is repeated for each delay and capture block, the signal is referred to as periodic. This classification is consistent with the definition of a periodic function and includes typical examples of sinusoids and chirps as well as a random signal that is repeated on the basis of the observation period (T). The periodic classification does not define whether the same signal is repeated for each successive group of contiguous delay and capture blocks.

3.1.8. Burst length

Burst length is the percentage (0–100%) of the average or ensemble time that the excitation signal is present. Burst length is normally adjusted in order to achieve a signal that is a totally observed transient. The decay of the signal is a function of the system damping and the characteristics of the excitation hardware. Burst length can be defined as the percentage of the total number of contiguous delay and capture blocks or of a percentage of just the capture blocks. For the purpose of this paper, the burst length refers to the percentage of the total number of contiguous delay and capture blocks.

3.1.9. RMS spectral averages

The number of RMS spectral averages is the number of auto- and cross-spectra that are averaged together to estimate the FRF measurements. The actual amount of test time contributing to each RMS spectral average is a function of the number of contiguous delay and capture blocks.

In order to clarify the preceding terminology, Fig. 9 is a schematic representation of the number of contiguous blocks of time domain data contributing to one RMS spectral average. In this example, the two blocks marked "D" represent delay blocks and the four blocks marked "C" represent capture blocks. The total time for each RMS spectral average is, therefore, six contiguous blocks of time data ($6 \times T$ seconds of data).

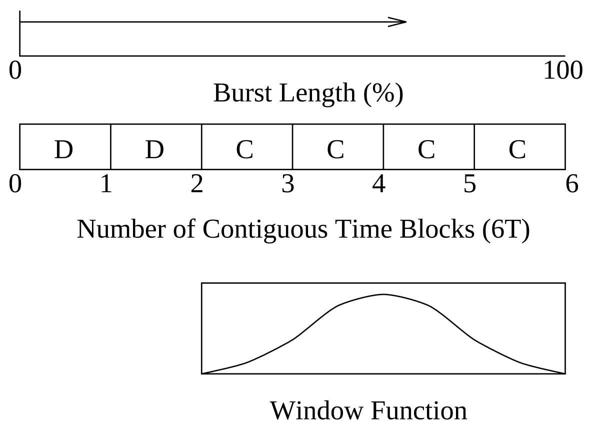

Fig. 9. Total contiguous time per RMS spectral average (ensemble).

3.2. Excitation methods: old, new, and hybrid

The excitation methods demonstrated in the following example include traditional methods (pure, periodic, pseudo, and burst random) [7,8], as well as methods that have been recently documented (burst random with cyclic averaging) [9,10]. Several of the methods are hybrid methods involving combinations of burst random and pseudorandom, burst random, and periodic random together with cyclic averaging. These hybrid methods have not previously been documented. Fig. 10 shows the energy content of a hybrid excitation method that combines pseudorandom with burst random. This excitation signal would be combined with cyclic averaging.

Fig. 11 shows the energy content of a hybrid excitation method that combines periodic random with burst random. This excitation signal would be combined with cyclic averaging.

4. Structural example

The following example presents a single FRF measurement on an H-frame test structure in a test lab environment as a representative example. The configuration of the test involved two shaker locations (inputs) and eight response accelerometers (outputs). The test results are representative of all data taken on the H-frame structure. This H-frame test structure is very lightly damped and has been the subject of many previous studies.

For all FRF measurement cases, the same test configuration was used. Sensors were installed and left in place; no additions or changes were made to the test configuration other than altering

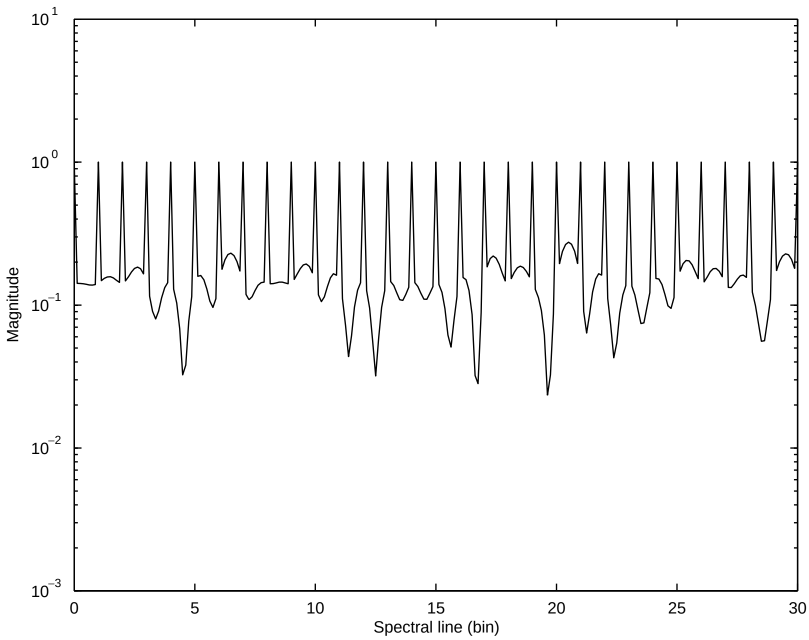

Fig. 10. Signal energy content—burst pseudorandom.

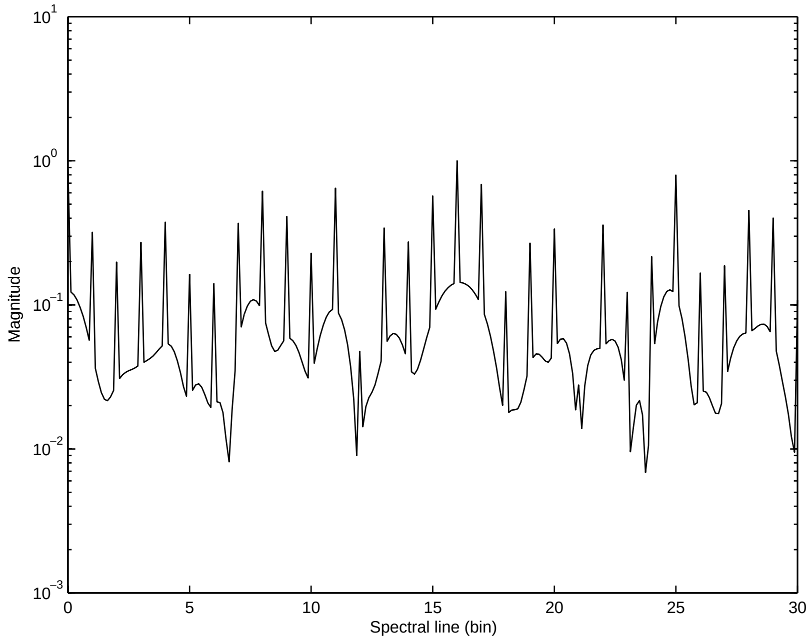

Fig. 11. Signal energy content—burst periodic random.

the excitation, averaging and digital signal processing parameters. Therefore, any changes in the FRF measurements are assumed to be due to the change in measurement technique and not due to a test set-up variation. The test results were repeated to be certain that the results are representative.

All FRF measurements are estimated using the H_1 estimation algorithm using 1024 spectral (frequency) lines of information. The frequency bandwidth is from 0 to 250 Hz for the 1024 spectral lines; only the first 80% of the spectral lines (0–200 Hz) are displayed in order to exclude the data affected by the anti-aliasing filters.

The FRF data are plotted with phase above log magnitude. The log magnitude portion of the plot also contains the relevant multiple coherence plotted on a linear scale in the background. The log magnitude scaling is annotated on the left side of the plot and the multiple coherence scaling is annotated on the right side of the plot.

Fourteen representative cases were measured on this structure. The relevant excitation and digital signal processing characteristics of each case are shown in Table 1. Keep in mind that the focus of this measurement exercise is to demonstrate the influence of the various measurement parameters on the qualitative nature of the resulting FRF. Because this exercise utilizes a MIMO-FRF formulation, the multiple coherence must be used. (Ordinary coherence is not meaningful.) As such the anti-resonance near 132 Hz demonstrates significant variation without apparent change in the multiple coherence. This occurs because there is a large mode nearby, but almost completely uncoupled from the input chosen for presentation. This anomaly may be safely ignored as it does not affect the point or conclusion of the exercise.

Table 1
Test cases—excitation/averaging/DSP parameters

Case	Signal type	Frequency shaping	Periodic function	Burst length	Window function	N_d	N_c	N_{avg}	Total
Case 1	Random	No	No	No	Hanning	0	1	20	20
Case 2	Random	No	No	No	Hanning	0	5	4	20
Case 3	Random	No	No	Yes (75%)	Uniform	0	5	4	20
Case 4	Random	Pseudo	No	No	Uniform	4	1	4	20
Case 5	Random	No	Yes	No	Uniform	4	1	4	20
Case 6	Random	Pseudo	No	No	Uniform	3	1	5	20
Case 7	Random	No	Yes	No	Uniform	3	1	5	20
Case 8	Random	Pseudo	No	Yes (75%)	Uniform	0	5	4	20
Case 9	Random	No	Yes	Yes (75%)	Uniform	0	5	4	20
Case 10	Random	No	No	Yes (75%)	Uniform	0	8	12	20
Case 11	Random	No	No	No	Hanning	0	1	96	96
Case 12	Random	No	No	No	Hanning	0	8	12	96
Case 13	Random	Pseudo	No	No	Uniform	3	2	4	20
Case 14	Random	No	Yes	No	Uniform	3	2	4	20

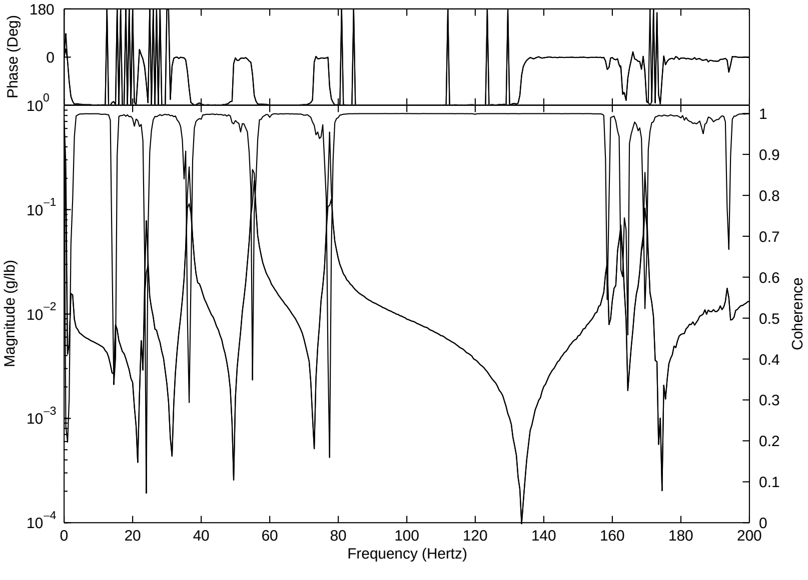

Fig. 12. Case 1: random excitation with Hann window.

Case 1 (Fig. 12) is considered a baseline case since this is a very popular method for making a FRF measurement and it can be easily made on all data acquisition equipment. However, it is clear that in this measurement situation, there is a significant drop in the multiple coherence function at frequencies consistent with the peaks in the FRF measurement. This characteristic drop in multiple (or ordinary) coherence is often an indication of a leakage problem. This can be

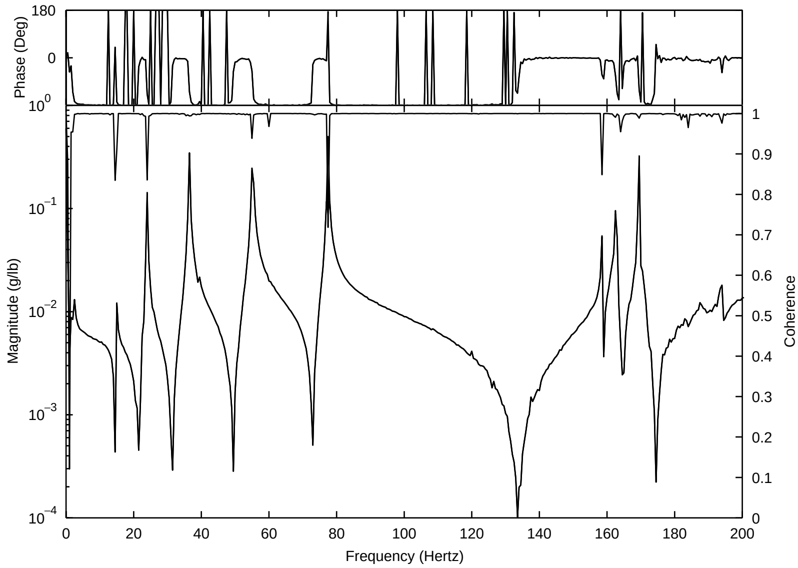

Fig. 13. Case 2: random excitation with Hann window and cyclic averaging.

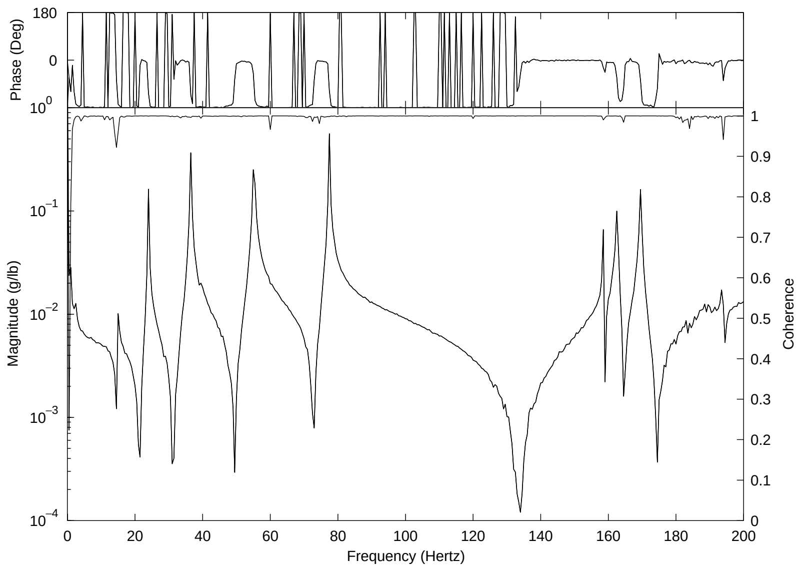

Fig. 14. Case 3: burst random excitation with cyclic averaging.

confirmed if a leakage reduction method reduces or eliminates the problem when the measurement is repeated. In all subsequent cases, the test configuration was not altered in any way—data was acquired simply using different excitation, averaging, and digital signal processing combinations.

Case 2 (Fig. 13) demonstrates an improvement over Case 1 when the same total measurement time is used but cyclic averaging is used to reduce the leakage error. Case 3 (Fig. 14) further

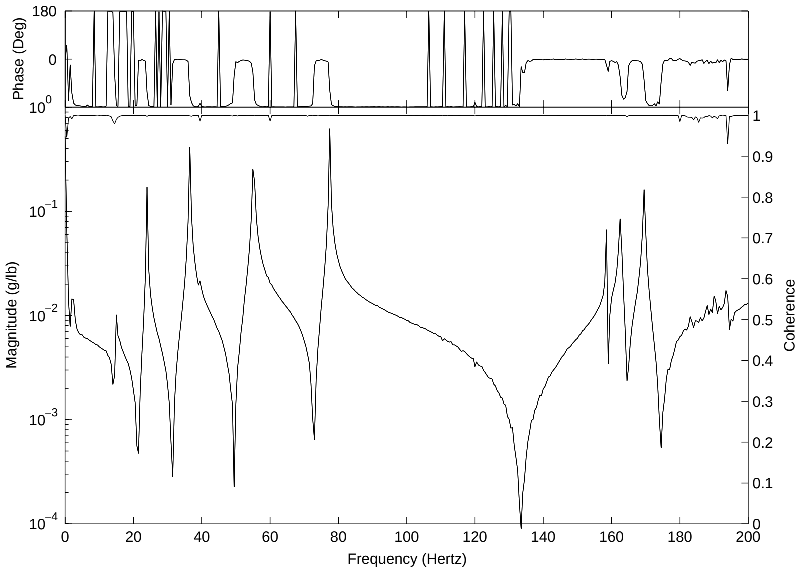

Fig. 15. Case 4: pseudorandom excitation.

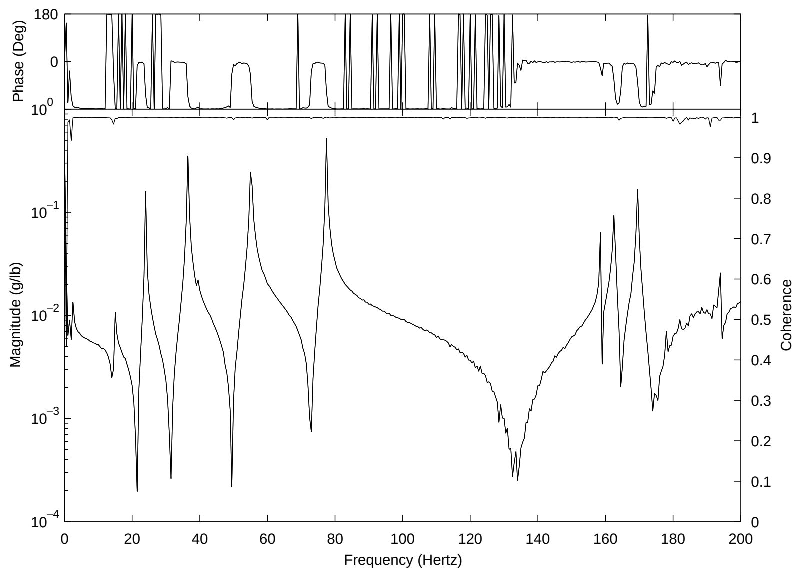

Fig. 16. Case 5: periodic random excitation.

demonstrates that burst random with cyclic averaging improves the measurement further. Again the total measurement time remains the same.

Cases 4–7 (Figs. 15–18) demonstrate the quality of FRF measurements that can be achieved with pseudo and periodic random excitation methods with very few RMS spectral averages.

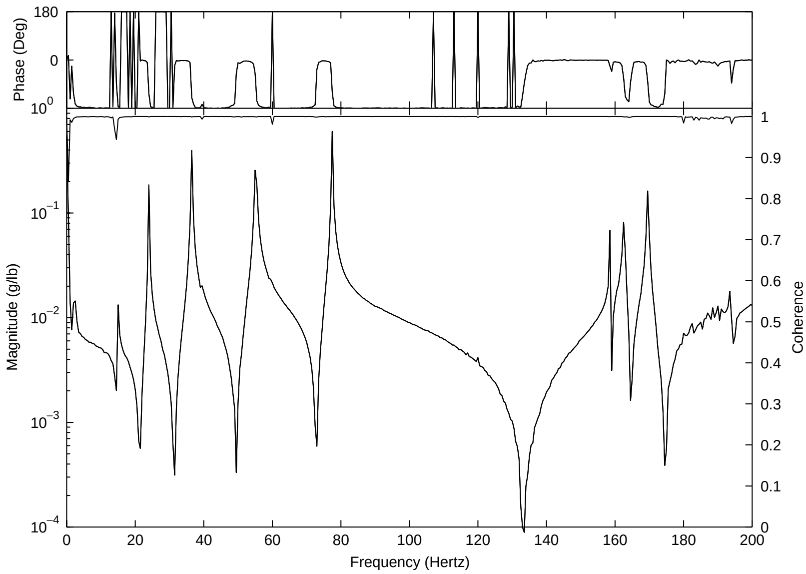

Fig. 17. Case 6: Pseudorandom excitation.

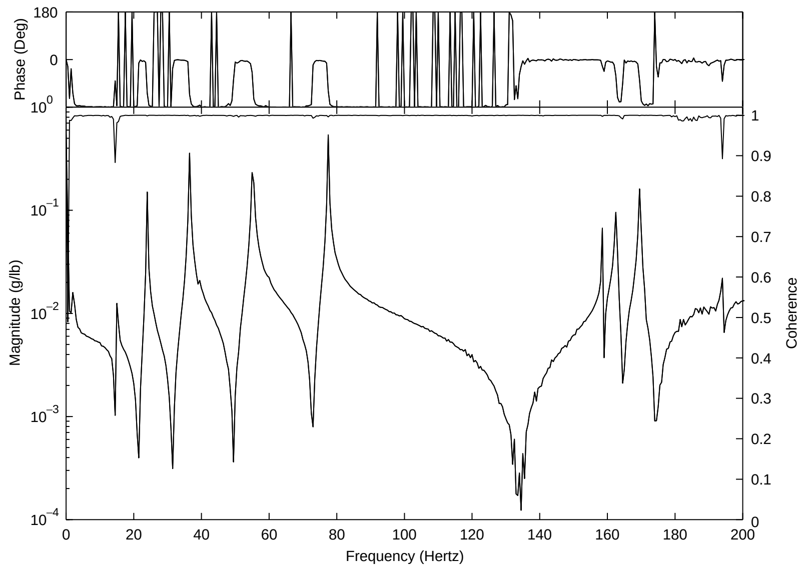

Fig. 18. Case 7: periodic random excitation.

Cases 8 and 9 (Figs. 19 and 20) are hybrid techniques involving the combination of burst random with pseudo and periodic random excitation together with cyclic averaging.

Case 10 (Fig. 21) demonstrates that Case 3 can be marginally improved with more averages, both cyclic and RMS spectral averages. However, Case 11 (Fig. 22) demonstrates that Case 1 (random with Hann window) cannot be improved by adding RMS spectral averages. This is a

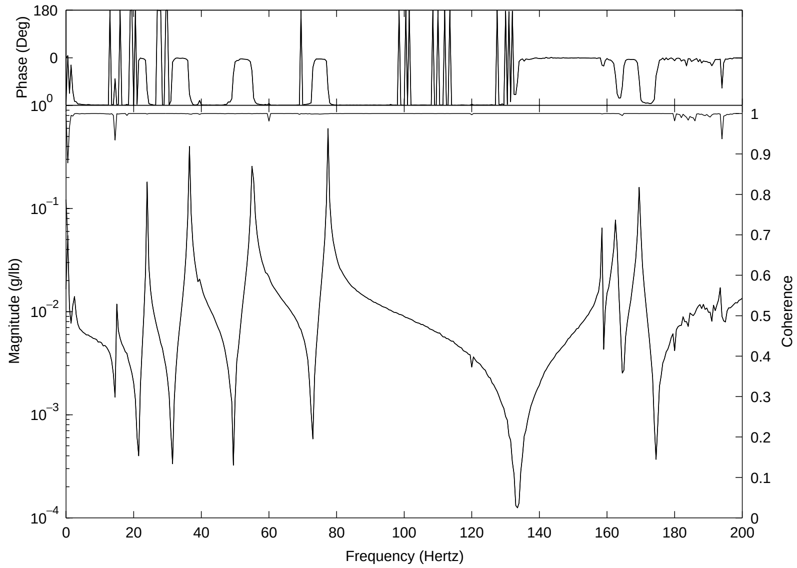

Fig. 19. Case 8: burst pseudorandom excitation with cyclic averaging.

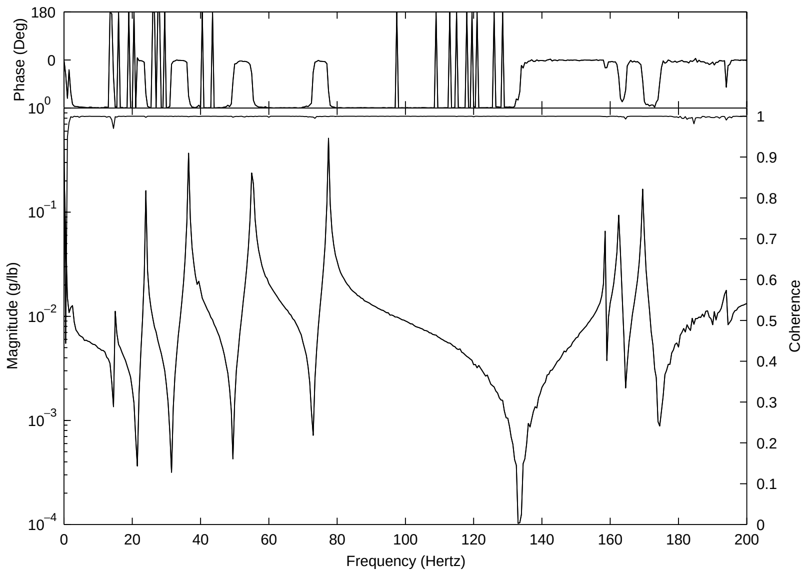

Fig. 20. Case 9: burst periodic random excitation with cyclic averaging.

popular misconception that adding RMS spectral averages will improve the FRF estimate. This is clearly not true for this case.

Case 12 (Fig. 23) demonstrates that additional cyclic averages, together with RMS spectral averages, is an improvement over Case 2 but the improvement is not significant considering the additional measurement time.

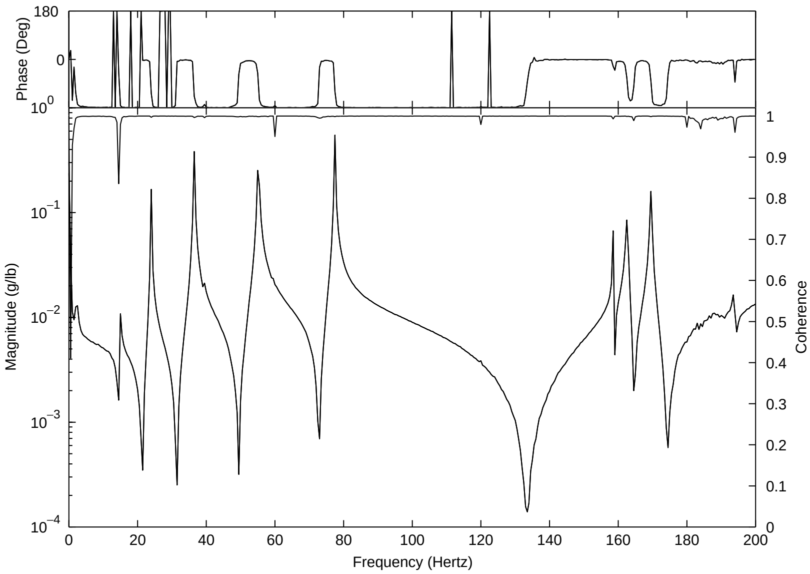

Fig. 21. Case 10: burst random excitation with cyclic averaging.

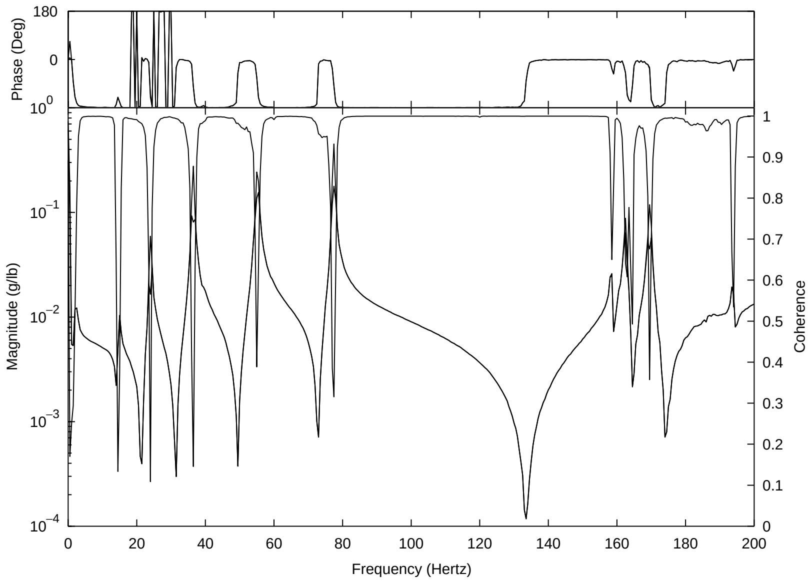

Fig. 22. Case 11: random excitation with Hann window.

Finally, Cases 13 and 14 (Figs. 24 and 25) demonstrate that, when pseudo and periodic random excitation is coupled with cyclic averaging, a nearly perfect (with respect to the removal of the leakage error) FRF measurement results. Note also that in almost every case where high-quality FRF measurements have been achieved, window functions are not required so correction for the window characteristics is unnecessary.

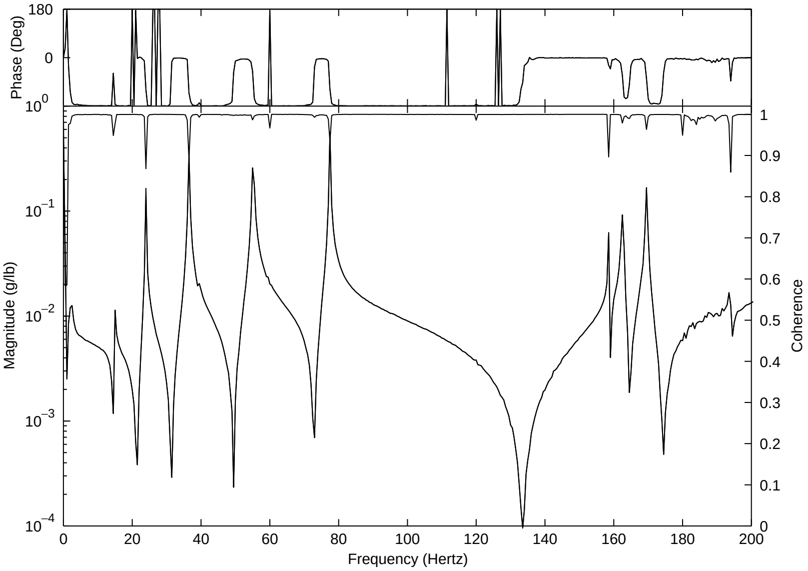

Fig. 23. Case 12: random excitation with Hann window and cyclic averaging.

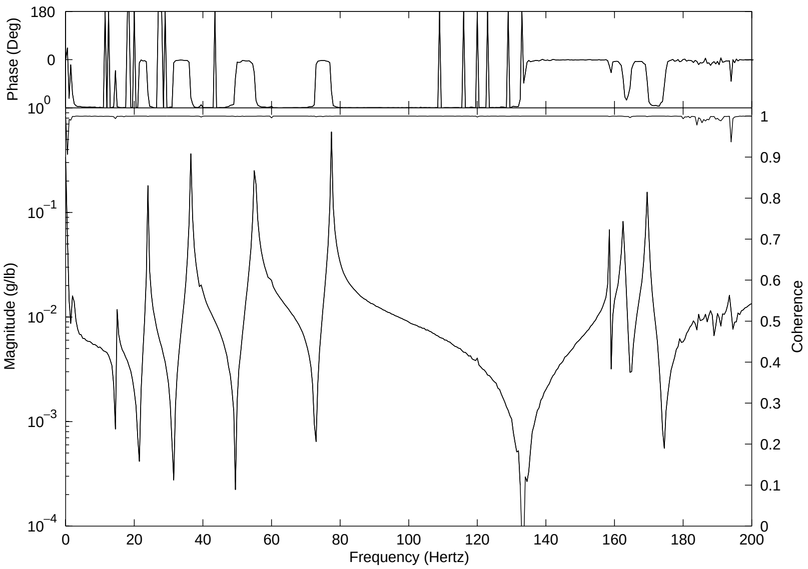

Fig. 24. Case 13: pseudorandom excitation with cyclic averaging.

It is clear that in many of the measurement cases, the multiple coherence can be improved dramatically using simple excitation, averaging and digital signal processing methods. Note that, as the multiple coherence improves, dramatic changes in the FRF magnitude accompany the improvement (factors of 2 to more than 10). When estimating modal parameters, the frequency

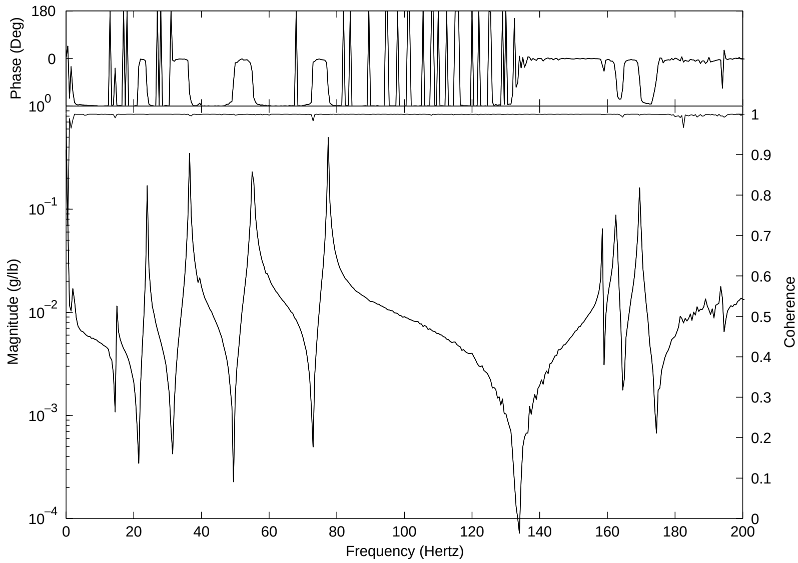

Fig. 25. Case 14: periodic random excitation with cyclic averaging.

and mode shape would probably be estimated reasonably in all cases. However, the damping and modal scaling would be distorted (over estimating damping and under estimating modal scaling). Using these results for model prediction or FE correction would bias the predicted results.

5. Conclusions

The most important conclusion that can be drawn from the results of this measurement exercise on a lightly damped mechanical system is that quality of the data in a given test situation is an indirect function of measurement time or number of averages but is a direct function of measurement technique. The leakage problem associated with utilizing fast Fourier transform (FFT) methodology to estimate frequency response functions on a mechanical system with light damping is a serious problem that can be managed with proper measurement techniques, like periodic and pseudorandom excitation or cyclic averaging with burst random excitation. Hybrid techniques demonstrated in this paper clearly show that a number of measurement techniques are acceptable but some commonly used techniques are clearly unacceptable.

It is also important to note that while ordinary/multiple coherence can indicate a variety of input/output problems, a drop in the ordinary/multiple coherence function, at the same frequency as a lightly damped peak in the frequency response function, is often a direct indicator of a leakage problem. Frequently, comparisons are made between results obtained with narrowband (sinusoid) excitation and broadband (random) excitation when the ordinary/multiple coherence function clearly indicates a potential leakage problem. It is important that good measurement technique be an integral part of such comparisons.

Appendix A. Nomenclature

N_{avg} number of RMS spectral averages
N_c number of cyclic averages
N_d number of periodic delay blocks
N_i number of inputs
N_o number of outputs
F_{min} minimum frequency (Hz)
F_{max} maximum frequency (Hz)
Δf frequency resolution (Hz)
T observation period (s)
$X(\omega)$ linear spectrum of the response
$F(\omega)$ linear spectrum of the excitation
H_1 FRF estimator—response error minimized
H_2 FRF estimator—excitation error minimized
H_v FRF estimator—both excitation and response error minimized
H_s FRF estimator—noise floor weighted form of H_v

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