Available online at www.sciencedirect.com
SCIENCE DIRECT

Journal of Sound and Vibration 262 (2003) 1235–1241

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Letter to the Editor

An approximate method of modelling scattering by composite bodies

I. Ihlárova[a,*], F. Jacobsen[b]

[a] Slovak Academy of Sciences, Institute of Materials and Machine Mechanics, Branch Martin, Severná 14, SK-03601 Martin, Slovakia

[b] Acoustic Technology, Ørsted.DTU, Technical University of Denmark, Building 352, Ørsteds Plads, DK-2800 Lyngby, Denmark

Received 17 July 2002; accepted 8 October 2002

1. Introduction

The boundary element method (BEM) is a powerful tool for solving exterior sound field problems such as, e.g., determining how an acoustic transducer disturbs the sound field [1–3]. However, unless the problem can be reduced to two dimensions the required computer memory and calculation time increase dramatically with the frequency [4].

The present study has been motivated by the need of modelling how a large array of identical microphones placed in a regular pattern disturbs the sound field. Such measurement arrays are used, e.g., for near-field acoustic holography [5,6]. However, modelling such a large array with BEM will require a very long CPU time and an enormous amount of RAM per frequency, unless some simplifying approximations can be made.

An approximate analytical method of calculating scattering from two rigid spheres has been used by Seybert et al. [7] and Juhl [8] as a test case for numerical calculations using BEM. An analytical solution is available for scattering of a plane wave by a rigid sphere [9]. The approximate method involves adding the scattered sound field caused by one sphere in the absence of the other to the sound field on the surface of the other sphere in the absence of the first one. The approximation, which ignores interacting scattering effects, was found to give fairly accurate results for the case of scattering by the two spheres exposed to plane wave incidence both in the direction of the lines between the spheres [7] and in the perpendicular direction [8] at low frequencies and for sufficiently large distances between the spheres.

To save memory and calculation time, a similar approximation might be used in connection with BEM for more general scattering problems. However, ignoring all interacting scattering effects by dealing with each part of a multipart body separately might not be sufficiently accurate under general sound field conditions. On the other hand, it might be possible to improve the

*Corresponding author.
E-mail addresses: ivona@savmt.sk (I. Ihlárova), fja@oersted.dtu.dk (F. Jacobsen).

doi:10.1016/S0022-460X(02)01433-5

accuracy by dealing with two bodies at a time. Thus, the purpose of this note is to examine an approximate method of modelling the scattering caused by composite bodies using BEM.

2. Numerical calculation of scattering by rigid bodies

Consider the case where a given sound field in free space, defined by the sound pressure p^I, is disturbed by a rigid body defined by the surface S. The resulting sound pressure p satisfies the Helmholtz integral equation [4,10,11]

$$C(P)p(P) = \int_S p(Q)\frac{\partial G(R)}{\partial n}\,\mathrm{d}S + 4\pi p^I(P), \tag{1}$$

where Q is a point on the surface S, R is the distance between P and Q, $G(R) = \mathrm{e}^{-\mathrm{i}kR}/R$ is the free-space Green function, and C is a geometrical constant that equals the solid angle at P measured from outside the scattering body. The operator $\partial/\partial n$ gives the component of the gradient normal to the surface S, pointing away from the body. The time factor $\mathrm{e}^{\mathrm{i}\omega t}$ has been omitted.

The boundary element method involves solving Eq. (1) numerically by dividing the surface of the body into j elements and placing the point P successively at each of the nodes [4]. The mesh must be fine enough to represent the shape of the body, and at least four elements are required per wavelength for linear elements or two elements per wavelength for quadratic elements [4]. The result is a matrix equation

$$(\mathbf{H} - \mathbf{C})\mathbf{p} = \mathbf{A}\mathbf{p} = -4\pi\mathbf{p}^I, \tag{2}$$

where the complex vector $\mathbf{p}$ contains the unknown nodal sound pressures, $\mathbf{p}^I$ is the nodal sound pressures of the incoming wave in absence of the scattering body, and element h_{ij} of the matrix $\mathbf{H}$ is the integral of the normal derivative of the Green function over the jth element with respect to the ith calculation point. When the collocation point P coincides with Q the integral equation is singular ($R = 0$) [4,12]. One method of solving this problem involves the introduction a local polar co-ordinate system (r, θ), such that the singularity is placed at $r = 0$. The singularity is neutralized because of the factor r in the Jacobian of the transformation [13].

The matrix $\mathbf{A}$ has the size $n \times n$, where n is number of nodes, and it depends only on the frequency and on the shape of the scattering body. The number of floating point operations (flops) needed for computing the matrix $\mathbf{A}$ is proportional to n^2, and the number of flops needed for solving Eq. (2) is proportional to n^3. If "double precision" is used (8 bytes) both for the real and the imaginary part of each element of $\mathbf{A}$ the storage requirement amounts to $16 \times n^2$ bytes.

2.1. *An approximate method for determining scattering caused by multipart bodies*

Full three-dimensional scattering problem may well require thousands of nodes, so a very large RAM memory is needed and very long calculation times can occur. However, if the scattering object consists of several unconnected parts it might be possible to obtain satisfactory accuracy by adding the scattered sound field generated by each part in the absence of the others, as demonstrated for the case of two spheres in Refs. [7,8].

To test this hypothesis, scattering by two parallel, cylindrical microphones has been examined. In Fig. 1 the relative mean square error (the square of the difference between the "exact" pressure

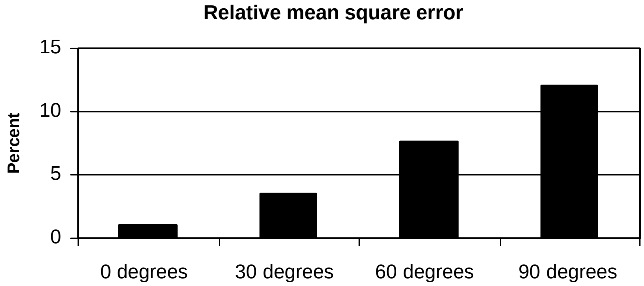

Fig. 1. Relative mean square error of simple approximate method for two parallel microphones, averaged over frequencies up to $ka = 1$, where a is the radius of the microphones, and calculated for plane waves of four different angles of incidence.

determined with a full BEM model of the two microphones and the approximate pressure computed using an approximate BEM technique similar to the approximate analytical method used in Refs. [7,8]) as a function of the angle of incidence of a plane wave is shown. It is apparent that the approximate method is quite accurate for axial incidence, but it can also be seen that its performance deteriorates with the angle of incidence.

2.2. *An improved approximate method*

A less efficient but potentially more accurate method for multipart bodies is described in what follows. First the vector of nodal sound pressures $\mathbf{p}_B$ on a given body B in the absence of the other bodies is calculated. In the next step, one of the neighbouring bodies, say no 1, is taken into account, and the nodal sound pressures on the combined body are determined. From this vector, the sound pressures that correspond to the nodes of the body B, $\mathbf{p}_{B1}(n_B)$, are selected, and $\mathbf{p}_B$ is subtracted from $\mathbf{p}_{B1}(n_B)$, which gives the sound pressure increase on the body B caused by body no. 1. This technique is used for calculating the sound pressure increase $\mathbf{p}_{Bm}$ on B caused by all surrounding bodies, one at a time. Eventually, the total pressure increase on the body B is determined as the sum of the sound pressure increase $\mathbf{p}_B$ caused by the body itself and the additional contributions from the surrounding bodies $(1, 2, 3, \ldots, M)$

$$\mathbf{p}_{tot} = \mathbf{p}_B + \sum_{m=1}^{M} \mathbf{p}_{Bm}. \tag{3}$$

2.3. *Discussion*

The simplest method ignores all interactions between bodies. The improved method described in Section 2.2 deals with the body under test and each of the surrounding bodies separately, and ignores reflections between the bodies that surround the body under investigation. Both methods are particularly economical for large arrays of identical bodies placed in a regular arrangement and having the same orientations in space, such as a typical microphone array. The accuracy can be expected to be better for plane wave incidence perpendicular to the plane of the array than for any other sound field, and better the longer the distance between adjoining bodies.

3. Results

The approximate method has been tested on an array consisting of nine 1-in microphones placed in a regular pattern as shown in Fig. 2. Each microphone had a radius of 11.9 mm and a height of 108 mm. The surface of each microphone was divided into 418 nodes, and the full model, which was needed as a reference, consisted of 3762 nodes. The calculation of the matrix **A** for just one frequency required $3762^2 \times 16$ bytes = 226 MB of RAM. By comparison, calculating the matrix for the centre microphone and one of the eight surrounding microphones required an RAM memory of $(2 \times 418)^2 \times 16$ bytes. Four of the matrices are identical, and the other four are also identical, so all in all the storage requirements amounted to $2 \times (2 \times 418)^2 \times 16$ bytes = 22.4 MB. The number of flops needed for setting up the equations was also about ten times less that for the full model ($3762^2/2 \times (2 \times 418)^2 \cong 10$), and the number of flops needed for solving the system for one frequency was about 11 times less than that for the full model ($3762^3/8 \times (2 \times 418)^3 \cong 11.4$).

In Figs. 3 and 4 the results obtained with the full model consisting of nine microphones are compared with the results of the approximate method for plane waves of four different angles of incidence, 0°, 30°, 60° and 90°, measured from the axis of one of the microphones. In Fig. 3 the distance between the centres of the closest microphones is 50 mm, and in Fig. 4 it is 35 mm. The calculations have been carried out for frequencies up to 4.6 kHz, corresponding to the dimensionless frequency $ka = 1$, where a is the radius of the microphones.

As can be seen in Fig. 3, the approximate method performs quite well in the entire frequency range when the distance between the microphones is 50 mm. Up to 1100 Hz the results are almost perfect at all angles of incidence. At higher frequencies small differences caused by reflections between the surrounding microphones occur. The maximum error is approximately 0.5 dB.

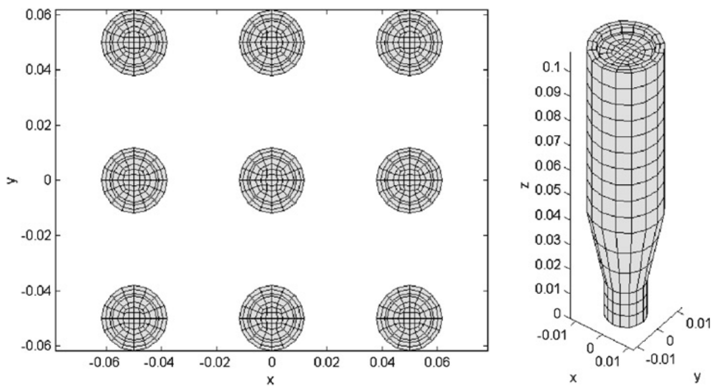

Fig. 2. Cross-section of microphone array and one of the microphones.

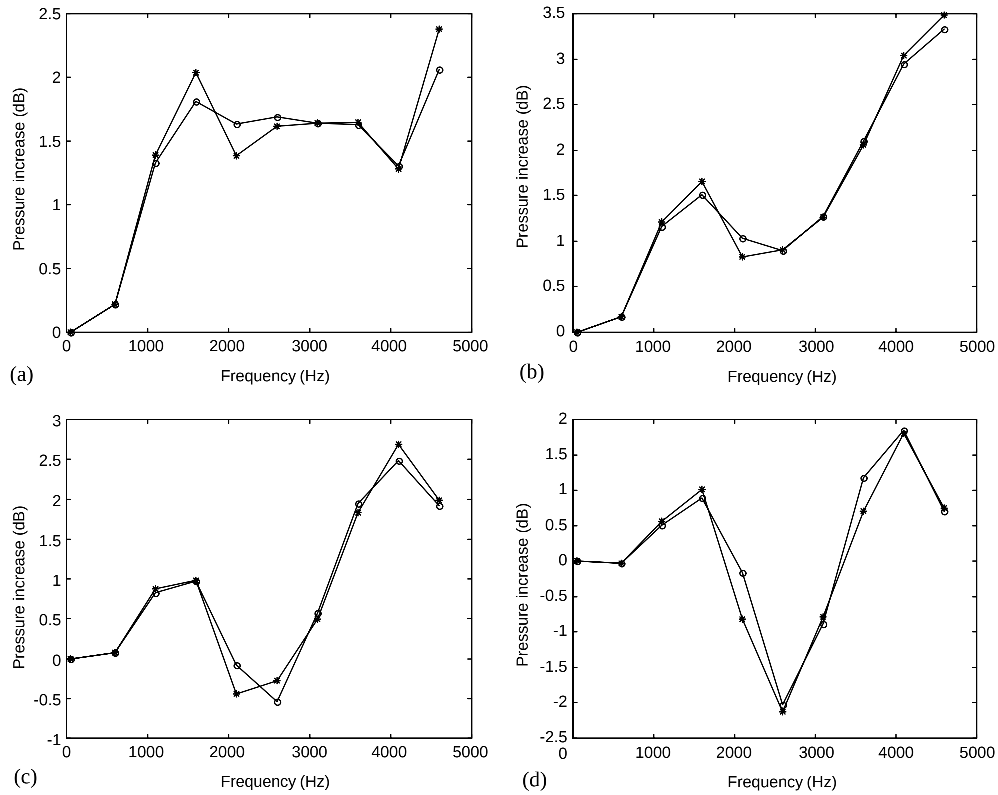

Fig. 3. Sound pressure increase on the membrane of the centre microphone computed with the full model (–∗–) and with the approximate model (–○–) for different angles of incidence (a) 0°, (b) 30°, (c) 60°, and (d) 90°. The distance between neighbouring microphones is 50 mm.

The approximation is somewhat less accurate when the distance between the centres of the microphones is 35 mm, corresponding to three times the microphone radius. At low frequency the method is quite accurate, but at higher frequencies deviations of up to 2 dB occur. Note the difference between the "true" curves shown in Figs. 3 and 4.

4. Conclusions

An approximate method of calculating the scattering of composite bodies has been examined. The method, which involves determining the increase of the sound pressure on one body caused by each of the surrounding bodies one at a time, has been used for determining how an array of cylindrical microphones placed in a regular grid disturbs a plane wave propagating in an arbitrary direction. There is a very significant reduction in the required computer resources if the bodies are identical, have the same orientation, and are placed in a regular pattern, which is the case with the microphone array. In the test case both the required RAM memory and the number of flops needed for calculating the matrix **A** were reduced by a factor of ten, whereas the number of flops needed for solving the system were reduced by a factor of 11.

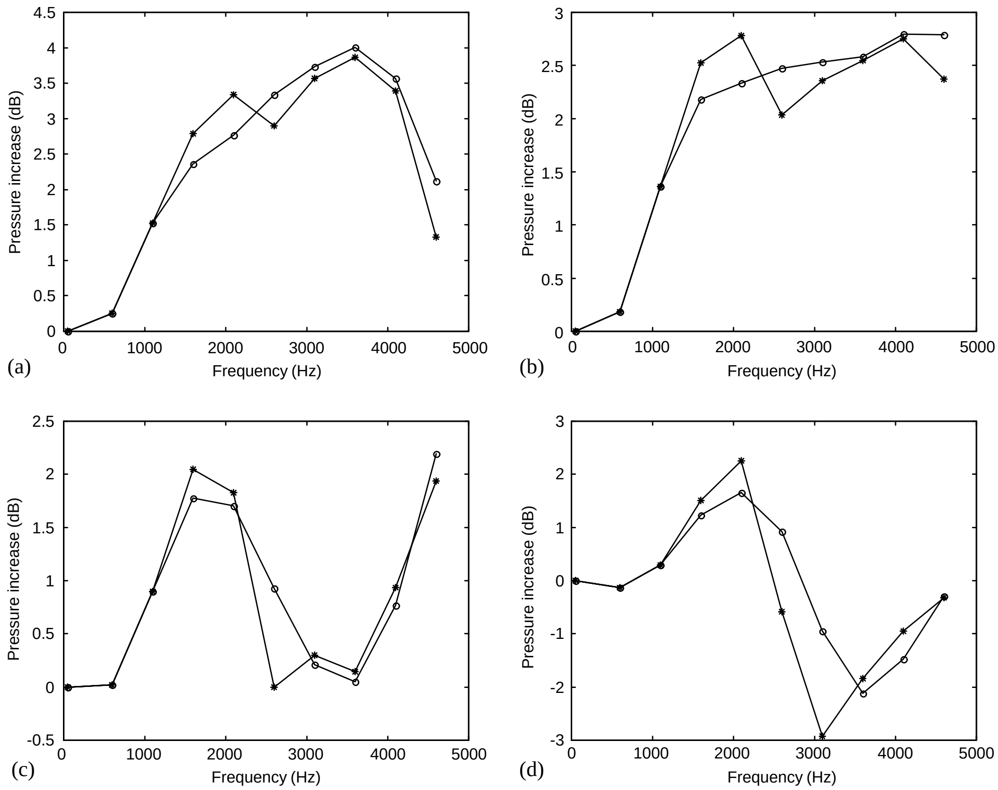

Fig. 4. Sound pressure increase on the membrane of the centre microphone computed with the full model (–∗–) and with the approximate model (–○–) for different angles of incidence (a) 0°, (b) 30°, (c) 60°, and (d) 90°. The distance between neighbouring microphones is 35 mm.

In general, the method performs better for sound incidence almost perpendicular to the plane of the array than for large angles of incidence, and better the larger the distance between the microphones. The method has been found to be accurate up to dimensionless frequency $ka = 1$ when the distance between the microphones is more than $3a$, where a is the radius of the cylindrical microphones.

Acknowledgements

This work was made possible by a Marie Curie Fellowship from the European Community.

References

[1] J.C. Rébillat, S.R. Rifai, Détermination expérimentale de la diffraction acoustique autour de sondes intensimétriques, Acustica 70 (1990) 83–88.

[2] P.M. Juhl, A numerical investigation of standard condenser microphones, Journal of Sound and Vibration 177 (1994) 433–446.
[3] F. Jacobsen, V. Cutanda, P.M. Juhl, A numerical and experimental investigation of the performance of sound intensity probes at high frequencies, Journal of the Acoustical Society of America 103 (1998) 953–961.
[4] A.F. Seybert, T.W. Wu, Acoustic modeling: boundary element methods, in: M.J. Crocker (Ed.), Encyclopedia of Acoustics, Wiley, New York, 1997 (Chapter 15)..
[5] J.D. Maynard, E.G. Williams, Y. Lee, Nearfield acoustic holography: I, theory of generalized holography and the development of NAH, Journal of the Acoustical Society of America 78 (1985) 1395–1413.
[6] E.G. Williams, Fourier Acoustics, Academic Press, San Diego, 1999.
[7] A.F. Seybert, B. Soenarko, F.J. Rizzo, D.J. Shippy, A special integral equation formulation for acoustic radiation and scattering for axisymmetric bodies and boundary conditions, Journal of the Acoustical Society of America 80 (1986) 1241–1247.
[8] P.M. Juhl, An axisymmetric integral equation formulation for free space non-axisymmetric radiation and scattering of a known incident wave, Journal of Sound andVibration 163 (1993) 397–406.
[9] P.M. Morse, K.U. Ingard, Theoretical Acoustics, McGraw-Hill, New York, 1968.
[10] B. Baker, E.T. Copson, The Mathematical Theory of Huygen's Principle, 2nd Edition, Oxford University Press, Oxford, 1953.
[11] A.D. Pierce, Acoustics. An Introduction to Its Physical Principles and Applications, McGraw-Hill, New York, 1981.
[12] A.F. Seybert, B. Soenarko, F.J. Rizzo, D.J. Shippy, An advanced computation method for radiation and scattering of acoustic waves in three-dimensions, Journal of the Acoustical Society of America 77 (1985) 362–368.
[13] F.J. Rizzo, D.J. Shippy, An advanced boundary integral equation method for three-dimensional thermoelasticity, International Journal for Numerical Methods in Engineering 11 (1977) 1753–1769.