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Natural vibration of rectangular plates with an internal line hinge using the first order shear deformation plate theory

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Abstract

This paper presents exact solutions for vibration of rectangular plates with an internal line hinge. The rectangular plate is simply supported on two parallel edges and the remaining two edges may take any combination of support conditions. The line hinge is perpendicular to the two simply supported parallel edges. The Lévy type solution method and the state-space technique are employed in connection with the first order shear deformation plate theory (FSDT) to study natural vibration of rectangular plates with an internal line hinge. In particular, exact vibration frequencies are obtained for rectangular plates of different aspect ratios and edge support conditions. The influence of the internal line hinge on the vibration behavior of rectangular plates is studied.

1. Introduction

Plates are one of the most important structural elements that are widely encountered in aerospace, marine, mechanical and civil engineering structures. The vibration behavior of plates and plate structures is one of the major concerns in designing plate type structures. Vibration characteristics of plates have been extensively studied over the last 100 years. Plates of all sorts of shapes, boundary conditions and subjected to various applied in-plane force distributions were considered and the frequency parameters were documented in monographs [1–4], standard texts [5–7], and technical papers [8–12].

In engineering applications, plates with various complications, i.e., stiffeners [13,14], interior openings [15] and line hinge [16], are used to serve different purposes required in a structure. For

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instance, a line hinge in a plate can be used to facilitate folding of gates, or the opening of doors and hatches. The hinge can also be used to simulate a through crack prior to the edge misalignment. It is interesting to note that apparently there is no study found in open literature for vibration of thick plates with internal line hinge.

The present study provides the first-known solutions based on the first order shear deformation theory for vibration of rectangular plates with an internal line hinge. The Lévy type solution method and the state-space technique [6,10,17–18] are employed in connection with the first order shear deformation plate theory (FSDT) to study natural vibration of rectangular plates with an internal line hinge. In treating a plate with an internal line hinge, a domain decomposition technique developed [19] is applied in this study. The plate is divided into two subdomains and the essential and natural continuity conditions along the interface of the two subdomains are imposed. Exact vibration frequencies are obtained for six possible sets of boundary conditions with different plate aspect ratios, plate thickness ratios, and various locations of the line hinge. The influence of the internal line hinge on the vibration behavior of rectangular plates with two parallel edges simply supported while the other edges having various boundary conditions is studied. The extensive exact solutions presented in this paper may serve as reference solutions for checking the accuracy of potential numerical solutions of thick rectangular plates with internal hinges.

2. Formulation

Consider an isotropic rectangular plate of length aL and width L as shown in Fig. 1. The plate is simply supported on the two edges parallel to the x-axis. An internal line hinge that is parallel to the y-axis, exists in the plate (see Fig. 1). The origin of the co-ordinate system (x, y) is set at the middle point of the plate bottom edge, so that $-aL/2 \leqslant x \leqslant aL/2$ and $0 \leqslant y \leqslant L$. The problem at

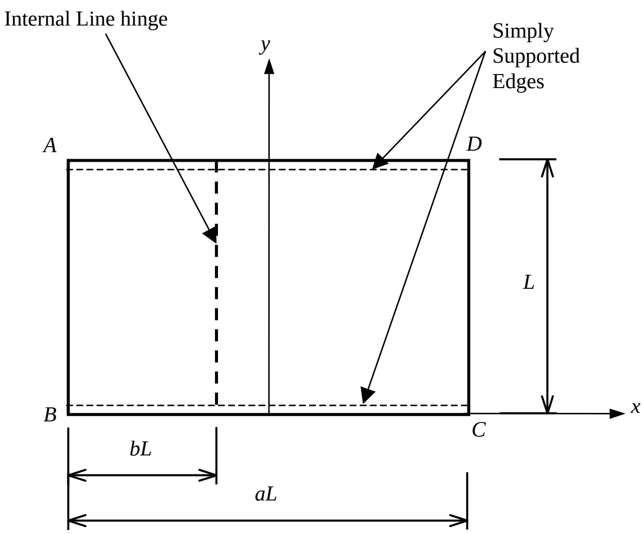

Fig. 1. A Lévy plate with an internal line hinge.

hand is to determine the vibration frequencies of the plate with an internal line hinge. We wish to use the FSDT [6,7] to study the problem.

The plate is considered to have two spans that are separated by the line hinge. The governing differential equations of the isotropic plate based on the first order plate theory for a span in harmonic vibration can be written as [7]

$$\kappa^2 Gh\left[\frac{\partial}{\partial x}\left(\frac{\partial w^i}{\partial x}+\phi_x^i\right)+\frac{\partial}{\partial y}\left(\frac{\partial w^i}{\partial y}+\phi_y^i\right)\right]+\rho h\omega^2 w^i=0, \tag{1}$$

$$D\left[\frac{\partial}{\partial x}\left(\frac{\partial \phi_x^i}{\partial x}+\nu\frac{\partial \phi_y^i}{\partial y}\right)\right]+\frac{(1-\nu)D}{2}\left[\frac{\partial}{\partial y}\left(\frac{\partial \phi_y^i}{\partial x}+\frac{\partial \phi_x^i}{\partial y}\right)\right] - \kappa^2 Gh\left(\frac{\partial w^i}{\partial x}+\phi_x^i\right)+\frac{\rho h^3}{12}\omega^2\phi_x^i=0, \tag{2}$$

$$D\left[\frac{\partial}{\partial y}\left(\frac{\partial \phi_y^i}{\partial y}+\nu\frac{\partial \phi_x^i}{\partial x}\right)\right]+\frac{(1-\nu)D}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \phi_y^i}{\partial x}+\frac{\partial \phi_x^i}{\partial y}\right)\right] - \kappa^2 Gh\left(\frac{\partial w^i}{\partial y}+\phi_y^i\right)+\frac{\rho h^3}{12}\omega^2\phi_y^i=0, \tag{3}$$

where the superscript i (=1, 2) denotes the ith span, h is the plate thickness, E is Young's modulus, $G=E/[2(1+\nu)]$ is the shear modulus, ν is the Poisson ratio, κ^2 is the shear correction factor, $D=Eh^3/[12(1-\nu^2)]$ is the flexural rigidity of the plate, ρ is the mass density of the plate, ω is the vibration frequency of the plate, w is the transverse displacement, and ϕ_x and ϕ_y are rotations of a transverse normal about the y and x directions, respectively.

The essential (geometric) and natural (force) boundary conditions for the two parallel edges (at $y=0$ and L) in the ith span are

$$w^i=0, \qquad M_y^i=0, \qquad \phi_x^i=0, \tag{4a-c}$$

where M_y^i is the bending moment and is defined by

$$M_y^i=D\left(\frac{\partial \phi_y^i}{\partial y}+\nu\frac{\partial \phi_x^i}{\partial x}\right). \tag{5}$$

The general Lévy-type solution approach is employed to solve the governing differential equations for the ith span. The displacement fields can be expressed as

$$\begin{Bmatrix} w^i(x,y) \\ \phi_x^i(x,y) \\ \phi_y^i(x,y) \end{Bmatrix}=\begin{Bmatrix} \Phi_w^i(x)\sin\dfrac{m\pi y}{L} \\ \Phi_x^i(x)\sin\dfrac{m\pi y}{L} \\ \Phi_y^i(x)\cos\dfrac{m\pi y}{L} \end{Bmatrix}, \tag{6}$$

where $\Phi_w^i(x)$, $\Phi_x^i(x)$ and $\Phi_y^i(x)$ are unknown functions to be determined, and m $(=1,2,\ldots,\infty)$ is the number of halfwaves in the y direction for a vibrating mode. Eq. (6) satisfies the simply supported boundary conditions on edges $y=0$ and L as defined in Eqs. (4a–c).

Substituting Eq. (6) into Eqs. (1)–(3), the following differential equation system can be derived:

$$\frac{\mathrm{d}\boldsymbol{\psi}^i}{\mathrm{d}x} = \mathbf{H}^i\boldsymbol{\psi}^i, \tag{7}$$

where $\boldsymbol{\psi}^i = [\Phi_w^i\ (\mathrm{d}\Phi_w^i/\mathrm{d}x)\ \Phi_x^i\ (\mathrm{d}\Phi_x^i/\mathrm{d}x)\ \Phi_y^i\ (\mathrm{d}\Phi_y^i/\mathrm{d}x)]^{\mathrm{T}}$ and $\mathbf{H}^i$ is a 6×6 matrix with the following non-zero elements:

$$H_{12}^i = H_{34}^i = H_{56}^i = 1, \quad H_{21}^i = \frac{-(m\pi/L)^2\kappa^2 Gh + \rho h\omega^2}{-\kappa^2 Gh} \tag{8,9}$$

$$H_{24}^i = -1, \quad H_{25}^i = \frac{m\pi}{L}, \quad H_{42} = \frac{\kappa^2 Gh}{D}, \quad H_{46} = \frac{(m\pi/b)(1+v)}{2}, \tag{10-13}$$

$$H_{43}^i = \frac{D(1-v)(m\pi/L)^2/2 + \kappa^2 Gh - \rho h^3\omega^2/12}{D}, \tag{14}$$

$$H_{61}^i = \frac{(m\pi/L)\kappa^2 Gh}{[D(1-v)/2]}, \qquad H_{64}^i = -\frac{(m\pi/L)(1+v)}{1-v}, \tag{15,16}$$

$$H_{65}^i = \frac{D(m\pi/L)^2 + \kappa^2 Gh - \rho h^3\omega^2/12}{[D(1-v)/2]}. \tag{17}$$

A general solution of Eq. (7) can be obtained as

$$\boldsymbol{\psi}^i = \mathbf{e}^{\mathbf{H}x}\mathbf{c}^i, \tag{18}$$

where $\mathbf{c}^i$ is a constant column vector that can be determined by the plate boundary conditions of the two edges parallel to the y-axis and/or interface conditions between the two spans and $\mathbf{e}^{\mathbf{H}x}$ is the general matrix solution of Eq. (7). The detailed procedure in determining Eq. (18) has been given by Reddy and Khdeir [10], Liew et al. [17] and Xiang et al. [18].

The two edges parallel to the y-axis at $x = -aL/2$ and $x = aL/2$ may have the following boundary conditions:

$$M_x^i = 0, \quad M_{xy}^i = 0, \quad Q_x^i = 0, \quad \text{if the edge is free;} \tag{19a-c}$$

$$w^i = 0, \quad M_x^i = 0, \quad \phi_y^i = 0, \quad \text{if the edge is simply supported;} \tag{20a-c}$$

$$w^i = 0, \quad \phi_x^i = 0, \quad \phi_y^i = 0, \quad \text{if the edge is clamped;} \tag{21a-c}$$

where the superscript i takes the value 1 or 2, M_x^i, M_{xy}^i and Q_x^i are bending moment, twist moment and transverse shear force in the plate, respectively, and defined by

$$M_x^i = D\left(\frac{\partial\phi_x^i}{\partial x} + v\frac{\partial\phi_y^i}{\partial y}\right),$$

$$M_{xy}^i = D\frac{(1-v)}{2}\left(\frac{\partial\phi_x^i}{\partial y} + \frac{\partial\phi_y^i}{\partial x}\right), \tag{22,23}$$

$$Q_x^i = \kappa^2 Gh\left(\frac{\partial w^i}{\partial x} + \phi_x^i\right). \tag{24}$$

To ensure the continuity and support condition along the internal line hinge, the essential and natural boundary conditions for the interface between the two spans are defined as

$$w^1 = w^2, \quad \phi_y^1 = \phi_y^2, \quad M_x^1 = 0, \quad M_x^2 = 0, \quad M_{xy}^1 = M_{xy}^2, \quad Q_x^1 = Q_x^2. \tag{25a-f}$$

In view of Eq. (18), a homogeneous system of equations can be derived by implementing the boundary conditions of the plate along the two edges parallel to the y-axis (Eqs. (19)–(21)) and the interface conditions between the two spans (Eq. (25)) when assembling the two spans to the whole plate [15]

$$\mathbf{K}\begin{Bmatrix} \mathbf{c}^1 \\ \mathbf{c}^2 \end{Bmatrix} = \{\mathbf{0}\}. \tag{26}$$

The vibration frequency ω is determined when the determinant of $\mathbf{K}$ in Eq. (26) is equal to zero. As the vibration frequency ω is imbedded in matrix $\mathbf{H}$, it cannot be obtained directly from Eq. (26). A numerical iteration procedure was developed by Xiang et al. [18] to solve the eigenvalue problem defined by Eq. (26) and the procedure in Ref. [18] is applied in this paper to obtain the vibration frequency ω of rectangular Mindlin plates with an internal line hinge.

3. Results and discussions

The analytical method discussed in the previous section is applied here to obtain exact vibration solutions for rectangular plates with an internal line hinge. The vibration frequency is expressed in term of a non-dimensional frequency parameter $\lambda = (\omega L^2/\pi^2)\sqrt{\rho h/D}$. For convenience, we shall use letters F, S and C to denote free, simply supported and clamped edges, respectively. A two-letter symbol is used to describe the plate boundary conditions on the two edges parallel to the y-axis. For instance, a CS Lévy plate has the edge AB clamped and the edge DC simply supported (see Fig. 1). The Poisson ratio $v = 0.3$ and the shear correction factor $\kappa^2 = \frac{5}{6}$ are used in the calculation.

3.1. Comparison studies

Table 1 presents vibration frequencies obtained by the authors and by Wang et al. [16] for a simply supported square plate with an internal line hinge. The results by Wang et al. [16] were based on the thin (Kirchhoff) plate theory, and the Ritz method was used to obtain the numerical solutions. In order to compare the present results with the ones by Wang et al. [16], the plate thickness ratio $h/L = 0.01$ is used to simulate a thin plate. We observe that the present analytical solutions are in close agreement with the Ritz solutions of Wang et al. [16]. It confirms the correctness of the present analytical solution.

3.2. Vibration of square Mindlin plates with an internal line hinge

Tables 2 and 3 present the first 10 exact frequency parameters for square Mindlin plates with symmetric Lévy-type supporting edges (i.e., SS, FF and CC plates) and asymmetric Lévy-type

Table 1
Comparison study of frequency parameters $\lambda = (\omega L^2/\pi^2)\sqrt{\rho h/D}$ for *SS* square plate with an internal line hinge (h/L = 0.01)

b	Source	Mode sequence		
		1	2	3
0.1	[16]	1.9068	3.8317	4.9614
	Present	1.90382	3.80991	4.95555
0.3	[16]	1.7011	3.9603	4.8047
	Present	1.69719	3.95086	4.79543
0.5	[16]	1.6348	4.7356	5.0000
	Present	1.63090	4.72531	4.99545

Table 2
Frequency parameters $\lambda = (\omega L^2/\pi^2)\sqrt{\rho h/D}$ for symmetric Lévy square plates with an internal line hinge

Cases	h/L	b	Mode sequence									
			1	2	3	4	5	6	7	8	9	10
SS plate	0.01	0.1	1.90382	3.80991	4.95555	7.33240	7.46706	9.94774	11.1622	12.6424	13.5464	16.7066
		0.3	1.69719	3.95086	4.79543	7.27309	9.73778	9.78077	12.4102	12.7881	15.2020	16.7189
		0.5	1.63090	4.72531	4.99545	7.60420	7.98838	9.70361	11.1952	12.9694	16.5951	16.6301
	0.10	0.1	1.81825	3.40685	4.56169	6.40675	6.51544	8.57906	9.24011	10.4759	11.1278	13.1240
		0.3	1.62114	3.65973	4.40241	6.42570	8.41913	8.42499	10.3268	10.6654	12.3424	13.3317
		0.5	1.55926	4.33580	4.60836	6.70711	7.07165	8.34976	9.38364	10.8093	13.1387	13.2592
CC plate	0.01	0.1	2.71926	5.48208	6.91335	9.54065	10.3276	13.0027	14.1608	15.5300	17.2034	19.8677
		0.3	2.70482	5.32798	5.44718	8.45427	10.1185	12.2193	13.3455	15.0119	16.9627	19.7284
		0.5	2.30503	5.12708	7.01083	9.56080	9.97574	10.0685	13.3203	14.1615	16.8293	18.4161
	0.10	0.1	2.52934	4.93671	5.95326	7.98161	8.79593	10.1942	11.3476	11.9157	13.6366	14.3547
		0.3	2.49830	4.77743	4.79457	7.14570	8.59887	9.83828	10.7586	11.7382	13.4272	14.4643
		0.5	2.14258	4.60484	5.99924	7.98542	8.16928	8.48497	10.4379	11.3539	13.3313	13.8487
FF plate	0.01	0.1	0.971257	1.59685	3.70470	3.92117	4.66004	7.05288	7.07353	8.84712	9.60088	9.83944
		0.3	0.965294	1.60460	3.16978	3.90124	4.67883	5.70215	6.95085	8.82211	9.64297	9.78045
		0.5	0.963332	1.63090	2.77180	3.89845	4.72531	6.50446	7.60420	8.82380	9.70361	10.6414
	0.10	0.1	0.953036	1.53509	3.42434	3.66758	4.29111	5.12983	6.27755	7.62369	7.72962	8.28533
		0.3	0.946998	1.53799	2.84085	3.64484	4.29692	5.07039	6.07996	7.69518	8.18742	8.30045
		0.5	0.944888	1.55926	2.52554	3.64070	4.33580	5.71208	6.70711	7.69526	8.34976	9.05350

supporting edges (i.e., *SF*, *CF* and *CS* plates). The location parameter of the internal line hinge b varies from 0.1 to 0.5 for the symmetric plates (*SS*, *FF* and *CC* plates) and from 0.1 to 0.9 for the asymmetric plates (*SF*, *CF* and *CC* plates). The plate thickness ratios $h/L = 0.01$ and 0.1 are used for the plates. All results in Tables 2 and 3 are presented with 6 significant digits.

The influence of the internal line hinge and the plate thickness ratio on the frequency parameters may be observed from Tables 2 and 3. As expected, the frequency parameters decrease

Table 3
Frequency parameters $\lambda = (\omega L^2/\pi^2)\sqrt{\rho h/D}$ for asymmetric Lévy square plates with an internal line hinge

Cases	h/L	b	Mode sequence									
			1	2	3	4	5	6	7	8	9	10
SF plate	0.01	0.1	1.17944	2.51224	4.16538	4.62791	5.83976	8.65324	8.81268	9.12514	10.9057	12.8255
		0.3	1.16556	2.18666	4.14343	5.33910	5.54343	8.91145	9.09445	10.6205	11.6783	14.2202
		0.5	1.14963	2.29980	4.11076	5.67713	5.85831	9.04624	9.17965	9.54854	10.7571	13.4143
		0.7	1.14473	2.69347	4.08803	4.46971	5.96659	8.48238	9.00120	10.2552	11.0054	13.6493
		0.9	1.16261	2.77981	4.11669	5.88822	6.09278	8.89361	9.02396	9.53011	10.8891	13.9835
	0.10	0.1	1.14853	2.32092	3.87530	4.09382	5.25035	7.34533	7.56908	7.93605	9.24035	10.3991
		0.3	1.13512	2.04281	3.85356	4.83569	4.98483	7.66626	7.90740	8.99140	9.85429	11.5463
		0.5	1.12025	2.13995	3.82181	5.10093	5.30073	7.86255	7.92300	8.22738	9.10514	10.9246
		0.7	1.11704	2.49201	3.80330	3.99784	5.38051	7.21697	7.82458	8.77187	9.33948	11.1663
		0.9	1.13580	2.64421	3.83896	4.84089	5.35300	6.54299	7.86709	8.19462	9.28407	10.6125
CF plate	0.01	0.1	1.22112	3.16016	4.20564	6.31451	7.23400	9.15857	10.4057	11.2977	13.2276	15.4034
		0.3	1.28051	2.97197	4.20994	5.81851	5.97981	9.13703	9.34764	10.9192	12.5219	14.6237
		0.5	1.26971	2.58102	4.16640	5.91753	7.28607	9.08147	10.3066	10.3778	10.9561	14.1510
		0.7	1.24394	2.99688	4.13260	5.31502	6.32836	9.02801	9.06092	11.2952	12.2140	14.4803
		0.9	1.25657	3.32858	4.16020	6.28131	6.86010	9.04851	9.66393	10.4129	11.1789	15.3586
	0.10	0.1	1.18313	2.93688	3.90351	5.61999	6.32253	7.95510	8.70283	9.49069	10.5871	12.2352
		0.3	1.23756	2.73248	3.90420	5.19975	5.29298	7.91616	7.93244	9.14914	10.3691	11.7153
		0.5	1.22842	2.37231	3.86204	5.25014	6.29664	7.88150	8.57886	8.66272	9.19700	11.2458
		0.7	1.20607	2.70423	3.83454	4.64130	5.60353	7.55250	7.83820	9.48074	9.86558	11.5026
		0.9	1.21998	3.08103	3.87007	5.03396	5.61537	7.26012	7.88018	8.63953	9.43337	10.8584
CS plate	0.01	0.1	2.21604	5.18244	5.79699	8.67880	10.1330	11.4427	13.5194	14.1945	17.0650	18.2219
		0.3	2.31282	4.59602	5.09580	7.75921	9.9715	10.2875	12.7916	13.3982	16.8607	18.4243
		0.5	1.94563	4.90690	5.71618	8.62362	9.10394	9.82889	12.3739	13.4822	16.7235	17.5473
		0.7	1.91116	4.90107	4.95277	7.95945	9.89318	11.4158	12.9333	14.1924	16.0577	16.8018
		0.9	2.20059	4.34688	5.16777	7.98521	8.53336	10.0949	12.1093	13.0926	15.4064	17.0210
	0.10	0.1	2.11457	4.73762	5.20686	7.49888	8.69791	9.41664	11.0671	11.3384	13.5850	13.6897
		0.3	2.18322	4.17085	4.63178	6.72742	8.53093	8.78176	10.5019	10.9796	13.3934	14.0408
		0.5	1.83580	4.46066	5.14851	7.46318	7.59897	8.41392	9.97201	11.0560	13.2941	13.5134
		0.7	1.80104	4.37674	4.50626	6.85203	8.47459	9.39838	10.5787	11.3682	12.7301	13.3613
		0.9	2.05540	3.80671	4.70556	6.80949	7.18863	8.65591	9.74040	10.6770	12.0400	13.4476

when a plate becomes thick due to the effect of transverse shear deformation and rotary inertia. This effect becomes more significant for a plate vibrating in higher modes and for a plate with higher degree of edge constraints (i.e., in the order of free to simply supported to clamped edges).

Fig. 2 shows the relationship between the fundamental frequency parameter and the location parameter of the hinge for the three symmetric square plates (Fig. 2(a)) and the three asymmetric square plates (Fig. 2(b)). There are 50 equi-spaced sampling points along the x-axis for curves in Fig. 2(a) and 99 sampling points for curves in Fig. 2(b). The starting and ending points are at $b = 0.01$ and 0.5 for curves in Fig. 2(a) and $b = 0.01$ and 0.99 for curves in Fig. 2(b), respectively. It is observed that the optimal location of the hinge in increasing the fundamental frequency

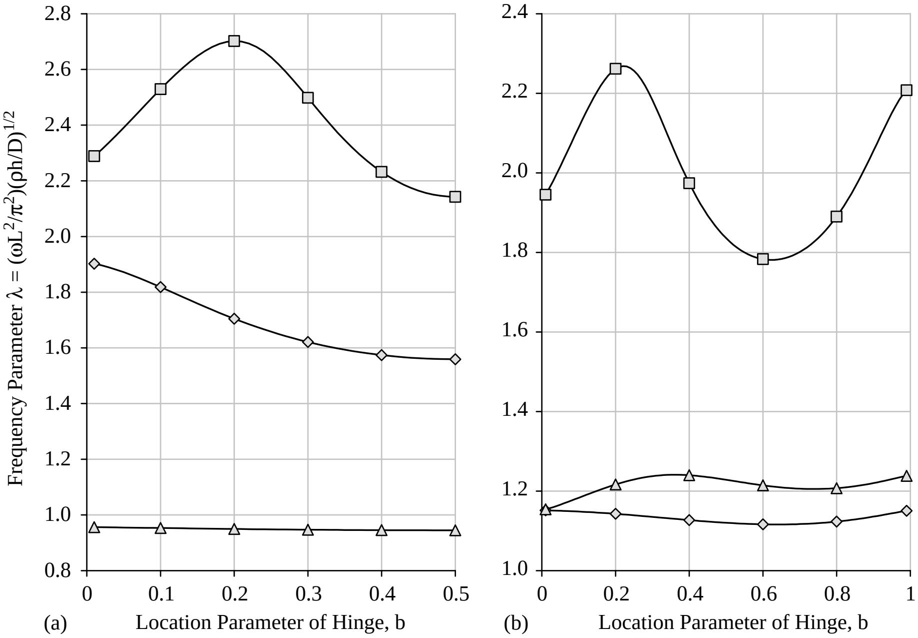

Fig. 2. Fundamental frequency parameters $\lambda = (\omega L^2/\pi^2)\sqrt{\rho h/D}$ versus the location parameter of the internal line hinge b for (a) symmetric plates: $\diamond$, *SS* plate; $\triangle$, *FF* plate; $\square$, *CC* plate and (b) asymmetric plates: $\diamond$, *SF* plate; $\triangle$, *CF* plate; $\square$, *CS* plate.

parameter is at $b \approx 0.2$ for the *CC* and *CS* plates, $b = 0.01$ for the *FF*, *SS* and *SF* plates, and $b \approx 0.35$ for the *CF* plate, respectively. The frequency parameters have a very small variation for the *FF* plate when the location parameter of the hinge changes from 0.01 to 0.5.

It is evident from Tables 1 and 2 and Fig. 2 that the inclusion of the internal line hinge decreases the frequency parameters of the plates. For instance, the fundamental frequency parameter for a simply supported thick square plate ($h/L = 0.1$) without the line hinge is 1.93. The frequency parameter for the same plate decreases to 1.82, 1.62 and 1.56 when the line hinge is placed in the plate at the location of $b = 0.1$, 0.3 and 0.5, respectively. We observe that the optimal location of the internal line hinge in maximizing the frequency parameters varies from plate to plate and from mode to mode.

Figs. 3 and 4 present the modal shapes of the first three modes for *SS* and *CF* square plates with an internal line hinge at various locations. The discontinuity of rotation ϕ_x along the hinge is evident for all cases in Figs. 3 and 4.

3.3. Vibration of rectangular Mindlin plates with an internal line hinge

Tables 4 and 5 show the first 10 exact frequency parameters for thick rectangular plates with symmetric edges (i.e., *SS*, *FF* and *CC* plates) and asymmetric edges (i.e., *SF*, *CF* and *CS* plates). The plate aspect ratio a is taken equal to 2 and 3. The influence of the internal hinge and the plate thickness ratio on the vibration behavior of the plate may be observed from the results presented in Tables 4 and 5.

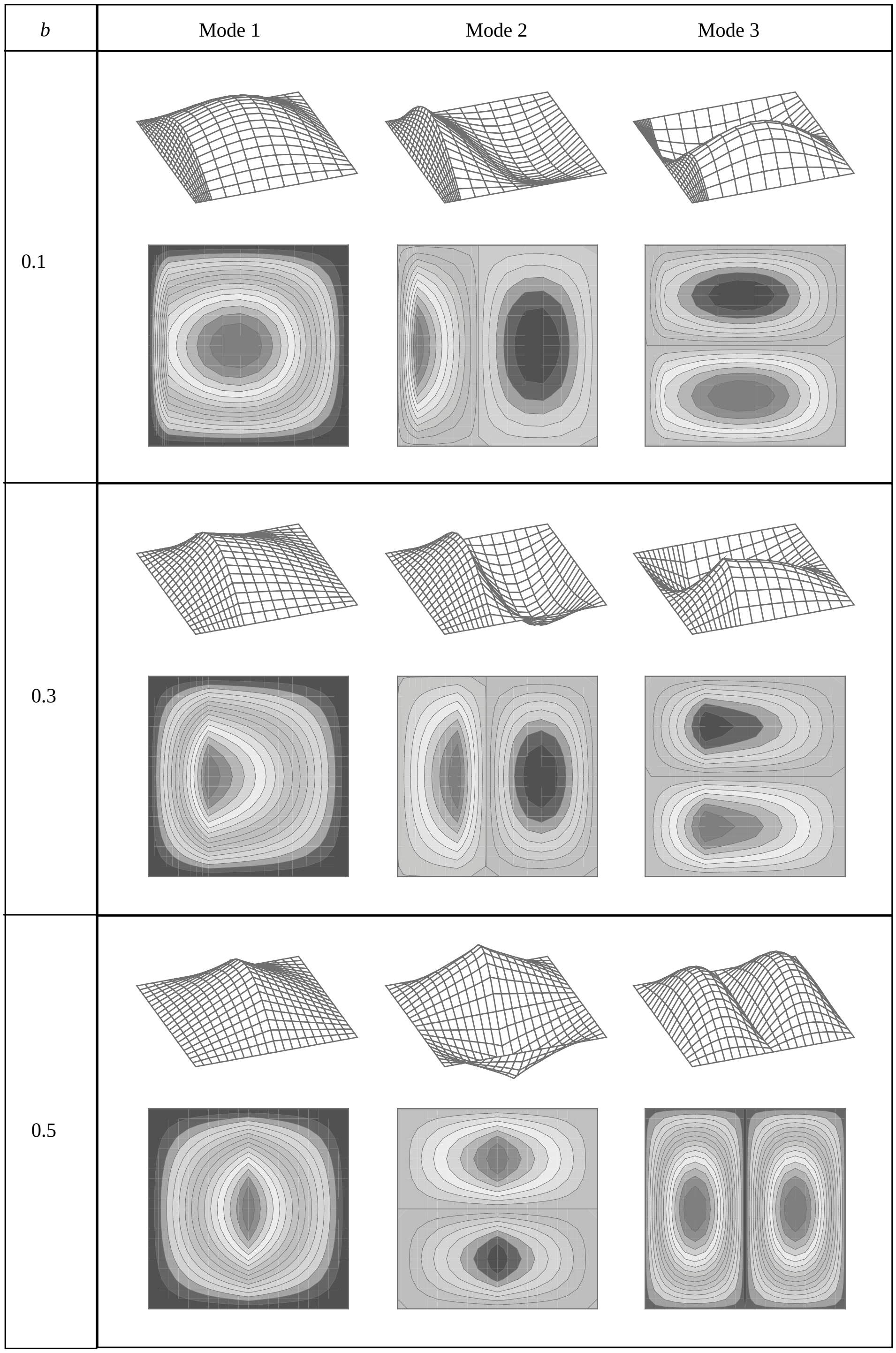

Fig. 3. The first three modal shapes for *SS* square Mindlin plate with an internal line hinge ($a = 1$, $h/L = 0.1$).

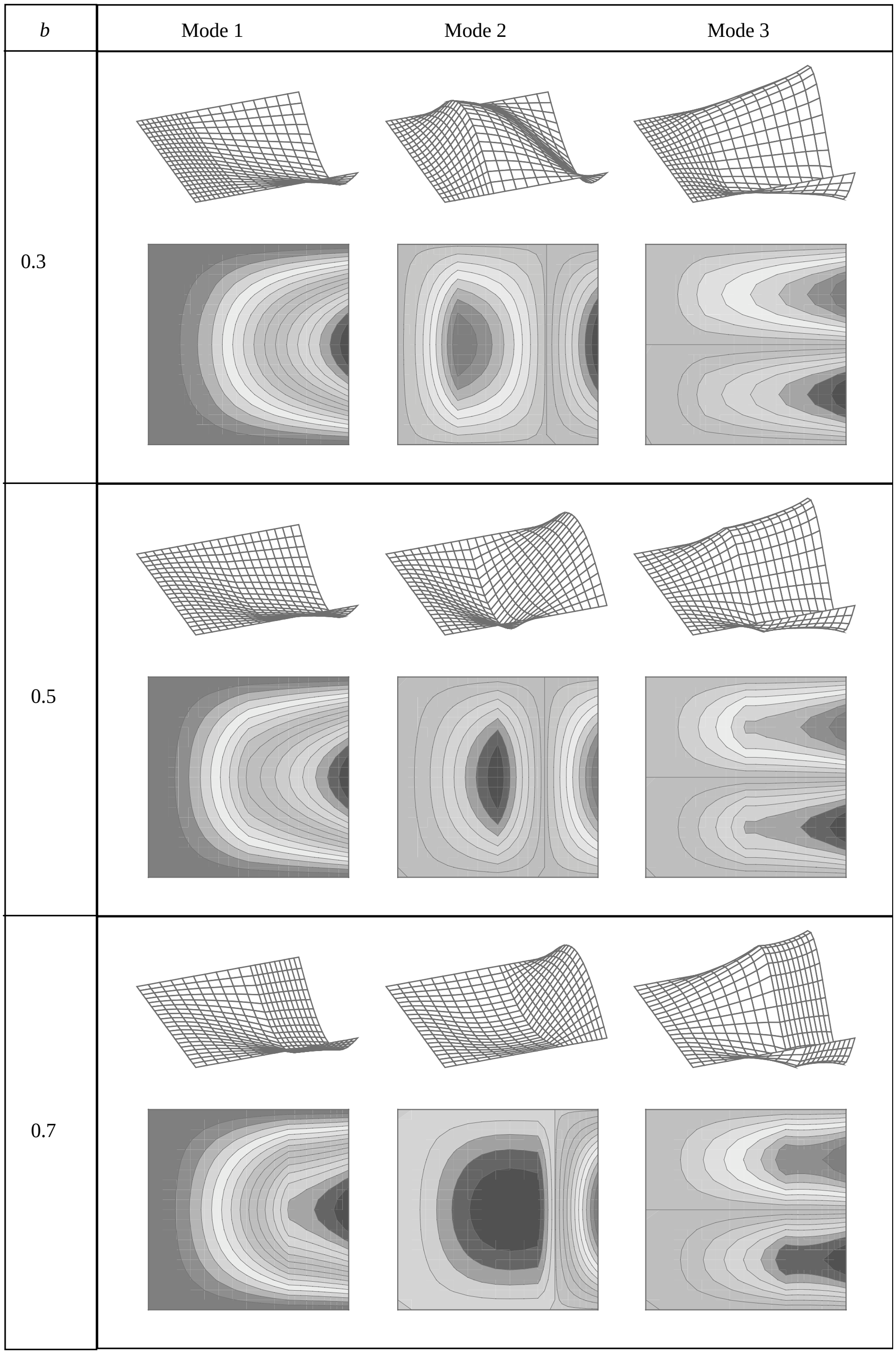

Fig. 4. The first three modal shapes for CF square Mindlin plate with an internal line hinge ($a = 1$, $h/L = 0.1$).

Table 4
Frequency parameters $\lambda = (\omega L^2/\pi^2)\sqrt{\rho h/D}$ for symmetric Lévy rectangular plates with an internal line hinge

Cases	a	h/L	b	Mode sequence 1	2	3	4	5	6	7	8	9	10
SS plate	2	0.01	1/3	1.19500	1.85048	3.24808	4.18422	4.36965	4.89344	6.24290	6.61480	7.60378	9.14930
			1/2	1.18286	1.99927	2.80691	4.16892	4.99545	4.99545	5.97187	6.25433	7.98838	9.12931
		0.1	1/3	1.16467	1.78163	3.07621	3.89448	4.03669	4.50595	5.65797	5.94863	6.71465	7.95850
			1/2	1.15234	1.93169	2.65000	3.87919	4.60836	4.60836	5.39497	5.63583	7.07165	7.94018
	3	0.01	1/3	1.08653	1.39316	1.99927	2.56196	3.53515	4.07065	4.39678	4.99545	4.99545	5.64805
			1/2	1.08062	1.44406	1.85192	2.77637	3.40896	4.06183	4.44085	4.89802	4.99545	5.77171
		0.1	1/3	1.06330	1.35533	1.93169	2.43658	3.32206	3.79922	4.08503	4.60836	4.60836	5.14166
			1/2	1.05719	1.40819	1.78179	2.64905	3.20011	3.79037	4.13026	4.50923	4.60836	5.26556
CC plate	2	0.01	1/3	1.31869	2.14871	3.88849	4.24263	5.08688	5.35235	6.69932	7.54215	8.33046	9.18658
			1/2	1.28399	2.39445	3.34267	4.21960	5.23013	5.93399	6.36860	7.31663	8.71101	9.16085
		0.1	1/3	1.27527	2.03560	3.58517	3.93628	4.63830	4.75939	5.95839	6.56329	7.15867	7.97857
			1/2	1.24111	2.26844	3.08104	3.91447	4.77263	5.28386	5.65390	6.35547	7.50842	7.95640
	3	0.01	1/3	1.12220	1.49965	2.24141	2.93821	3.92077	4.08732	4.45674	5.14312	5.57429	5.89472
			1/2	1.11130	1.57786	2.05749	3.14608	3.83891	4.07597	4.51145	5.03119	5.60568	6.01759
		0.1	1/3	1.09543	1.44878	2.14002	2.74845	3.62362	3.81129	4.12750	4.71283	5.03267	5.30990
			1/2	1.08439	1.52725	1.95790	2.95296	3.54048	3.80026	4.18165	4.60211	5.05345	5.43282
FF plate	2	0.01	1/3	0.975832	1.17408	1.71388	2.48683	3.94040	4.15569	4.21921	4.80640	5.74668	5.76605
			1/2	0.975409	1.18286	1.63090	2.80691	3.70925	3.94172	4.16892	4.72531	5.97187	6.25433
		0.1	1/3	0.957060	1.14381	1.63825	2.34181	3.68218	3.86635	3.90440	4.41390	5.18133	5.20410
			1/2	0.956534	1.15234	1.55926	2.65000	3.43073	3.68373	3.87919	4.33580	5.39497	5.63583
	3	0.01	1/3	0.982095	1.07567	1.34966	1.73677	2.49836	3.24720	3.95670	4.05432	4.13326	4.35748
			1/2	0.982252	1.08062	1.31357	1.85192	2.32053	3.40896	3.95830	4.06183	4.07900	4.31844
		0.1	1/3	0.963450	1.05216	1.30888	1.66645	2.37701	3.04078	3.69888	3.78306	3.82797	4.04219
			1/2	0.963617	1.05719	1.27270	1.78179	2.20089	3.20011	3.70088	3.77270	3.79037	4.00305

4. Conclusions

In this paper, the first known analytical solutions based on the FSDT for the natural vibration of rectangular plates with an internal line hinge are presented. Rectangular plates with two parallel edges simply supported while the other having a combination of free, simply supported and clamped boundary conditions are studied. The Lévy solution method and a domain decomposition technique are employed to investigate the vibration behavior of rectangular plates with an internal line hinge. The influence of the line hinge and its location and the effects of transverse shear deformation and rotary inertia on the vibration behavior are studied. It is noted that the optimal location of the line hinge in maximizing the frequency parameters varies from mode to mode and from plate to plate. The increase of plate thickness ratio, on the other hand, always decreases the frequency parameters in plates. The analytical solutions tabulated in the paper may serve as benchmark values for checking the validity and accuracy of numerical solutions of thick rectangular plates with internal hinges.

Table 5
Frequency parameters $\lambda = (\omega L^2/\pi^2)\sqrt{\rho h/D}$ for asymmetric Lévy rectangular plates with an internal line hinge

Cases	a	h/L	b	Mode sequence 1	2	3	4	5	6	7	8	9	10
SF plate	2	0.01	1/3	1.03546	1.38706	2.28812	3.68016	4.01218	4.40759	4.83624	5.39404	6.80445	7.54690
			1/2	1.02865	1.42204	2.29976	3.36594	3.99888	4.44505	5.38144	5.43142	6.64403	7.12273
			2/3	1.02346	1.48802	2.10816	3.56334	3.98511	4.50374	5.28076	5.44166	6.69975	6.94098
		0.1	1/3	1.01410	1.33976	2.17932	3.44557	3.74679	4.07655	4.42419	4.92106	6.09035	6.68985
			1/2	1.00708	1.37391	2.19673	3.14030	3.73312	4.11290	4.91463	4.96327	5.92862	6.33189
			2/3	1.00199	1.44142	1.99911	3.34118	3.71898	4.17434	4.80706	4.95149	6.00010	6.19447
	3	0.01	1/3	1.01156	1.18286	1.60626	2.22599	2.80691	3.98382	3.99309	4.16892	4.62798	5.26979
			1/2	1.00711	1.19863	1.60314	2.13069	3.00847	3.83066	3.98429	4.18848	4.62117	5.21964
			2/3	1.00287	1.22433	1.54695	2.17612	3.04818	3.74031	3.97454	4.21939	4.58256	5.22844
		0.1	1/3	0.99200	1.15234	1.55379	2.13393	2.65000	3.70968	3.73251	3.87919	4.28125	4.82984
			1/2	0.987328	1.16830	1.55257	2.03499	2.84913	3.56472	3.72340	3.89854	4.27664	4.77602
			2/3	0.982922	1.19548	1.49238	2.08683	2.87749	3.48623	3.71314	3.93175	4.23365	4.79150
CF plate	2	0.01	1/3	1.05190	1.48126	2.39257	4.01995	4.09942	4.45950	5.44475	5.48938	7.12527	7.73194
			1/2	1.04262	1.48280	2.58279	3.55239	4.00534	4.48584	5.55909	6.08376	6.81042	7.47265
			2/3	1.03503	1.57068	2.27719	3.98999	3.99569	4.55837	5.39895	5.68670	6.99750	7.60842
		0.1	1/3	1.02926	1.42332	2.26369	3.75264	3.78531	4.11292	4.85581	4.98172	6.30699	6.80255
			1/2	1.01973	1.42526	2.43887	3.28294	3.73778	4.14007	5.04023	5.44047	6.02892	6.55150
			2/3	1.01230	1.51131	2.14358	3.68478	3.72233	4.21241	4.88681	5.12160	6.19657	6.64202
	3	0.01	1/3	1.01623	1.21274	1.65247	2.39173	3.02660	3.99542	4.07666	4.18455	4.66134	5.37696
			1/2	1.01107	1.22121	1.70140	2.20943	3.26530	3.97410	3.98610	4.20058	4.67735	5.28270
			2/3	1.00603	1.25486	1.60859	2.33511	3.17145	3.97575	3.98934	4.23588	4.62217	5.32620
		0.1	1/3	0.996313	1.17895	1.59244	2.27821	2.82922	3.73428	3.77691	3.89028	4.30353	4.90759
			1/2	0.990909	1.18777	1.64046	2.09957	3.06249	3.67285	3.72471	3.90665	4.31816	4.81732
			2/3	0.985725	1.22238	1.54620	2.22412	2.97289	3.68575	3.71394	3.94342	4.26152	4.86109
CS plate	2	0.01	1/3	1.26378	1.95469	3.47946	4.21743	4.96628	5.11699	6.43554	6.83628	8.08408	9.17207
			1/2	1.22854	2.15904	3.10364	4.19266	5.09992	5.34149	6.17292	6.90188	8.29510	9.14443
			2/3	1.23423	2.03828	3.53454	4.20510	4.67576	5.00544	6.45505	7.35031	7.85518	9.16202
		0.1	1/3	1.22820	1.86877	3.26494	3.91945	4.55383	4.59734	5.78752	6.08310	7.02007	7.97164
			1/2	1.19285	2.06960	2.89103	3.89594	4.68265	4.86828	5.52775	6.07242	7.26270	7.94807
			2/3	1.19883	1.94535	3.30773	3.90882	4.26397	4.58615	5.80002	6.44712	6.86485	7.96479
	3	0.01	1/3	1.10624	1.43225	2.09573	2.82672	3.64324	4.08080	4.41967	5.06152	5.19464	5.79888
			1/2	1.09473	1.50166	1.95918	2.92474	3.65676	4.06861	4.47339	4.96417	5.22457	5.88249
			2/3	1.09906	1.45614	2.11029	2.68513	3.82295	4.07636	4.43157	5.06527	5.25715	5.74073
		0.1	1/3	1.08153	1.38882	2.01509	2.66139	3.40077	3.80695	4.10048	4.65555	4.75942	5.24856
			1/2	1.06979	1.45967	1.87448	2.77244	3.39717	3.79515	4.15415	4.55579	4.78114	5.34183
			2/3	1.07434	1.41200	2.02790	2.53750	3.55288	3.80309	4.11078	4.65824	4.80398	5.20295

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