ACADEMIC PRESS

Available online at www.sciencedirect.com

SCIENCE DIRECT

Journal of Sound and Vibration 263 (2003) 1005–1016

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Layerwise optimization for the maximum fundamental frequency of laminated composite plates

Y. Narita*

Department of Mechanical Systems Engineering, Hokkaido Institute of Technology, 7-15 Maeda, Teineku, Sapporo 006-8585, Japan

Accepted 4 February 2003

Abstract

A new concept of a layerwise optimization approach (LOA) is proposed to optimize vibration behavior for the maximum natural frequency of laminated composite plates. Design variables are taken to be the fiber orientation angles in all N layers. This usually causes a rapid increase in computation time due to the search for optimum solutions in the N-dimensional space. The LOA makes possible this multi-dimensional optimization into only N times repetition in a one-dimensional search. The idea is based on the physical consideration that the outer layer has more stiffening effect than the inner layer in the bending of plates and is more influential in determining the natural frequency. In numerical examples, a Ritz method is employed to calculate the natural frequencies of laminated rectangular plates under any combination of the three classical edge conditions. Results are corroborated by comparing with other optimum solutions available in the literature.

1. Introduction

Laminated fibrous composite plates are finding a wide range of applications in structural design, especially for light-weight structures that have tight stiffness and strength requirements. They are attractive replacements for conventional metal plates but their analysis and design are more complex than isotropic metal plates due to material anisotropy. Furthermore, the light-weight structures are often exposed to severe vibration circumstances and the consideration for

*Fax: +81-11-681-3622.
E-mail address: narita@hit.ac.jp (Y. Narita).

doi:10.1016/S0022-460X(03)00270-0

optimizing anti-resonance performance (e.g., by maximizing the natural frequency) becomes more important than before in composite structural design.

The literature survey shows that the free vibration of flat plates has been extensively studied in the past, and various textbooks and monographs have appeared on vibration of isotropic plates [1–3] and on composite plates [4,5]. However, previous references on the optimization of vibrating composite plates are quite limited [6–9] and deal with only certain stacking and boundary conditions. The reason for the limited references is that the problem involves two complicated aspects of the plate vibration analysis and the optimization technique used. Particularly, the computational difficulty consists in that, when one relates a fiber orientation angle of each layer directly to a corresponding design variable in the N-layered plate, one has N design variables and must look for optimum solutions in the N-dimensional space. This requires a larger amount of computation time as the plate has an increased number of layers. Although there is a different approach to introducing a set of the lamination parameters [10] as intermediate parameters to avoid an increase of solution search time, this requires another complicated mathematical process, or alternatively a graphical technique in a simple case, in order to determine specific fiber orientation angles from the parameters.

A layerwise optimization approach (LOA) presented here is approximate and simple, yet quite an effective, optimization approach. This is based on the physical observation that the outer layer has more stiffening effect than the inner layer in bending of laminated plates, and therefore the outer layer is considered to be a more influential factor in determining the maximum natural frequency of the plate. Then the assumption for LOA is introduced as "The optimal stacking sequence for the maximum natural frequency of laminated plate can be determined sequentially in the order from the outermost to the innermost layer".

On the other hand, it is known that natural frequencies of flat plates are significantly affected by edge constraints, which are modelled typically by one of the boundary conditions of free, simply supported and clamped edges. There are a wide variety of combinations along the entire boundary of a rectangle when an edge constraint is independently assumed along each of the four edges [11]. Since LOA is independent of methods for calculating natural frequencies, one can use any method including the finite element method. In the present numerical examples, a modified Ritz method [12,13] is employed to calculate the natural frequencies of laminated rectangular plates under various combinations of the three boundary conditions. Accuracy of the present Ritz-based approach in calculating frequencies has already been established [14]. The present optimum solutions are corroborated by comparing with reference frequencies for typical stacking sequences and other optimum solutions obtained by the complex method [15].

2. Definition of the problem

Fig. 1 shows a laminated rectangular plate in the co-ordinate system o-xy and in each layer the major and minor principal material axes are denoted by the L and T axes. The dimension of the whole plate is given by $a \times b \times h$ (thickness). The plate considered is limited to symmetric laminates, and the total number of layers is redefined as $2N$ (i.e., N layers in the upper (lower) half cross-section). Free vibration of such symmetrically laminated thin plates is governed in the

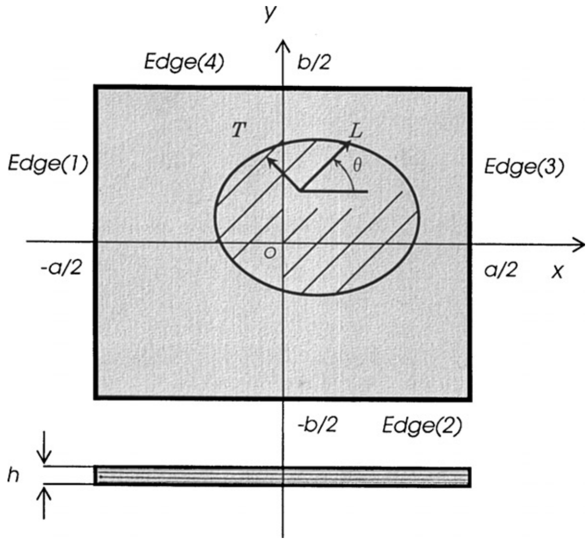

Fig. 1. Laminated composite rectangular plate and co-ordinate system.

classical plate theory by

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}+\rho\frac{\partial^2 w}{\partial t^2}=0, \tag{1}$$

where w is the deflection and ρ is a mean mass per unit area of the plate. The D_{ij} $(i,j=1,2,6)$ are the bending rigidities of the symmetric laminate defined by

$$D_{ij}=\frac{2}{3}\sum_{k=1}^{N}\bar{Q}_{ij}^{(k)}(z_k^3-z_{k-1}^3) \tag{2}$$

with z_k being a thickness co-ordinate measured from the middle surface. The $\bar{Q}_{ij}^{(k)}$ are elastic constants in the kth layer, obtained from

$$Q_{11}=\frac{E_L}{(1-v_{LT}v_{TL})},\quad Q_{12}=\frac{E_L v_{TL}}{(1-v_{LT}v_{TL})},\quad Q_{22}=\frac{E_T}{(1-v_{LT}v_{TL})},\quad Q_{16}=G_{LT} \tag{3}$$

(superscript (k) is omitted) by considering a fiber orientation angle $\theta^{(k)}$ in the layer [4,5]. The E_L and E_T are moduli of longitudinal elasticity in the L and T directions, respectively, G_{LT} is a shear modulus and v_{LT} is the Poisson ratio.

Natural frequency is normalized as a frequency parameter

$$\Omega=\omega a^2\left(\frac{\rho}{D_0}\right)^{1/2}, \tag{4}$$

where ω is a radian frequency of vibration and $D_0=E_T h^3/12(1-v_{LT}v_{TL})$ is a reference bending rigidity. The frequency parameter for the lowest (fundamental) mode is used as the objective function and is maximized in the present optimization. The design variables are taken to be a set of fiber orientation angles in the N layers of the upper (lower) half of the cross-section:

$$[\theta^{(1)}/\theta^{(2)}/\ldots/\theta^{(k)}/\ldots/\theta^{(N)}]_s, \tag{5}$$

where $\theta^{(k)}$ is a fiber orientation angle in the kth layer ($k=1$: outermost, $k=N$: innermost) and the subscript "s" denotes symmetric lamination. This approach of using each fiber orientation angle directly as a design variable is straightforward but the number of design variables increases in proportion to the number of layers, resulting in the optimization of the multi-dimensional search problem.

The present LOA attempts to avoid this intensive computational problem by making use of the following physical observation:

> In bending of plates, the outer layer has more stiffening effect than the inner layer and is more influential on the natural frequency

This well-known physical fact suggests that the outer layer plays a more decisive role in determining the maximum frequency of laminated plates. Based on this consideration, the next assumption in optimization is proposed:

> The optimum stacking sequence $[\theta^{(1)}/\theta^{(2)}/\ldots/\theta^{(N)}]_{S,opt}$ for the maximum fundamental natural frequency of laminated plates can be determined sequentially in the order from the outermost to the innermost layer.

The following algorithm is designed on this assumption and is proposed:

Step 0. Assume a laminated plate made of N layers in the upper (also in the lower) half with an elastic constant of $E=E_T$ in every direction (the value of an elastic constant in the direction perpendicular to the fiber in an orthotropic lamina).

Step 1. Find $\theta^{(1)}_{opt}$ in one-dimensional search to make the maximum fundamental frequency $\Omega^{(1)}_{opt}$ of the laminated plate with an orthotropic lamina in the 1st layer (outermost). The other inner ($N-1$) layers are still those with $E=E_T$ in every direction.

Step 2. Find $\theta^{(2)}_{opt}$ in one-dimensional search to make the maximum fundamental frequency $\Omega^{(2)}_{opt}$ of the laminated plate with an orthotropic lamina in the 2nd layer and that in the 1st orthotropic layer with $\theta^{(1)}=\theta^{(1)}_{opt}$. The inner ($N-2$) layers other than the two surface layers are still those with $E=E_T$.

....(this process is applied repeatedly to $\theta^{(3)}$,$\theta^{(4)}$,...)

Step N. Find $\theta^{(N)}_{opt}$ to make the maximum $\Omega^{(N)}_{opt}$ of the laminated plate with an orthotropic lamina in the Nth layer (innermost). This step determines an optimum solution $[\theta^{(1)}/\theta^{(2)}/\ldots/\theta^{(N)}]_{S,opt}$ that yields the maximum frequency $\Omega_{opt}=\Omega^{(N)}_{opt}$ of the optimized plate.

Solutions thus obtained are not guaranteed to be globally optimum in the exact mathematical sense, but are expected to be optimum or nearly optimum. Steps 1–N may be repeated to seek for the improved set of solutions by using $[\theta^{(1)}/\theta^{(2)}/\ldots/\theta^{(N)}]_{S,opt}$ as an initial solution in Step 0.

3. Free vibration analysis of laminated plates

Since the present design approach (LOA) is independent of vibration analysis methods, one can use any analytical or numerical method, such as the finite element method including commercial

FEM codes, in calculating natural frequencies. In this paper, a semi-analytical solution is used [12,13], because this solution has a low computational cost and ease in varying design parameters. It also has good accuracy and fast convergence behavior for various sets of boundary conditions [14].

For the small amplitude (linear) free vibration of a thin plate, the deflection w may be written as

$$w(x, y, t) = W(x, y) \sin \omega t, \tag{6}$$

where W is the amplitude. Then, the maximum strain energy due to the bending is expressed by

$$U_{max} = \frac{1}{2} \iint_A \{\boldsymbol{\kappa}\}^{\mathrm{T}} \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \{\boldsymbol{\kappa}\} \, \mathrm{d}A, \tag{7}$$

where $\{\boldsymbol{\kappa}\}$ is a curvature vector

$$\{\boldsymbol{\kappa}\} = \left\{ -\frac{\partial^2 W}{\partial x^2} \quad -\frac{\partial^2 W}{\partial y^2} \quad -2\frac{\partial^2 W}{\partial x \partial y} \right\}^{\mathrm{T}}. \tag{8}$$

The maximum kinetic energy is given by

$$T_{max} = \frac{1}{2} \rho \omega^2 \iint_A W^2 \, \mathrm{d}A. \tag{9}$$

In the Ritz method, the amplitude is assumed in the form

$$W(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn} X_m(x) Y_n(y), \tag{10}$$

where A_{mn} are unknown coefficients, and $X_m(x)$ and $Y_n(y)$ are the functions modified so that any kinematical boundary conditions are satisfied at the edges with "boundary indices" [12,13].

After substituting Eq. (10) into the sum of energies (7) and (9), the stationary value is obtained by

$$\frac{\partial}{\partial A_{\overline{mn}}} (T_{max} - U_{max}) = 0 \quad (\bar{m} = 0, 1, 2, \ldots; \ \bar{n} = 0, 1, 2, \ldots). \tag{11}$$

The minimizing process gives a set of linear simultaneous equations in terms of the coefficients A_{mn}, and the eigenvalues Ω may be extracted by using existing computer subroutines. This analytical procedure is a standard routine of the Ritz method but the special form of polynomials can satisfy kinematical boundary conditions [12,13].

4. Results and discussions

4.1. Numerical example

Frequency parameters are calculated by the frequency equation derived from Eq. (11). The elastic constants used in the following examples taken for graphite/epoxy composite [5] are

$$\text{G/E material}: \quad E_L = 138 \text{ GPa}, \ E_T = 8.96 \text{ GPa}, \ G_{LT} = 7.1 \text{ GPa}, \ \nu_{LT} = 0.30. \tag{12}$$

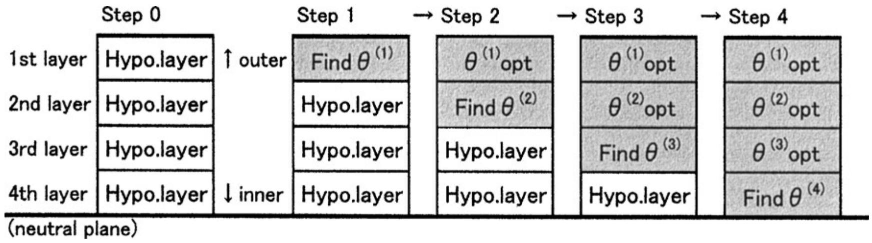

Fig. 2. Optimization process by the layerwise optimization approach for symmetric 8-layered plate (Hypo.layer: use $E = E_T$ of G/E material, shaded area: use E_L and E_T of G/E material).

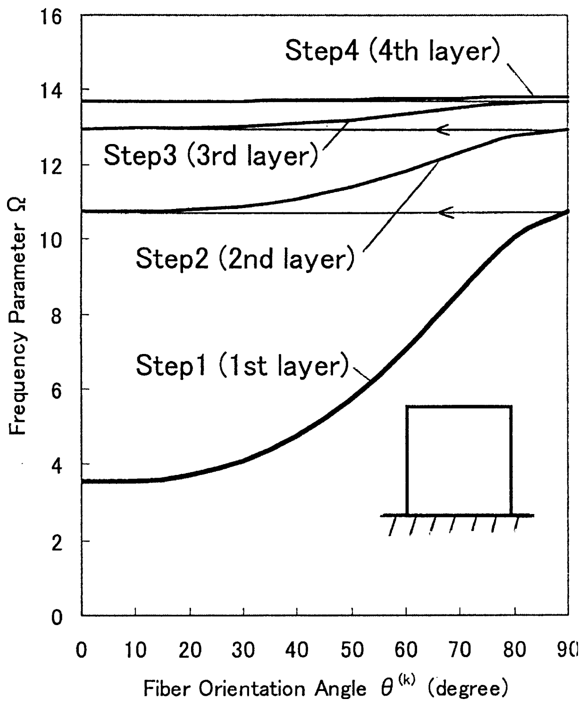

Fig. 3. Variation of the frequency parameter in the optimization process of symmetric 8-layered square plate (FCFF(cantilever), $a/b = 1$).

In the convergence tests previously reported [13,14], it was clearly seen that the frequencies monotonically decrease from above as the number of terms is increased in Eq. (10) and converge almost within four significant figures when $M = N = 10$ in the series are taken. The frequencies were therefore calculated thereafter by using $M \times N = 10 \times 10$ solutions.

Numerical examples are given for symmetrically laminated 8-layered square ($a/b = 1$) and rectangular ($a/b = 2$) plates. The design variables are presented in the usual notation as $[\theta^{(1)}/\theta^{(2)}/\theta^{(3)}/\theta^{(4)}]_s$, where $\theta^{(1)}$ is the fiber orientation angle of the 1st layer (outermost) and $\theta^{(4)}$ is that of the 4th layer (innermost) located on the middle surface of the plate. The boundary conditions of plates are given for free (F), simply supported (S) and clamped (C) edges, and the notations F, S and C are used in the order they occur in Edge(1), (2), (3) and (4) (see Fig. 1 to check the edge notation). For example, the SCFF plate indicates a rectangular plate with simply supported edge at Edge(1), clamped edge at Edge(2) and free edges on the remaining two edges.

In the application of LOA, as shown in Step 0 of Fig. 2, the upper (lower) four layers are first assumed to have the constants

$$E = E_T = 8.96\ \text{GPa}, \quad G_{LT} = 7.1\ \text{GPa}, \quad \nu_{LT} = 0.30. \tag{13}$$

Those hypothetical layers have identical elastic constant $E = E_T = 8.96$ GPa in every direction but cannot be real *isotropic* material due to lack of relation $G = E/2(1+\nu)$.

In Step 1, the first layer is replaced by the orthotropic G/E layer with the constants (12) and the optimum fiber orientation angle $\theta^{(1)}_{opt}$ is searched sequentially by changing $\theta^{(1)}$ with an increment

Table 1
Examples in the optimization process for symmetric 8-layered square plates with various boundary conditions ($a/b = 1$, increment 5°)

Present	$[\theta^{(1)}/\theta^{(2)}/\theta^{(3)}/\theta^{(4)}]_s$	Ω
(a) *SSFF plate*		
Step 1	$[-45/*/*/*]_s$	7.782
Step 2	$[-45/45/*/*]_s$	10.71
Step 3	$[-45/45/-45/*]_s$	11.20
Step 4	$[-45/45/-45/45]_s$	11.28
Ref. [15]	$[-45/45/-45/45]_s$	11.28
(b) *SSCF plate*		
Step 1	$[-5/*/*/*]_s$	48.33
Step 2	$[-5/0/*/*]_s$	57.90
Step 3	$[-5/0/-5/*]_s$	61.05
Step 4	$[-5/0/-5/-5]_s$	61.49
Ref. [15]	$[-2/-4/-2/-5]_s$	61.54
(c) *SSSS plate*		
Step 1	$[45/*/*/*]_s$	38.62
Step 2	$[45/-45/*/*]_s$	52.70
Step 3	$[45/-45/-45/*]_s$	55.90
Step 4	$[45/-45/-45/-45]_s$	56.32
Ref. [15]	$[45/-45/45/-45]_s$	55.30
(s) *CCCC plate*		
Step 1	$[0/*/*/*]_s$	75.66
Step 2	$[0/90/*/*]_s$	88.73
Step 3	$[0/90/90/*]_s$	93.06
Step 4	$[0/90/90/90]_s$	93.67
Ref. [15]	$[0/90/0/90]_s$	93.69

$\theta = 5°$ from $-90°$ to $90°$. One can use finer increment, e.g., $\theta = 2.5°$ or $1°$, depending upon the solution accuracy desired. In Step 2, the $\theta^{(2)}_{opt}$ is searched for the plate where the first layer has the fiber angle of $\theta^{(1)}_{opt}$ and the 2nd layer is replaced by the G/E layer. This process continues and the optimization is terminated after $\theta^{(4)}_{opt}$ is determined in Step 4.

The advantage of LOA in computation time is obvious in the numerical example. When one looks for the maximum fundamental frequency Ω_{opt} by examining all combinations of $[\theta^{(1)}/\theta^{(2)}/\theta^{(3)}/\theta^{(4)}]_s$ with an increment $\theta = 5°$, one needs to calculate frequencies for $37^4 = 1\,874\,161$ combinations. In LOA, one calculates for only $37 \times 4 = 148$ combinations. This ends up with the reduction to $148/37^4 = 4/37^3 \doteqdot 0.008$ percent of the computation time.

4.2. Square plate

Fig. 3 presents for illustrative purposes a set of variations in the frequency parameter during the optimization process. This symmetric 8-layered square plate is clamped at Edge(2) and is free on other edges. Because the lowest mode of the cantilever plate is the first bending mode of beam type, it is clear to have the optimum solution $[90°/90°/90°/90°]_s$ ("°" is omitted hereafter). It is seen in the figure that the frequency rapidly increases in Step 1 as $\theta^{(1)}$ approaches from 0 to 90, and then the increase in frequency is reduced in Step 2. The 4th layer (innermost) practically does not change the frequency value.

Table 2
Optimum solutions for symmetric 8-layered square plates with various boundary conditions (BC) ($a/b = 1$, increment 5°)

BC	$[\theta^{(1)}/\theta^{(2)}/\theta^{(3)}/\theta^{(4)}]_{S,opt}$	Ω_{opt}
(1) FFFF	$[65/-50/20/25]_s$	35.83
(2) SFFF	$[55/-45/-55/35]_s$	20.88
(3) CFFF	$[0/0/0/0]_s$	13.79
(4) SSFF	$[-45/45/-45/45]_s$	11.28
(5) SCFF	$[75/-50/65/65]_s$	16.28
(6) CCFF	$[65/-35/40/40]_s$	18.80
(7) SFSF	$[0/0/0/0]_s$	38.69
(8) SFCF	$[0/0/0/0]_s$	60.47
(9) CFCF	$[0/0/0/0]_s$	87.77
(10) SSSF	$[0/0/0/0]_s$	39.84
(11) SCSF	$[0/0/0/0]_s$	40.28
(12) SSCF	$[-5/0/-5/-5]_s$	61.49
(13) SCCF	$[-5/-5/0/0]_s$	61.88
(14) CSCF	$[0/0/0/0]_s$	88.41
(15) CCCF	$[0/0/0/0]_s$	88.63
(16) SSSS	$[45/-45/-45/-45]_s$	56.32
(17) SSSC	$[90/75/-60/-60]_s$	65.27
(18) SSCC	$[0/45/-45/-45]_s$	68.72
(19) SCSC	$[90/90/90/90]_s$	90.89
(20) CCCS	$[0/0/0/0]_s$	91.99
(21) CCCC	$[0/90/0/90]_s$	93.67

The optimization process of determining $\theta^{(k)}$ sequentially from outer layers is also seen, in Table 1, for (a) SSFF, (b) SSCF, (c) SSSS and (d) CCCC plates. The same tendency, as found in Fig. 3, that the outer layer is more effective in increasing the frequency is obvious for plates with those boundary conditions. It is also seen that the present optimum solutions coincide well with those of Ref. [15]. The complex method, which needs considerable amount of computation time, was employed in this reference.

Table 2 presents optimum solutions obtained by LOA for symmetric 8-layered square plates with 21 different combinations of boundary conditions. Since constraints are generally added to the edges from cases (1)–(21), the maximum fundamental frequencies Ω_{opt} generally increase in order. The first two cases (1) FFFF and (2) SFFF are exceptional in the sense that they show rigid body motions and the lowest elastic vibration modes have higher frequency values than the cases in (3) CFFF ~ (6) CCFF.

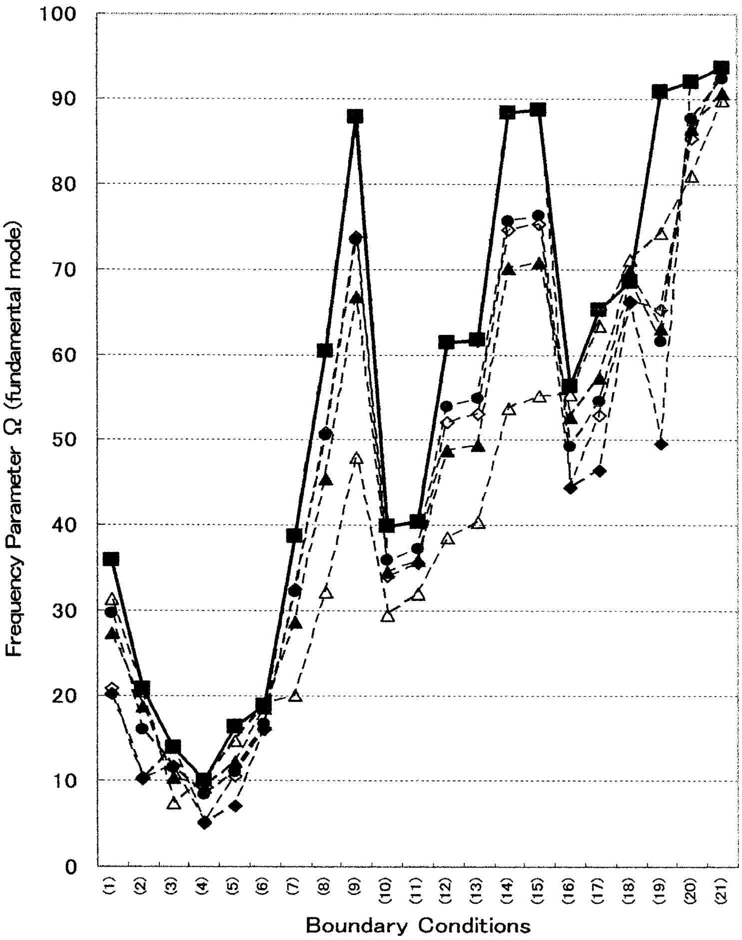

Fig. 4. Comparison between the optimum frequency Ω_{opt} and frequencies of symmetric 8-layered square plate for various stacking sequences ($a/b = 1$): ■, present optimum frequency Ω_{opt}; ♦, $[0/0/0/0]_s$; ◇, $[0/90/0/90]_s$; ▲, $[30/-30/30/-30]_s$; △, $[45/-45/45/-45]_s$; ●, $[0/-45/45/90]_s$ (quasi-isotropic).

To validate optimality of the obtained solutions, comparison is made in Fig. 4 to see that the plates with the present optimum solutions $[\theta^{(1)}/\theta^{(2)}/\theta^{(3)}/\theta^{(4)}]_{S,opt}$ actually give higher frequencies than those with other stacking sequences. Typical stacking sequences of the symmetric 8-layered plates are chosen for comparison as $[0/0/0/0]_s$, $[0/90/0/90]_s$, $[30/-30/30/-30]_s$, $[45/-45/45/-45]_s$ and $[0/-45/45/90]_s$. The first two (i.e., $[0/0/0/0]_s$ are $[0/90/0/90]_s$) are macroscopically specially orthotropic, and are denoted by ♦ and ◇, respectively. The next two ($[30/-30/30/-30]_s$ and $[45/-45/45/-45]_s$) are alternating angle-ply sequence, denoted by ▲ and △. The last one ($[0/-45/45/90]_s$) is a quasi-isotropic case, denoted by •. It is observed that almost all the present optimum solutions (denoted by ■) yield higher frequencies than those of plates with the five typical stacking sequences.

Only one exception in the figure is the case of (18) SSCC. The present solution [0/45/−45/−45] gives $\Omega = 68.72$ while an angle-ply sequence $[45/-45/45/-45]_s$ gives slightly higher value of $\Omega = 71.21$. This fact indicates that there is a slight possibility that the present LOA may fall into local optimums.

4.3. Rectangular plate

Table 3 presents the present solutions for symmetric 8-layered rectangular plates ($a/b = 2$) with 21 different combinations in boundary conditions, in the same format as Table 2. Tendency of

Table 3
Optimum solutions for symmetric 8-layered rectangular plates with various boundary conditions (BC) ($a/b = 2$, increment 5°)

BC	$[\theta^{(1)}/\theta^{(2)}/\theta^{(3)}/\theta^{(4)}]_{S,opt}$	Ω_{opt}
(1) FFFF	$[0/-35/45/40]_s$	61.79
(2) SFFF	$[5/-40/50/45]_s$	32.23
(3) CFFF	$[0/0/0/0]_s$	13.79
(4) SSFF	$[-35/45/-45/45]_s$	21.89
(5) SCFF	$[85/85/85/85]_s$	57.06
(6) CCFF	$[85/85/85/85]_s$	57.71
(7) SFSF	$[0/0/0/0]_s$	38.66
(8) SFCF	$[0/0/0/0]_s$	60.44
(9) CFCF	$[0/0/0/0]_s$	87.74
(10) SSSF	$[0/-30/40/35]_s$	45.26
(11) SCSF	$[90/70/-55/-55]_s$	61.94
(12) SSCF	$[-10/0/-5/25]_s$	64.84
(13) SCCF	$[-10/65/-35/-35]_s$	69.88
(14) CSCF	$[0/0/0/0]_s$	90.28
(15) CCCF	$[0/0/0/0]_s$	92.28
(16) SSSS	$[90/90/90/90]_s$	159.9
(17) SSSC	$[90/90/90/90]_s$	245.7
(18) SSCC	$[90/90/90/90]_s$	246.4
(19) SCSC	$[90/90/90/90]_s$	353.9
(20) CCCS	$[90/90/90/90]_s$	247.1
(21) CCCC	$[90/90/90/90]_s$	354.9

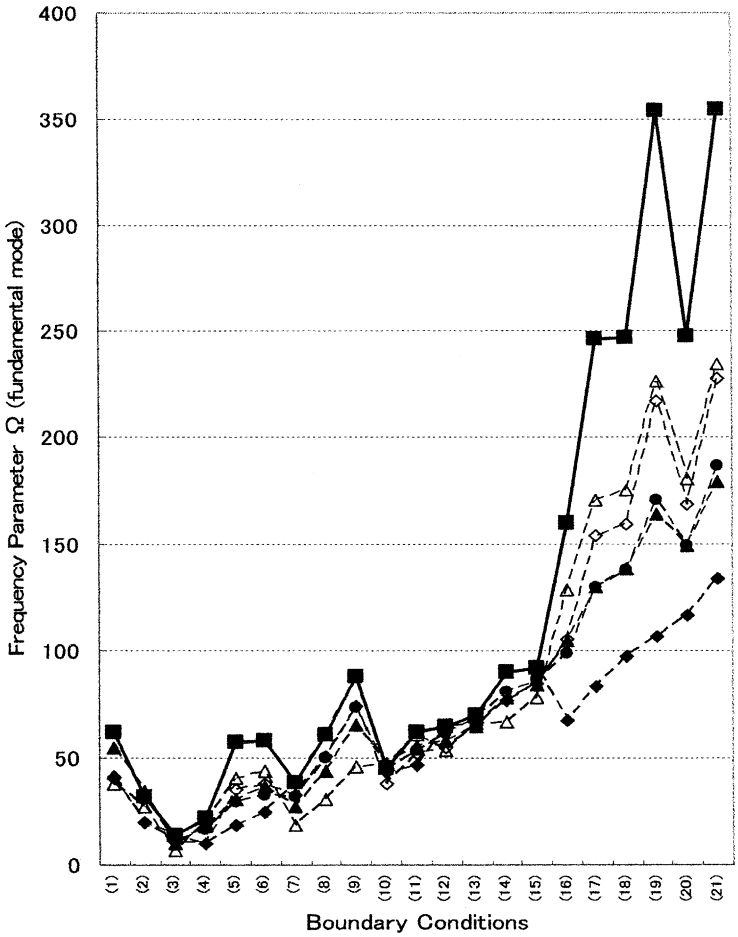

Fig. 5. Comparison between the optimum frequency Ω_{opt} and frequencies of symmetric 8-layered rectangular plate for various stacking sequences ($a/b=2$): ■, present optimal frequency Ω_{opt}; ♦, $[0/0/0/0]_s$; ◇, $[0/90/0/90]_s$; ▲, $[30/-30/30/-30]_s$; △, $[45/-45/45/-45]_s$; ●$[0/-45/45/90]_s$ (quasi-isotropic).

increasing frequencies, moving from case (1) to (21), is the same as in Table 2. Also (1) FFFF and (2) SFFF have rigid body motions. It is noted that cases of (16) SSSS ~ (21) CCCC with strong constraints have an identical solution of $[90/90/90/90]_s$, because it is effective to stiffen the plates by bridging the fibers (major principal material axis) between a short span of the opposite simply supported or clamped edges.

The comparison is also made in Fig. 5 to see that the rectangular plates with present optimum solutions $[\theta^{(1)}/\theta^{(2)}/\theta^{(3)}/\theta^{(4)}]_{S,opt}$ give higher frequencies than those with other typical stacking sequences. The same notations ♦, ◇, ▲, △ and ●, as in Fig. 4, are used to indicate $[0/0/0/0]_s$, $[0/90/0/90]_s$, $[30/-30/30/-30]_s$, $[45/-45/45/-45]_s$ and $[0/-45/45/90]_s$, respectively. Unlike the square plate there are no exceptions and all the present optimum solutions are higher than the reference frequencies.

5. Conclusions

A layerwise optimization approach (LOA) was proposed in a certain class of optimum design problems to yield the maximum fundamental frequency of symmetrically laminated plate. For the purpose, an assumption is introduced that the optimum stacking sequence for the maximum frequency is determined sequentially in the order from the outermost to the innermost layer. With this idea of LOA, the design algorithm is proposed. Numerical experiment is conducted to show the validity and usefulness of LOA and examples are given for different aspect ratios and boundary conditions. Only one exception was found where the LOA yields local solution but this local solution is found to be still very close to the global solution. In all other numerical examples, the present solutions gave the higher frequencies than the reference values. It is hoped that the LOA will be extended as a practical optimization technique in the composite structural design.

References

[1] A.W. Leissa, Vibration of Plates, NASA-160, U.S. Government Printing Office, Washington, DC, 1969.
[2] R.D. Blevins, Formulas for Natural Frequency and Mode Shape, Van Nostrand Reinhold, New York, 1979.
[3] D.J. Gorman, Free Vibration Analysis of Rectangular Plates, Elsevier, New York, 1982.
[4] R.M. Jones, Mechanics of Composite Materials, Scripta, Washington, DC, 1975.
[5] J.R. Vinson, R.L. Sierakowski, The Behavior of Structures Composed of Composite Materials, Martinus Nijhoff, Dordrecht, 1986.
[6] C.W. Bert, Optimal design of a composite material plate to maximize its fundamental frequency, Journal of Sound and Vibration 50 (1977) 229–237.
[7] C.W. Bert, Design of clamped composite material plates to maximize fundamental frequency, Journal of Mechanical Design (1978) 274–278.
[8] R. Reiss, S. Ramachandran, Maximum frequency design of symmetric angle-ply laminates, in: I.H. Marshall (Ed.), Composite Structures 4, Vol. 1: Analysis and Design Studies, Elsevier, London, 1987, pp. 1476–1487.
[9] H. Fukunaga, H. Sekine, M. Sato, Optimal design of symmetrically laminated plates for the fundamental frequency, Journal of Sound and Vibration 171 (1994) 219–229.
[10] Z. Gurdal, R.T. Haftka, P. Hajela, in: Design and Optimization of Laminated Composite Materials, Wiley-Interscience, New York, 1999, pp. 297–310.
[11] Y. Narita, Combinations for the free-vibration behaviors of anisotropic rectangular plates under general edge conditions, Journal of Applied Mechanics 67 (2000) 568–573.
[12] Y. Narita, Y. Ohta, G. Yamada, Y. Kobayashi, Analytical method for vibration of angle-ply cylindrical shells having arbitrary edges, American Institute of Aeronautics and Astronautics Journal 30 (1992) 790–796.
[13] Y. Narita, Series and Ritz-type buckling analysis, in: G.J. Turvey, I.H. Marshall (Eds.), Buckling and Postbuckling of Composite Plates, Chapman & Hall, London, 1995, pp. 33–57 (Chapter 2).
[14] Y. Narita, Closure to "Discussion of 'Combinations for the free-vibration behavior of anisotropic rectangular plates under general edge conditions'", Journal of Applied Mechanics 68 (2001) 685–686.
[15] X. Zhao, Y. Narita, Maximization of fundamental frequency for generally laminated rectangular plates by the complex method, Transactions of JSME 63C (1997) 364–370 (in Japanese).