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A solution of a non-homogeneous orthotropic cylindrical shell for axisymmetric plane strain dynamic thermoelastic problems

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Abstract

A solution of a non-homogeneous orthotropic elastic cylindrical shell for axisymmetric plane strain dynamic thermoelastic problems is developed. Firstly, a new dependent variable is introduced to rewrite the governing equation, the boundary conditions as well as the initial conditions. Secondly, a special function is introduced to transform the inhomogeneous boundary conditions to the homogeneous ones. Then by virtue of the orthogonal expansion technique, the equation with respect to the time variable is derived, of which the solution can be obtained. The displacement solution is finally presented, which can degenerate in a rather straightforward way to the solution for a homogeneous orthotropic cylindrical shell and isotropic solid cylinder as well as that for a non-homogeneous isotropic cylindrical shell. Using the present method, integral transform can be avoided. It is fit for a cylindrical shell with arbitrary thickness subjected to arbitrary thermal loads. It is also very convenient to deal with dynamic thermoelastic problems for different boundary conditions. Besides, the numerical calculation involved is very easy to be performed. Several examples are presented.

1. Introduction

Cylindrical shell structure is a common structure type that can be used in applications involving aerospace, submarine structures, nuclear reactors as well as chemical pipes. When the structures are exposed to a temperature field, the thermal stresses are then induced. The research for thermoelastic problems, especially for dynamic thermoelastic problems, is of increasing interest in engineering science and many works have been done. For quasi-static thermoelastic problem, Parida and Das [1] studied the transient thermal stresses in a homogeneous orthotropic thin

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circular disc due to an instantaneous point heat source. Sugano [2] solved the transient thermal stresses in a homogeneous transversely isotropic, finite cylinder due to an arbitrary internal heat generation. Kardomateas [3,4] obtained the transient thermal stresses in a homogeneous cylindrically orthotropic hollow cylinder, due to a constant temperature imposed on one surface and heat convection into a medium at the other surface. For dynamic thermoelastic problem, Ho [5] obtained the dynamic thermal stress response in a uniformly heated, homogeneous isotropic, infinite cylindrical rod. Wang [6,7] studied the dynamic thermal stresses in thermally shocked, homogeneous isotropic solid cylinders and cylindrical shells. Abd-Alla [8] solved the thermal stresses in a homogeneous, transversely isotropic, infinite cylindrical shell subjected to an instantaneous heat source. Cho et al. [9] obtained the thermal stresses in a thermally shocked, homogeneous orthotropic cylindrical shell.

In recent years, many new type materials have been used in engineering and there are a lot of works have been done for non-homogeneous materials. Among them, the special case that Young's modulus has a power-law dependence on the radial co-ordinate, while the linear thermal expansion coefficient and the Poisson ratio are constant, have been considered by many scientists and engineers. For instance, Shaffer [10] has obtained the general solutions for a non-homogeneous orthotropic annular disk in-plane stress subjected to uniform pressures at the internal and external surfaces. Horgan and Chan [11,12] investigated the pressured FGM hollow cylinder and disk problems and the stress response of FGM isotropic linear elastic rotating disks recently. The rotation problem of a non-homogeneous orthotropic composite cylinder was considered by El-Naggar et al. [13]. Abd-Alla et al. [14,15] studied the transient thermal stresses in a rotating non-homogeneous cylindrically orthotropic composite tube and in a non-homogeneous spherically orthotropic elastic medium with spherical cavity, respectively. Tarn [16] obtained the exact solutions of functionally graded anisotropic cylinders subjected to thermal and mechanical loads for steady-state problem. Sarma [17] investigated the torsional oscillations of a finite non-homogeneous piezoelectric cylindrical shell, in which the analytical solution is only suitable for class 622 crystals, not for class 6 mm crystals that are usually met. In the above studies, the variation of material density is often assumed to be the same as that of Young's modulus ([13–17]). The non-homogeneous material has gained much attention because of its good heat-shielding character as well as other significant superiorities. While the study for dynamic thermoelastic problems for a special non-homogeneous orthotropic elastic cylindrical shell has yet not been reported.

The dynamic thermoelastic problems are usually solved using Laplace transform technique that the difficulty of inverse transform will be encountered in certain cases. In this paper, a theoretical solution of a non-homogeneous orthotropic cylindrical shell is developed for the axisymmetric plane strain dynamic thermoelastic problem. Firstly, a new dependent variable is introduced to rewrite the governing equation, the boundary conditions as well as the initial conditions. Secondly, the thermal load is treated as the inhomogeneous item in the boundary conditions and a special function is introduced to transform the inhomogeneous boundary conditions to the homogeneous ones. Thirdly, by using the orthogonal expansion technique, the equation with respect to the time variable is derived, of which the solution is easily obtained. The present method can avoid integral transform and is fit for an arbitrary thick-walled cylindrical shell subjected to general thermal loads. Several special cases are discussed and numerical examples are finally presented.

2. Mathematical formulations of the problem

In a cylindrical co-ordinate system (r, θ, z), for the axially symmetric problem, we have $u_\theta = 0$, $u_r = u_r(r, z, t)$ and $u_z = u_z(r, z, t)$. Furthermore, if only axisymmetric plane strain problem is considered, we have $u_\theta = u_z = 0$ and $u_r = u_r(r, t)$. The strain–displacement relations are

$$\gamma_{rr} = \frac{\partial u_r}{\partial r}, \quad \gamma_{\theta\theta} = \frac{u_r}{r}, \quad \gamma_{zz} = \gamma_{zr} = \gamma_{r\theta} = \gamma_{\theta z} = 0, \tag{1}$$

where u_i and γ_{ij} are the displacement components and strain components, respectively. The stress–strain relations are

$$\begin{aligned} \sigma_{rr} &= c_{11}\gamma_{rr} + c_{12}\gamma_{\theta\theta} - \beta_1 T(r, t), \\ \sigma_{\theta\theta} &= c_{12}\gamma_{rr} + c_{22}\gamma_{\theta\theta} - \beta_2 T(r, t), \\ \sigma_{zz} &= c_{13}\gamma_{rr} + c_{23}\gamma_{\theta\theta} - \beta_3 T(r, t), \end{aligned} \tag{2}$$

where $\sigma_{ij}, c_{ij}, \beta_i$ and $T(r, t)$ are the stress components, elastic constants, stress–temperature constants and the reference temperature, respectively. The stress–temperature constants can be expressed in terms of elastic constants c_{ij} and coefficients of linear thermal expansion α_i as follows:

$$\begin{aligned} &\beta_1 = c_{11}\alpha_r + c_{12}\alpha_\theta + c_{13}\alpha_z, \quad \beta_2 = c_{12}\alpha_r + c_{22}\alpha_\theta + c_{23}\alpha_z, \\ &\beta_3 = c_{13}\alpha_r + c_{23}\alpha_\theta + c_{33}\alpha_z. \end{aligned} \tag{3}$$

The equation of motion is

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \frac{\partial^2 u_r}{\partial t^2}, \tag{4}$$

where ρ is the mass density.

In this study, we assume the non-homogeneous property of the material is characterized by

$$c_{ij} = (r/b)^{2N} A_{ij}, \quad \rho = (r/b)^{2N} \rho_0, \tag{5}$$

where b, A_{ij}, ρ_0 and N are known constants, while the coefficients of linear thermal expansion α_i are constant. Substituting the first equation in Eq. (5), Eqs. (1) and (3) into Eq. (2), we obtain

$$\begin{aligned} \sigma_{rr} &= (r/b)^{2N} \left[A_{11} \frac{\partial u_r}{\partial r} + A_{12} \frac{u_r}{r} - B_1 T(r, t) \right], \\ \sigma_{\theta\theta} &= (r/b)^{2N} \left[A_{12} \frac{\partial u_r}{\partial r} + A_{22} \frac{u_r}{r} - B_2 T(r, t) \right], \\ \sigma_{zz} &= (r/b)^{2N} \left[A_{13} \frac{\partial u_r}{\partial r} + A_{23} \frac{u_r}{r} - B_3 T(r, t) \right], \end{aligned} \tag{6}$$

where

$$\begin{aligned} &B_1 = A_{11}\alpha_r + A_{12}\alpha_\theta + A_{13}\alpha_z, \quad B_2 = A_{12}\alpha_r + A_{22}\alpha_\theta + A_{23}\alpha_z, \\ &B_3 = A_{13}\alpha_r + A_{23}\alpha_\theta + A_{33}\alpha_{.z}. \end{aligned} \tag{7}$$

Substituting the second equation in Eqs. (5) and (6) into Eq. (4), yields the following governing equation:

$$\frac{\partial^2 u_r}{\partial r^2}+\frac{2N+1}{r}\frac{\partial u_r}{\partial r}-\frac{\mu_1^2}{r^2}u_r=\frac{1}{c_L^2}\frac{\partial^2 u_r}{\partial t^2}+g(r,t), \tag{8}$$

where

$$\mu_1^2=\frac{A_{22}-2NA_{12}}{A_{11}},\quad c_L=\sqrt{\frac{A_{11}}{\rho_0}},$$
$$g(r,t)=\frac{(2N+1)B_1-B_2}{A_{11}}\frac{T(r,t)}{r}+\frac{B_1}{A_{11}}\frac{\partial T(r,t)}{\partial r}. \tag{9}$$

The boundary conditions are

$$r=a\ \text{and}\ b:\quad A_{11}\frac{\partial u_r}{\partial r}+2A_{12}\frac{u_r}{r}-B_1T(r,t)=0, \tag{10}$$

where a and b are the inner and outer radii of the cylindrical shell, respectively. The initial conditions at $t=0$ are

$$u_r(r,0)=u_0(r),\quad \dot{u}_r(r,0)=v_0(r), \tag{11}$$

where a dot over the letter denotes its partial derivative with respect to t, and $u_0(r)$ and $v_0(r)$ are known functions.

3. Solving method and the theoretical solution

Firstly, a new dependent variable $w(r,t)$ is introduced as

$$u_r(r,t)=r^{-N}w(r,t). \tag{12}$$

Then Eqs. (8), (10) and (11) become

$$\frac{\partial^2 w(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial w(r,t)}{\partial r}-\frac{\mu^2}{r^2}w(r,t)=\frac{1}{c_L^2}\frac{\partial^2 w(r,t)}{\partial t^2}+g_1(r,t), \tag{13}$$

$$r=a:\quad \frac{\partial w(r,t)}{\partial r}+h\frac{w(r,t)}{r}=p_a(t), \tag{14a}$$

$$r=b:\quad \frac{\partial w(r,t)}{\partial r}+h\frac{w(r,t)}{r}=p_b(t), \tag{14b}$$

$$w(r,0)=u_1(r),\quad \dot{w}(r,0)=v_1(r), \tag{15}$$

where

$$\mu=\sqrt{\mu_1^2+N^2},\quad g_1(r,t)=r^Ng(r,t),\quad h=A_{12}/A_{11}-N,$$
$$p_a(t)=a^NB_1T(a,t)/A_{11},\quad p_b(t)=b^NB_1T(b,t)/A_{11},$$
$$u_1(r)=r^Nu_0(r),\quad v_1(r)=r^Nv_0(r). \tag{16}$$

Secondly, we transform the inhomogeneous boundary conditions into the homogeneous ones by assuming

$$w(r,t) = w_1(r,t) + w_2(r,t), \tag{17}$$

where $w_2(r,t)$ satisfies the inhomogeneous boundary conditions, and can be taken as

$$w_2(r,t) = d_1(r-a)^m p_b(t) + d_2(r-b)^m p_a(t), \tag{18}$$

where

$$d_1 = \frac{b^{1-m}}{m(1-s)^{m-1} + h(1-s)^m}, \quad d_2 = \frac{b^{1-m}}{m(s-1)^{m-1} + h(s-1)^m/s}, \tag{19}$$

in which $m \geqslant 2$ is an arbitrary integer that should satisfy

$$[m(1-s)^{m-1} + h(1-s)^m][m(s-1)^{m-1} + h(s-1)^m/s] \neq 0. \tag{20}$$

Substituting Eq. (17) into Eqs. (13)–(15) gives

$$\frac{\partial^2 w_1(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial w_1(r,t)}{\partial r} - \frac{\mu^2}{r^2} w_1(r,t) = \frac{1}{c_L^2}\frac{\partial^2 w_1(r,t)}{\partial t^2} + g_2(r,t), \tag{21}$$

$$r = a \text{ and } b: \quad \frac{\partial w_1(r,t)}{\partial r} + h\frac{w_1(r,t)}{r} = 0, \tag{22}$$

$$w_1(r,0) = u_2(r), \quad \dot{w}_1(r,0) = v_2(r), \tag{23}$$

where

$$g_2(r,t) = g_1(r,t) + \frac{1}{c_L^2}\frac{\partial^2 w_2(r,t)}{\partial t^2} + \frac{\mu^2}{r^2} w_2(r,t) - \frac{1}{r}\frac{\partial w_2(r,t)}{\partial r} - \frac{\partial^2 w_2(r,t)}{\partial r^2},$$
$$u_2(r) = u_1(r) - w_2(r,0), v_2(r) = v_1(r) - \dot{w}_2(r,0). \tag{24}$$

By using the trial-and-error method, the solution of Eq. (21) can be assumed in the following form:

$$w_1(r,t) = \sum_i R_i(r) F_i(t), \tag{25}$$

where $F_i(t)$ are unknown functions about t, and $R_i(r)$ are given by

$$R_i(r) = \mathrm{J}_\mu(k_i r)\mathrm{Y}(\mu, k_i, a) - \mathrm{Y}_\mu(k_i r)\mathrm{J}(\mu, k_i, a), \tag{26}$$

in which $\mathrm{J}_\mu(k_i r)$ and $\mathrm{Y}_\mu(k_i r)$ are Bessel functions of the first and second kinds, respectively, and k_i, arranged in an ascending order, are a series of positive roots of the following eigenequation:

$$\mathrm{J}(\mu, k_i, a)\mathrm{Y}(\mu, k_i, b) - \mathrm{J}(\mu, k_i, b)\mathrm{Y}(\mu, k_i, a) = 0, \tag{27}$$

where

$$\mathrm{J}(\mu, k_i, r) = \frac{\mathrm{dJ}_\mu(k_i r)}{\mathrm{d}r} + h\frac{\mathrm{J}_\mu(k_i r)}{r}, \quad \mathrm{Y}(\mu, k_i, r) = \frac{\mathrm{dY}_\mu(k_i r)}{\mathrm{d}r} + h\frac{\mathrm{Y}_\mu(k_i r)}{r}. \tag{28}$$

It can be shown that $w_1(r,t)$ given in Eq. (25) satisfies the homogeneous boundary conditions in Eq. (22).

Substituting Eq. (25) into Eq. (21), gives

$$-c_L^2 \sum_i k_i^2 F_i(t) R_i(r) = \sum_i R_i(r) \frac{\mathrm{d}^2 F_i(t)}{\mathrm{d}t^2} + c_L^2 g_2(r,t). \tag{29}$$

By virtue of the orthogonal property of Bessel functions, it is easy to verify the following equation:

$$\int_a^b r R_i(r) R_j(r)\,\mathrm{d}r = N_i \delta_{ij}, \tag{30}$$

where δ_{ij} is the Kronecker delta, and

$$N_i = \left\{ b^2 \left[\frac{\mathrm{d}R_i(b)}{\mathrm{d}r}\right]^2 - a^2 \left[\frac{\mathrm{d}R_i(a)}{\mathrm{d}r}\right]^2 + k_i^2 [b^2 R_i^2(b) - a^2 R_i^2(a)] - \mu^2 [R_i^2(b) - R_i^2(a)] \right\} \Big/ 2k_i^2. \tag{31}$$

In the above equation, we denote $\mathrm{d}R_i(a)/\mathrm{d}r = \mathrm{d}R_i(r)/\mathrm{d}r\big|_{r=a}$ and $\mathrm{d}R_i(b)/\mathrm{d}r = \mathrm{d}R_i(r)/\mathrm{d}r\big|_{r=b}$. Utilizing Eq. (30), we can derive the following equation from Eq. (29)

$$\frac{\mathrm{d}^2 F_i(t)}{\mathrm{d}t^2} + \omega_i^2 F_i(t) = q_i(t), \tag{32}$$

where

$$\omega_i = k_i c_L, \quad q_i(t) = -\frac{c_L^2}{N_i} \int_a^b r g_2(r,t) R_i(r)\,\mathrm{d}r. \tag{33}$$

The solution of Eq. (32) is

$$F_i(t) = G_{1i} \cos \omega_i t + \frac{G_{2i}}{\omega_i} \sin \omega_i t + \frac{1}{\omega_i} \int_0^t q_i(\tau) \sin \omega_i (t-\tau)\,\mathrm{d}\tau, \tag{34}$$

where

$$G_{1i} = \frac{1}{N_i} \int_a^b r u_2(r) R_i(r)\,\mathrm{d}r, \quad G_{2i} = \frac{1}{N_i} \int_a^b r v_2(r) R_i(r)\,\mathrm{d}r. \tag{35}$$

Finally, the displacement solution can be obtained as follows:

$$u_r(r,t) = r^{-N} [w_1(r,t) + w_2(r,t)]. \tag{36}$$

4. Some particular cases

4.1. Isotropic material

If

$$\begin{gathered} A_{11} = A_{22} = A_{33} = E(1-\upsilon)/k, \quad A_{12} = A_{13} = A_{23} = E\upsilon/k, \\ k = (1+\upsilon)(1-2\upsilon), \quad \alpha_r = \alpha_\theta = \alpha_z = \alpha, \end{gathered} \tag{37}$$

where E and υ are Young's modulus and the Poisson ratio, respectively, the solution obtained above degenerates to that of a non-homogeneous isotropic cylindrical shell for the dynamic thermoelastic problem.

4.2. Homogeneous material

If $N = 0$, the solution degenerates to that of a homogeneous orthotropic cylindrical shell for the dynamic thermoelastic problem. Further, if the material constants satisfy Eq. (37), the solution becomes that of a homogeneous isotropic cylindrical shell for the dynamic thermoelastic problem.

4.3. Fixed boundary conditions

For a cylindrical shell fixed at the internal surface, the boundary condition at $r = a$ becomes

$$u_r(a,t) = 0. \tag{10a}$$

Consequently, instead of Eqs. (14a) and (22), we have

$$w(a,t) = 0, \tag{14a$'$}$$

$$w_1(a,t) = 0. \tag{22a}$$

Then, we can just set $p_a(t) = 0$, $\mathrm{J}(\mu, k_i, a) = \mathrm{J}_\mu(k_i a)$ and $\mathrm{Y}(\mu, k_i, a) = \mathrm{Y}_\mu(k_i a)$ in relevant formulations to obtain the solution of the dynamic thermoelastic problem for a non-homogeneous orthotropic cylindrical shell with fixed internal surface.

4.4. Homogeneous isotropic solid cylinder

For a homogeneous isotropic solid cylinder, we have $N = 0$, $a = 0$, and the material constants satisfy Eq. (37). We can just set $p_a(t) = 0$, $\mathrm{J}(\mu, k_i, a) = 0$ and $\mathrm{Y}(\mu, k_i, a) = 1$ in relevant formulations to obtain the solution of the dynamic thermoelastic problem for a homogeneous isotropic solid cylinder.

5. Numerical results and discussions

Both isotropic and orthotropic materials are considered. For the orthotropic cylindrical shell, we take $a = 50\,\mathrm{mm}$, $b = 100\,\mathrm{mm}$, $A_{11} = 17.075\,\mathrm{GPa}$, $A_{12} = 6.757\,\mathrm{GPa}$, $A_{13} = 7.289\,\mathrm{GPa}$, $A_{22} = 59.645\,\mathrm{GPa}$, $A_{23} = 6.752\,\mathrm{GPa}$, $A_{33} = 17.074\,\mathrm{GPa}$, $\alpha_r = 4.0 \times 10^{-5}/^\circ\mathrm{C}$, $\alpha_\theta = 1.0 \times 10^{-5}/^\circ\mathrm{C}$, $\alpha_z = 4.0 \times 10^{-5}/^\circ\mathrm{C}$, and for the isotropic cylindrical shell (solid cylinder) with the same size ($a = 0$ for cylinder), we take $E = 55.9\,\mathrm{GPa}$, $\upsilon = 0.277$, $\alpha = 1.0 \times 10^{-5}/^\circ\mathrm{C}$. In the result, the time and co-ordinate are normalized as follows:

$$\xi = \frac{r-a}{b-a}, \quad t^* = \frac{c_L}{b-a}t. \tag{38}$$

In the following calculation, we take $T_0 = 200^\circ\mathrm{C}$, $u_0(r) = 0$, $v_0(r) = 0$. Here T_0 is a reference temperature. Also, $H(t)$ denotes the Heaviside step function.

Example 1. Thermally shocked homogeneous isotropic cylindrical shell: $T(r,t) = T_0 H(t)$.

Figs. 1 and 2 give the dynamic stress responses of the homogeneous isotropic cylindrical shell due to a uniform temperature. The same problem has been solved by Cho et al. [9] in a different way, and Fig. 3 is the copy of Fig. 4b for isotropic case in Ref. [9]. For the sake of comparing with those obtained by Cho et al. [9], the stresses are not given in a dimensionless form in the two figures. It is found that Fig. 1 agree well with Fig. 3 (the amplitude has a little difference). Thus, the validation of the method developed in this paper is verified.

From Fig. 1, We find that the peaks appear periodically. Such phenomenon has been explained in Ref. [9]. Since the thermal loading is applied to the entire hollow cylinder instantaneously, the stress disturbance is simultaneously generated all over the wall. In the vicinity

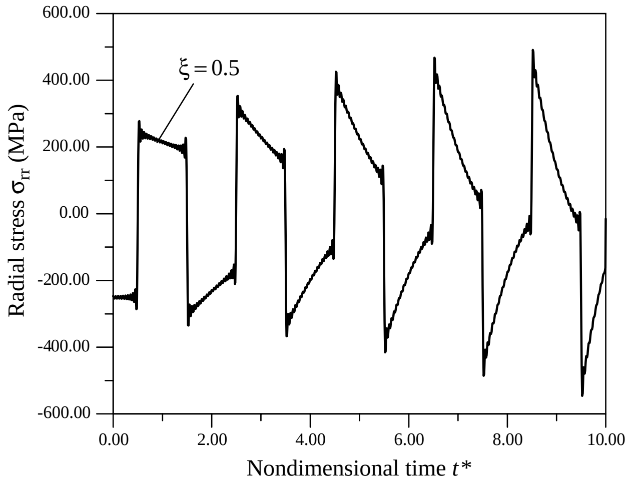

Fig. 1. History of dynamic stress σ_{rr}.

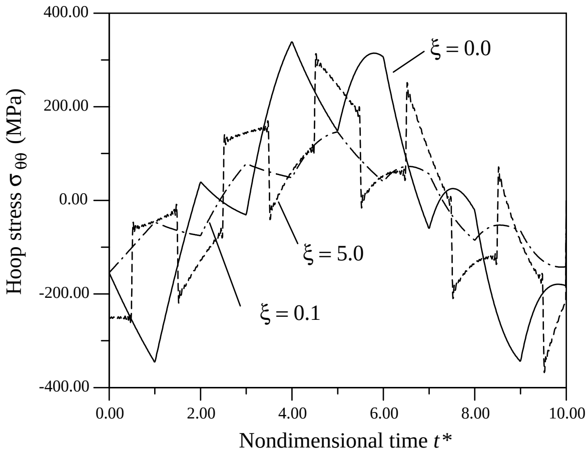

Fig. 2. History of dynamic stress $\sigma_{\theta\theta}$.

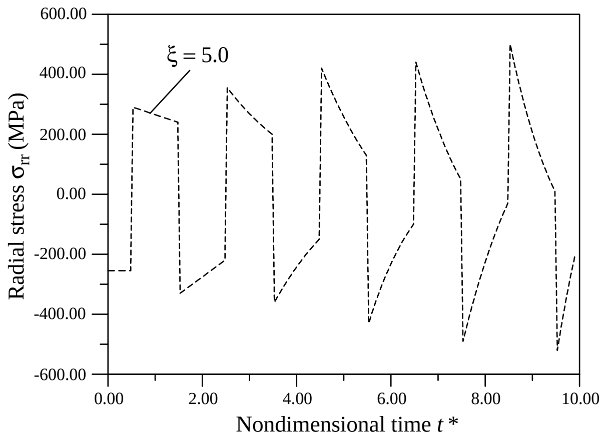

Fig. 3. History of dynamic stress σ_{rr} [9].

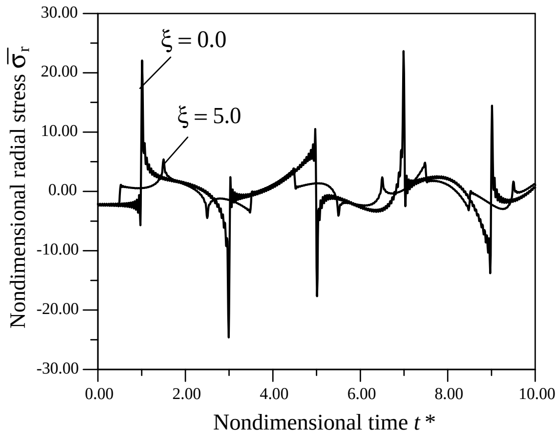

Fig. 4. History of non-dimensional dynamic stress $\bar{\sigma}_r$.

of the boundary, the radial stress feels the strong discontinuity (wave front) right after the loading. Each wave front moves inward and the waves meet at the center location and cause a stress reversion and sudden jump in magnitude. After that, the wave fronts proceed to the boundaries continuously. When the wave fronts reach the boundaries, they are reflected into to opposite direction [9].

Example 2. Thermally shocked homogeneous isotropic solid cylinder: $u_r(0, t) = 0$, $T(r, t) = T_0 H(t)$.

Figs. 4 and 5 depict the non-dimensional dynamic stress responses at $\xi = 0.0$ and 0.5 in the homogeneous isotropic solid cylinder due to a uniform temperature rise. The dimensionless

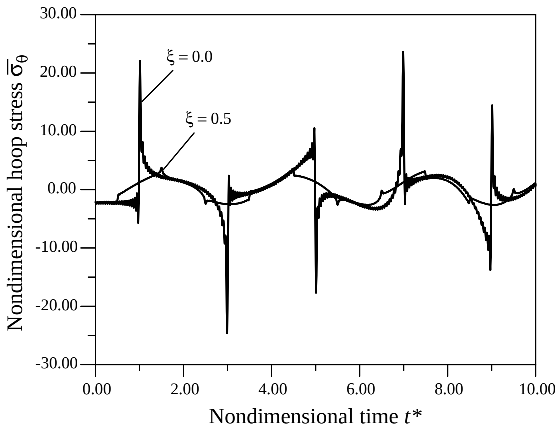

Fig. 5. History of non-dimensional dynamic stress $\bar{\sigma}_\theta$.

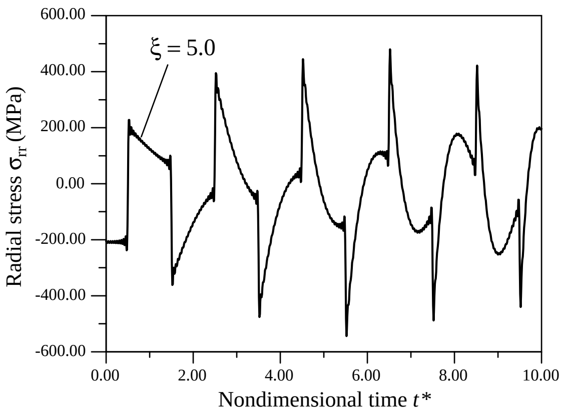

Fig. 6. History of dynamic stress σ_{rr}.

stresses are defined as

$$\bar{\sigma}_i = \frac{\sigma_{ii}}{\sigma_0} \quad (i = r, \theta, z), \qquad \sigma_0 = \alpha E T_0. \tag{39}$$

The dynamic stress concentration phenomenon occurs at the center. The first dynamic tensile stress peak values of $\bar{\sigma}_r$ and $\bar{\sigma}_\theta$ at $\xi = 0.0$ are 4.1 and 5.9 times, respectively, as large as those at $\xi = 0.5$. The agreement of Figs. 4 and 5 somewhat proofs that the result are correct.

Example 3. Thermally shocked homogeneous orthotropic cylindrical shell: $T(r, t) = T_0 H(t)$.

Figs. 6 and 7 give the dynamic stress responses of the homogeneous orthotropic cylindrical shell due to a uniform temperature rise. The same problem has also been solved by Cho [9] in a different way, and Figs. 8 and 9 are the copies of Fig. 4b for orthotropic case and Fig. 5 in Ref. [9],

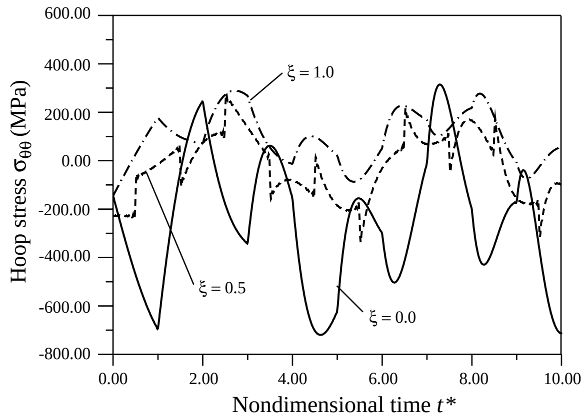

Fig. 7. History of dynamic stress $\sigma_{\theta\theta}$.

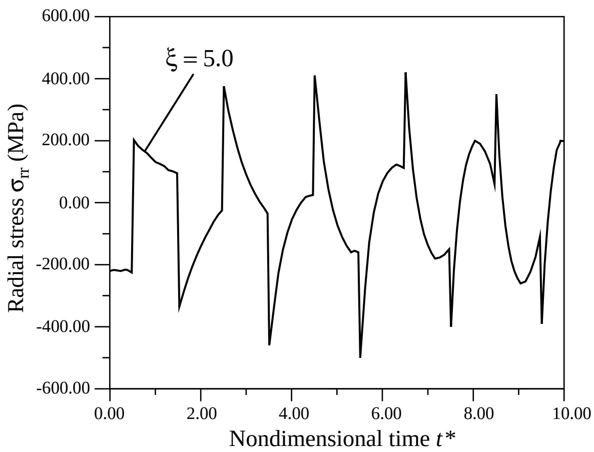

Fig. 8. History of dynamic stress σ_{rr} [9].

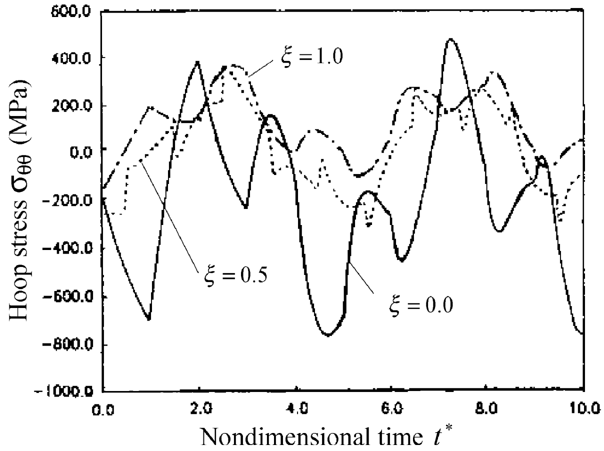

Fig. 9. History of dynamic stress $\sigma_{\theta\theta}$ [9].

respectively. It is found that our results agree well with those obtained in Ref. [9] (the amplitude has a little difference), see Figs. 8 and 9 in this paper. Thus, the validation of the method developed in this paper is further verified.

Example 4. Thermally shocked non-homogeneous orthotropic cylindrical shell: $T(r,t) = T_0H(t)$.

In order to compare the results with that of a homogeneous orthotropic cylindrical shell, the dynamic stress are not normalized in this example. Fig. 10 shows the radial stress responses at $\xi = 0.5$ in the thermally shocked non-homogeneous orthotropic cylindrical shell for different values of N. We can see that at $t^* = 0.5, 1.5, 2.5\ldots$, the radial dynamic thermal stress possesses a strong discontinuity and the peak values of the radial dynamic stress decrease slightly when N increases. Figs. 11–14 give the dynamic stress responses at the internal and external surfaces in the thermally shocked non-homogeneous orthotropic cylindrical shell for different values of N. We

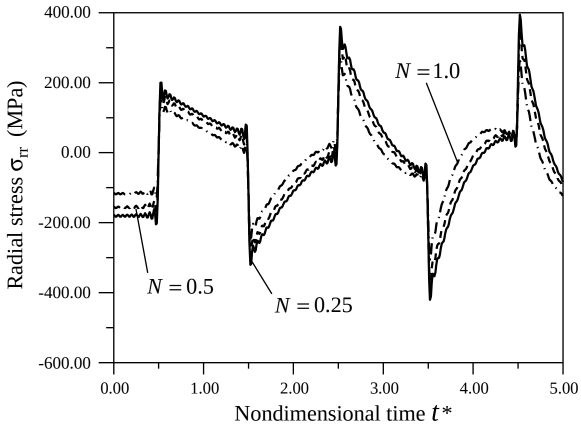

Fig. 10. History of dynamic stress σ_{rr} at $\xi = 0.5$ for different N's.

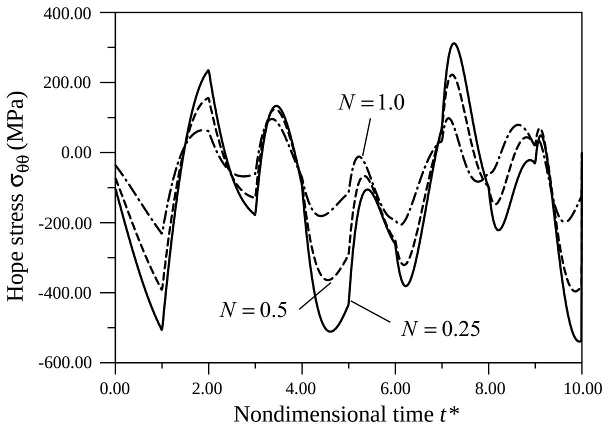

Fig. 11. History of dynamic stress $\sigma_{\theta\theta}$ at the inner surface for different N's.

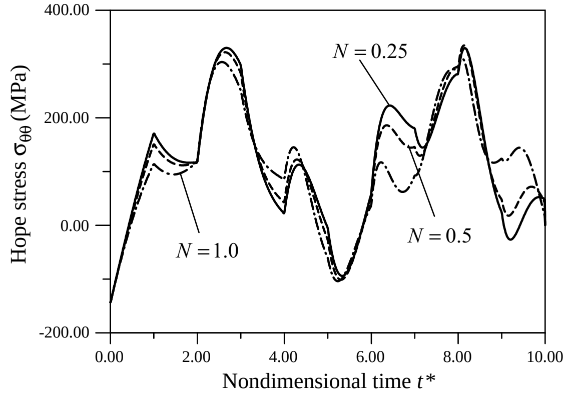

Fig. 12. History of dynamic stress $\sigma_{\theta\theta}$ at the outer surface for different N's.

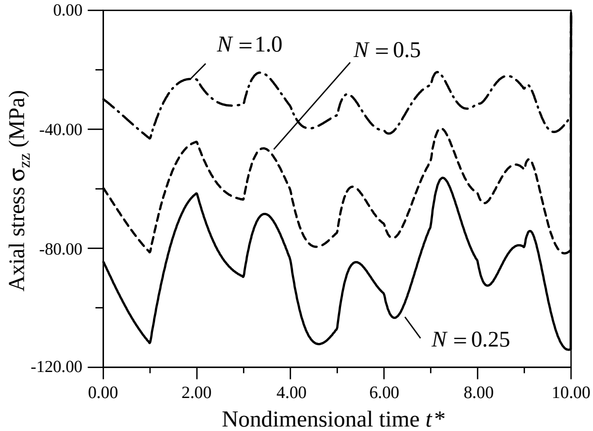

Fig. 13. History of dynamic stress σ_{zz} at the inner surface for different N's.

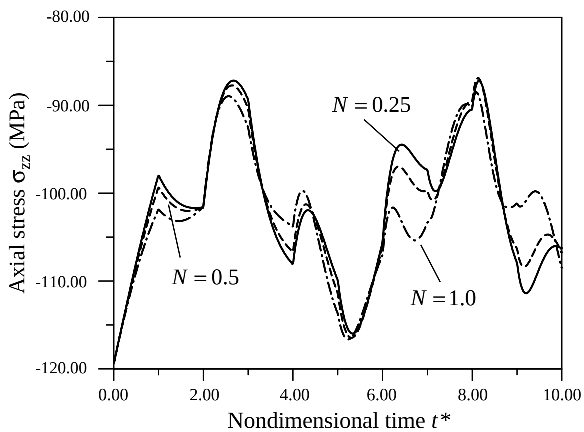

Fig. 14. History of dynamic stress σ_{zz} at the outer surface for different N's.

can see that the peak values of dynamic stresses at the internal surface decrease with the increase of N, while the peak values of dynamic stresses at the external surface vary slightly with N except at some particular moments, such as $t^* = 6.5, 9.5$, etc. The results also show that the axial stress is not significant in comparison with the radial and hoop stresses.

For $N = 0.5$, the distributions of the radial and hoop stresses in a thermally shocked non-homogeneous orthotropic cylindrical shell at different times are presented in Figs. 15 and 16. From the curves, we can clearly see the propagation phenomenon of the thermal stress wave in the uniformly heated cylindrical shell at the initial phase. The thermal stress waves are induced when a cylindrical shell subjected to a uniform temperature rise and propagate inward and outward, respectively. When they arrive at the internal surface and the external surface, they are reflected in

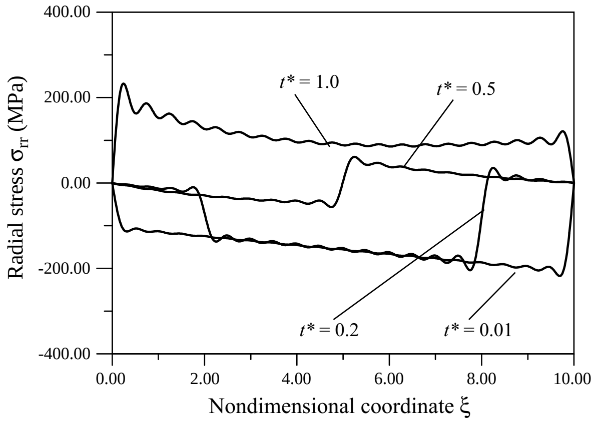

Fig. 15. Distribution of dynamic stress σ_{rr} at different time ($N = 0.5$).

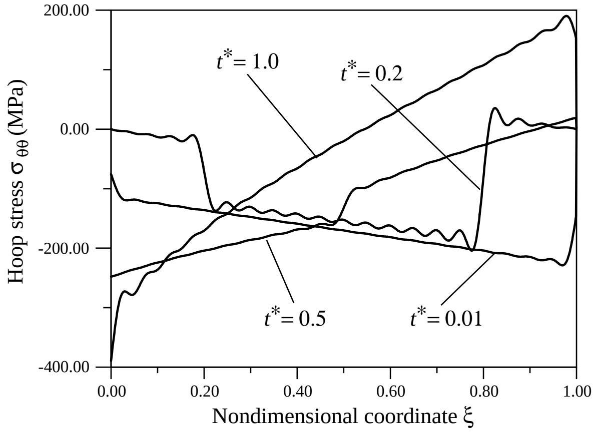

Fig. 16. Distribution of dynamic stress $\sigma_{\theta\theta}$ at different time ($N = 0.5$).

the opposite direction. It thus causes the dynamic stresses peaking periodically in a thermally shocked cylindrical shell.

Acknowledgements

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