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Forced transverse vibrations of an elastically connected complex simply supported double-beam system

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Abstract

The present paper is devoted to analyzing undamped forced transverse vibrations of an elastically connected complex double-beam system. The problem is formulated and solved in the case of simply supported beams. The classical modal expansion method is applied to ascertain dynamic responses of beams due to arbitrarily distributed continuous loads. Several cases of particularly interesting excitation loadings are investigated. The action of stationary harmonic loads and moving forces is considered. In discussing vibrations caused by exciting harmonic forces, conditions of resonance and dynamic vibration absorption are determined. The beam-type dynamic absorber is a new concept of a continuous dynamic vibration absorber (CDVA), which can be applied to suppress excessive vibrations of corresponding beam systems. A numerical example is presented to illustrate the theoretical analysis.

1. Introduction

Beam-type structures are widely used in many branches of modern civil, mechanical, and aerospace engineering. Therefore, the dynamics of single beam and beam systems is still a subject of great interest to many investigators. The vibration theory for simple one-dimensional continuous systems as beams and strings is developed in a number of monographs by e.g., Ziemba [1], Solecki and Szymkiewicz [2], Kaliski [3], Snowdon [4], Fryba [5], Nowacki [6], Timoshenko et al. [7], Craig [8], and Rao [9], among others. As is well known, there are four fundamental models for transversely vibrating beam. These are the Bernoulli–Euler, Rayleigh, shear (Flügge), and Timoshenko models [1–3,7,10]. In Ref. [11], the author has discussed free transverse vibrations of two simply supported Bernoulli–Euler beams connected by a Winkler elastic layer. The present paper deals with forced vibrations of this complex beam system. Different problems concerning forced responses of an elastically connected double-beam system (or sandwich beam)

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are considered by many scientists: Dublin and Friedrich [12], Seelig and Hoppmann II [13], Kessel [14], Kessel and Raske [15], Kozlov [16], Saito and Chonan [17,18], Lu and Douglas [19], Oniszczuk [20,24,35,40–42], Chonan [21,22], Jacquot and Foster [23], Douglas and Yang [25], Irie et al. [26], Dmitriev [27], Hamada et al. [28,29], Yamaguchi [30], Vu [31], Kokhmanyuk [32], Aida et al. [33], Chen and Sheu [34], Chen and Lin [36], Szcześniak [37], Kawazoe et al. [38], and Vu et al. [39]. The analysis of special cases of forced harmonic vibrations allows one to formulate conditions which generate the appearance of a dynamic vibration absorption in a double-beam system. This very interesting phenomenon is of great technical importance and therefore a beam is often applied as a continuous dynamic vibration absorber (CDVA). Theories of beam-type absorbers are presented in a few works by Kessel and Raske [15], Jacquot and Foster [23], Hamada et al. [28,29], Yamaguchi [30], Vu [31], Aida et al. [33], Chen and Sheu [34], Oniszczuk [35,40,41], Chen and Lin [36], Kawazoe et al. [38], and Vu et al. [39].

In the present paper being an extension of work described in reference [11], exact theoretical general solutions of undamped forced vibrations for a simply supported double-beam system are determined. Then, several cases of particular excitation loadings are studied. The forced vibration analysis of a similar system of elastically connected two strings presented by the author in Refs. [41,43,44] can be helpful in the investigation of the title system because of the same boundary conditions and the same mathematical procedures applied. The general vibration analysis of an elastically connected double-beam system is complicated and laborious in view of a large variety of possible combinations of boundary conditions [24,35]. Vibrations of a general system of two beams governed by arbitrary boundary conditions which are four fundamental homogeneous ones, will be discussed in a future publication.

2. Formulation of the problem

The scheme of vibratory system considered is depicted in Fig. 1. An elastically connected double-beam system consists of two parallel beams of the same length, which are joined by an elastic layer modelled as a Winkler massless foundation. Both beams are homogeneous, prismatic and slender, what makes it possible to apply the classical Bernoulli–Euler beam theory in deriving the equations of system motion. For simplicity of the analysis, all four ends of beams are assumed to be simply supported. The beams are subjected to arbitrarily distributed transverse continuous loads. Small undamped vibrations of the system are discussed.

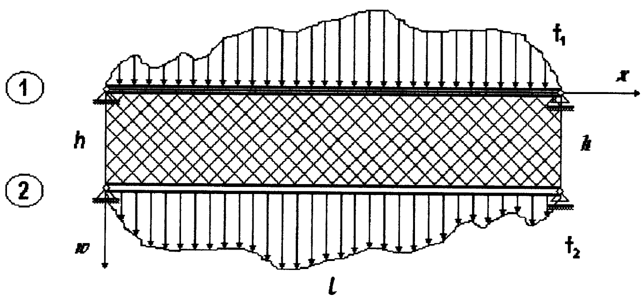

Fig. 1. The dynamic model of an elastically connected complex simply supported double-beam system.

The transverse vibrations of a generally loaded double-beam system are governed by the following differential equations [13–15,20,24,29,33,35,40,41], based on the Bernoulli–Euler theory:

$$K_1 w_1^{\mathrm{iv}} + m_1 \ddot{w}_1 + k(w_1 - w_2) = f_1(x,t),$$
$$K_2 w_2^{\mathrm{iv}} + m_2 \ddot{w}_2 + k(w_2 - w_2) = f_2(x,t), \tag{1}$$

where $w_i = w_i(x,t)$ is the transverse beam deflection; $f_i = f_i(x,t)$ is the exciting distributed continuous load; x, t are the spatial co-ordinate and the time, respectively; E_i is Young's modulus of elasticity; F_i is the cross-sectional area of the beam; J_i is the moment of inertia of the beam cross-section; K_i is the flexural rigidity of the beam; h, k are the thickness and the stiffness modulus of a Winkler elastic layer, respectively; l is the length of the beam; ρ_i is the mass density;

$$K_i = E_i J_i, \quad m_i = \rho_i F_i, \quad \dot{w}_i = \partial w_i/\partial t, \quad w_i' = \partial w_i/\partial x, \quad i = 1,2.$$

The boundary conditions for simply supported beams are:

$$w_i(0,t) = w_i''(0,t) = w_i(l,t) = w_i''(l,t) = 0, \quad i = 1,2. \tag{2}$$

3. Solution of the problem

The forced vibrations of beams subjected to arbitrarily distributed continuous loads are determined by applying the classical modal expansion method [24,35,40,41]. Particular solutions of non-homogeneous differential equations (1) representing forced vibrations of a double-beam system are assumed in the following form [24,35,40,41]:

$$w_1(x,t) = \sum_{n=1}^{\infty}\sum_{i=1}^{2} X_{1in}(x)S_{in}(t) = \sum_{n=1}^{\infty} X_n(x)\sum_{i=1}^{2} S_{in}(t) = \sum_{n=1}^{\infty}\sin(k_n x)\sum_{i=1}^{2} S_{in}(t),$$
$$w_2(x,t) = \sum_{n=1}^{\infty}\sum_{i=1}^{2} X_{2in}(x)S_{in}(t) = \sum_{n=1}^{\infty} X_n(x)\sum_{i=1}^{2} a_{in}S_{in}(t) = \sum_{n=1}^{\infty}\sin(k_n x)\sum_{i=1}^{2} a_{in}S_{in}(t), \tag{3}$$

where

$$a_{in} = (K_1 k_n^4 + k - m_1\omega_{in}^2)k^{-1} = k(K_2 k_n^4 + k - m_2\omega_{in}^2)^{-1} = \Omega_{10}^{-2}(\Omega_{11n}^2 - \omega_{in}^2)$$
$$= \Omega_{20}^2(\Omega_{22n}^2 - \omega_{in}^2)^{-1},$$
$$a_{1n}a_{2n} = -m_1 m_2^{-1} = -M_1 M_2^{-1}, \quad a_{1n} > 0, \quad a_{2n} < 0, \tag{4}$$

$$i = 1,2, \quad K = kl, \quad k_n = l^{-1}n\pi, \quad M_i = m_i l = \rho_i F_i l, \quad n = 1,2,3,\ldots$$

$$\Omega_{iin}^2 = (K_i k_n^4 + k)m_i^{-1} = [K_i l^{-3}(n\pi)^4 + K]M_i^{-1}, \quad \Omega_{i0}^2 = k m_i^{-1} = K M_i^{-1},$$
$$\Omega_{120}^4 = \Omega_{10}^2\Omega_{20}^2 = k^2(m_1 m_2)^{-1} = K^2(M_1 M_2)^{-1},$$
$$\omega_{1,2n}^2 = 0.5\{(\Omega_{11n}^2 + \Omega_{22n}^2) \mp [(\Omega_{11n}^2 - \Omega_{22n}^2)^2 + 4\Omega_{120}^4]^{1/2}\}, \quad \omega_{1n} < \omega_{2n}, \tag{5}$$

$$\omega_{1,2n}^2 = 0.5\{[(K_1k_n^4 + k)m_1^{-1} + (K_2k_n^4 + k)m_2^{-1}] \mp ([K_1k_n^4 + k)m_1^{-1} + (K_2k_n^4 + k)m_2^{-1}]^2 - 4k_n^4(m_1m_2)^{-1}[K_1K_2k_n^4 + k(K_1 + K_2)])^{1/2}\},$$
$$X_{1in}(x) = X_n(x), \quad X_{2in}(x) = a_{in}X_n(x), \quad X_n(x) = \sin(k_nx), \tag{6}$$

$X_n(x)$ is the known mode shape function for a simply supported single beam and $S_{in}(t)$ are the unknown time functions corresponding to the natural frequencies ω_{in} [11]. All quantities mentioned above are defined in Ref. [11], in which the free vibration problem of the title system is considered.

Substituting the assumed solutions (3) into system (1) results in the relationships

$$\sum_{n=1}^{\infty} X_n \sum_{i=1}^{2} [\ddot{S}_{in} + (\Omega_{11n}^2 - \Omega_{10}^2a_{in})S_{in}] = m_1^{-1}f_1,$$
$$\sum_{n=1}^{\infty} X_n \sum_{i=1}^{2} [\ddot{S}_{in} + (\Omega_{22n}^2 - \Omega_{20}^2a_{in}^{-1})S_{in}]a_{in} = m_2^{-1}f_2.$$

Taking expressions (4) into consideration gives

$$\sum_{n=1}^{\infty} X_n \sum_{i=1}^{2} (\ddot{S}_{in} + \omega_{in}^2S_{in}) = m_1^{-1}f_1, \quad \sum_{n=1}^{\infty} X_n \sum_{i=1}^{2}(\ddot{S}_{in} + \omega_{in}^2S_{in})a_{in} = m_2^{-1}f_2.$$

Multiplying the above relationships by the eigenfunction X_m then integrating them with respect to x from 0 to l and applying the classical orthogonality condition [11]

$$\int_0^l X_mX_n\,\mathrm{d}x = \int_0^l \sin(k_mx)\sin(k_nx)\,\mathrm{d}x = c\delta_{mn},$$
$$c = c_n^2 = \int_0^l X_n^2\,\mathrm{d}x = \int_0^l \sin^2(k_nx)\,\mathrm{d}x = 0.5l, \tag{7}$$

where δ_{mn} is the Kronecker delta function: $\delta_{mn} = 0$ for $m \neq n$, and $\delta_{mn} = 1$ for $m = n$, one gets a set of equations

$$\sum_{i=1}^{2} (\ddot{S}_{in} + \omega_{in}^2S_{in}) = 2M_1^{-1}\int_0^l f_1X_n\,\mathrm{d}x, \quad \sum_{i=1}^{2} (\ddot{S}_{in} + \omega_{in}^2S_{in})a_{in} = 2M_2^{-1}\int_0^l f_2X_n\,\mathrm{d}x$$

from which after some manipulation the following two infinite sequences of ordinary differential equations for the unknown time functions are obtained:

$$\ddot{S}_{in} + \omega_{in}^2S_{in} = H_{in}(t), \quad i = 1, 2, \quad n = 1, 2, 3, \ldots, \tag{8}$$

where

$$H_{1n}(t) = d_{1n}\int_0^l [a_{2n}M_1^{-1}f_1(x,t) - M_2^{-1}f_2(x,t)]\sin(k_nx)\,\mathrm{d}x,$$
$$H_{2n}(t) = d_{2n}\int_0^l [a_{1n}M_1^{-1}f_1(x,t) - M_2^{-1}f_2(x,t)]\sin(k_nx)\,\mathrm{d}x, \tag{9}$$
$$d_{1n} = -d_{2n} = 2(a_{2n} - a_{1n})^{-1} = 2\Omega_{10}^2(\omega_{1n}^2 - \omega_{2n}^2)^{-1}.$$

Particular solutions of Eq. (8) satisfying homogeneous initial conditions are [24,35,40–42]

$$S_{in}(t) = \omega_{in}^{-1} \int_0^l H_{in}(s)\sin[\omega_{in}(t-s)]\,\mathrm{d}s, \quad i = 1, 2. \tag{10}$$

Finally, the forced vibrations of an elastically connected simply supported double-beam system are found to be in the form

$$w_1(x,t) = \sum_{n=1}^{\infty} \sin(k_n x) \sum_{i=1}^{2} \omega_{in}^{-1} \int_0^t H_{in}(s)\sin[\omega_{in}(t-s)]\,\mathrm{d}s,$$

$$w_2(x,t) = \sum_{n=1}^{\infty} \sin(k_n x) \sum_{i=1}^{2} a_{in}\omega_{in}^{-1} \int_0^t H_{in}(s)\sin[\omega_{in}(t-s)]\,\mathrm{d}s. \tag{11}$$

Solutions (11) are sufficiently versatile to allow determination of the dynamic response of this system due to an arbitrary exciting transversal loading as well as stationary loads and moving forces. For simplicity of further analysis and discussion it is assumed that the exciting load acts only on the first beam (see Fig. 2), whilst the other one is not loaded; i.e., $f_1(x,t) \neq 0$, $f_2(x,t) = 0$. Thus the time functions (9) take the simpler form

$$H_{in}(t) = b_{in} \int_0^l f_1(x,t)\sin(k_n x)\,\mathrm{d}x, \quad i = 1, 2, \tag{12}$$

where $b_{1n} = a_{2n}d_{1n}M_1^{-1} = 2a_{2n}[(a_{2n} - a_{1n})M_1]^{-1}$, $b_{2n} = a_{1n}d_{2n}M_1^{-1} = 2a_{1n}[(a_{1n} - a_{2n})M_1]^{-1}$.

If the exciting load is a harmonic function of time $f_1(x,t) = f(x)\sin(pt)$, then one gets

$$H_{in}(t) = b_{in}\sin(pt) \int_0^l f(x)\sin(k_n x)\,\mathrm{d}x, \quad i = 1, 2, \tag{13}$$

where $f(x)$ is the arbitrary function of spatial co-ordinate x, and p is the frequency of the exciting harmonic load.

Next relationships (10) can be transformed to the form

$$\begin{aligned} S_{in}(t) &= b_{in}\omega_{in}^{-1} \int_0^l f(x)\sin(k_n x)\,\mathrm{d}x \int_0^t \sin(ps)\sin[\omega_{in}(t-s)]\,\mathrm{d}s \\ &= b_{in}F_n(\omega_{in}^2 - p^2)^{-1}[\sin(pt) - p\omega_{in}^{-1}\sin(\omega_{in}t)], \end{aligned} \tag{14}$$

where $F_n = \int_0^l f(x)\sin(k_n x)\,\mathrm{d}x$, $i = 1, 2$.

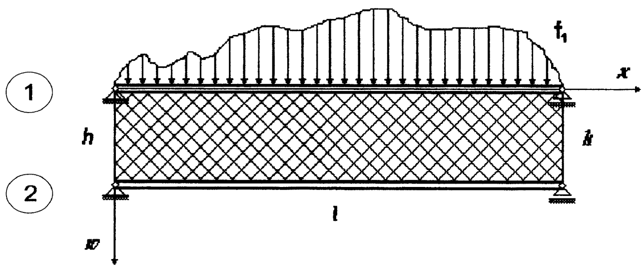

Fig. 2. An elastically connected complex double-beam system subjected to distributed continuous harmonic load.

A detailed forced vibration analysis is now performed for two interesting cases of the exciting harmonic loadings acting stationarily: uniform distributed continuous load and concentrated force. The classical case of a moving concentrated force is also discussed.

Case 1: *Stationary exciting harmonic loads*. To begin with the general case of distributed continuous harmonic load is considered. The first beam is subjected to arbitrarily distributed load $f_1(x,t) = f(x)\sin(pt)$ acting on the entire length of the beam as is shown in Fig. 2.

The forced vibrations of the beam system are determined from the general solutions (11) by using relations (13) and (14)

$$w_1(x,t) = \sum_{n=1}^{\infty} \sin(k_n x)\left[A_{1n}\sin(pt) + \sum_{i=1}^{2} B_{in}\sin(\omega_{in}t)\right],$$
$$w_2(x,t) = \sum_{n=1}^{\infty} \sin(k_n x)\left[A_{2n}\sin(pt) + \sum_{i=1}^{2} a_{in}B_{in}\sin(\omega_{in}t)\right], \tag{15}$$

where

$$A_{1n} = 2F_n M_1^{-1}(\Omega_{22n}^2 - p^2)[(\omega_{1n}^2 - p^2)(\omega_{2n}^2 - p^2)]^{-1},$$
$$A_{2n} = 2F_n K^{-1}\Omega_{120}^4[(\omega_{1n}^2 - p^2)(\omega_{2n}^2 - p^2)]^{-1},$$
$$B_{1n} = 2a_{2n}F_n M_1^{-1}p[(a_{1n} - a_{2n})\omega_{1n}(\omega_{1n}^2 - p^2)]^{-1},$$
$$B_{2n} = 2a_{1n}F_n M_1^{-1}p[(a_{2n} - a_{1n})\omega_{2n}(\omega_{2n}^2 - p^2)]^{-1},$$
$$F_n = \int_0^l f(x)\sin(k_n x)\,\mathrm{d}x, \quad k_n = l^{-1}n\pi, \; i = 1,2. \tag{16}$$

The solutions obtained using Eq. (15) are composed of two parts. The first part being a function of $\sin(pt)$ denotes the steady state (pure) forced vibrations of the system, and the other one containing the terms $\sin(\omega_{in}t)$ represents the free vibration produced by the application of exciting loading. Neglecting the free response, and assuming that only the steady state response has a practical significance, the forced vibrations of an elastically connected double-beam system are found to be in the form

$$w_1(x,t) = \sin(pt)\sum_{n=1}^{\infty} A_{1n}\sin(k_n x), \quad w_2(x,t) = \sin(pt)\sum_{n=1}^{\infty} A_{2n}\sin(k_n x). \tag{17}$$

Discussing the steady state vibration amplitudes A_{1n}, A_{2n} (16) a number of interesting and important conclusions may be drawn. This analysis leads to the fundamental conditions

(a) *Condition of resonance*: $p = \omega_{in}, \; i = 1,2, \; n = 1,2,3,\ldots$
(b) *Condition of dynamic variation absorption*:

$$p^2 = p_n^2 = \Omega_{22n}^2 = (K_2 k_n^4 + k)m_2^{-1} = [K_2 l^{-3}(n\pi)^4 + K]M_2^{-1},$$
$$A_{1n} = 0, \quad A_{2n} = -2F_n K^{-1} = -2K^{-1}\int_0^l f(x)\sin(k_n x)\,\mathrm{d}x. \tag{18}$$

With the application of harmonic forces, a dynamic vibration absorption phenomenon occurs and the second beam acts as a dynamic vibration absorber in relation to the first one. Relationship (18) is the basic condition of a dynamic vibration absorption which can be used to optimal design a complex system of two beams. Optimum values of tuning parameters of a dynamic absorber are found by a proper choice of the elastic layer stiffness modulus k, flexural rigidity $K_2 = E_2J_2$ and mass of the second beam $M_2 = m_2l = \rho_2F_2l$. The dynamic absorption eliminates any selected harmonic component A_{1n} of the first beam vibrations. The dynamical damper reduces forced vibrations of the first beam but never eliminates them completely [35,41].

Case 1.1: *Uniform distributed harmonic load.* The uniform distributed harmonic continuous load $f_1(x,t) = f\sin(pt)$ acts on the first beam (see Fig. 3). f and p are the amplitude and exciting frequency of the load, respectively.

After performing calculations on the basis of solutions (11), (13) and (14), the forced vibrations are received in the form

$$\begin{aligned} w_1(x,t) &= \sum_{(n)} \sin(k_nx)\left[A_{1n}\sin(pt) + \sum_{i=1}^{2} B_{in}\sin(\omega_{in}t)\right], \quad n = 1,3,5,\ldots, \\ w_2(x,t) &= \sum_{(n)} \sin(k_nx)\left[A_{2n}\sin(pt) + \sum_{i=1}^{2} a_{in}B_{in}\sin(\omega_{in}t)\right], \end{aligned} \tag{19}$$

where

$$\begin{aligned} A_{1n} &= 4F(M_1n\pi)^{-1}(\Omega_{22n}^2 - p^2)[(\omega_{1n}^2 - p^2)(\omega_{2n}^2 - p^2)]^{-1}, \\ A_{2n} &= 4F(Kn\pi)^{-1}\Omega_{120}^4[(\omega_{1n}^2 - p^2)(\omega_{2n}^2 - p^2)]^{-1}, \\ B_{1n} &= 4a_{2n}F(M_1n\pi)^{-1}p[(a_{1n} - a_{2n})\omega_{1n}(\omega_{1n}^2 - p^2)]^{-1}, \\ B_{2n} &= 4a_{1n}F(M_1n\pi)^{-1}p[(a_{2n} - a_{1n})\omega_{2n}(\omega_{2n}^2 - p^2)]^{-1}, \\ F &= fl, \quad i = 1,2, \quad k_n = l^{-1}n\pi, \quad n = 1,3,5,\ldots\ . \end{aligned} \tag{20}$$

Omitting the terms of free response, the steady state forced vibrations of beams are found to be

$$\begin{aligned} w_1(x,t) &= \sin(pt)\sum_{(n)} A_{1n}\sin(k_nx), \\ w_2(x,t) &= \sin(pt)\sum_{(n)} A_{2n}\sin(k_nx), \quad n = 1,3,5,\ldots\ . \end{aligned} \tag{21}$$

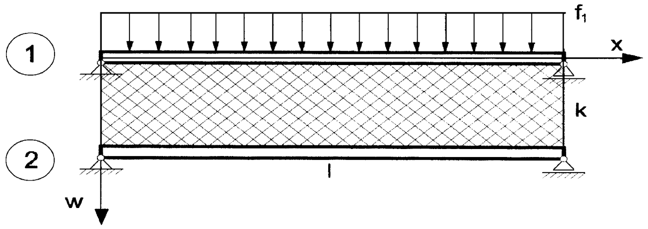

Fig. 3. An elastically connected complex double-beam system subjected to uniform distributed harmonic continuous load.

The analysis of the steady state vibration amplitudes A_{in} (20) leads to the fundamental conditions

(a) *Condition of resonance*: $p = \omega_{in}, \; i = 1, 2, \; n = 1, 3, 5, \ldots$.

(b) *Condition of dynamic vibration absorption*:

$$p^2 = p_n^2 = \Omega_{22n}^2 = (K_2 k_n^4 + k) m_2^{-1} = [K_2 l^{-3} (n\pi)^4 + K] M_2^{-1},$$
$$A_{1n} = 0, \quad A_{2n} = -4F(Kn\pi)^{-1}, \quad n = 1, 3, 5, \ldots \; . \tag{22}$$

Case 1.2: *Concentrated harmonic force*. The first beam is subjected to the concentrated harmonic force $f_1(x,t) = F(t)\delta(x - 0.5l) = F\sin(pt)\delta(x - 0.5l)$ applied for simplicity at the midspan of the beam (see Fig. 4). F and p are the amplitude and frequency of the exciting harmonic force, respectively, and $\delta(x)$ is the Dirac delta function.

For this case the forced vibrations of a two-beam system are received in analogous form as (19)

$$w_1(x,t) = \sum_{(n)} \sin(k_n x) \left[A_{1n} \sin(pt) + \sum_{i=1}^{2} B_{in} \sin(\omega_{in} t) \right], \quad n = 1, 3, 5, \ldots$$
$$w_2(x,t) = \sum_{(n)} \sin(k_n x) \left[A_{2n} \sin(pt) + \sum_{i=1}^{2} a_{in} B_{in} \sin(\omega_{in} t) \right], \tag{23}$$

where

$$A_{1n} = 2b_n F M_1^{-1} (\Omega_{22n}^2 - p^2)[(\omega_{1n}^2 - p^2)(\omega_{2n}^2 - p^2)]^{-1},$$
$$A_{2n} = 2b_n F K^{-1} \Omega_{120}^4 [(\omega_{1n}^2 - p^2)(\omega_{2n}^2 - p^2)]^{-1}, \tag{24}$$

$$B_{1n} = 2a_{2n} b_n F M_1^{-1} p[(a_{1n} - a_{2n})\omega_{1n}(\omega_{1n}^2 - p^2)]^{-1},$$
$$B_{2n} = 2a_{1n} b_n F M_1^{-1} p[(a_{2n} - a_{1n})\omega_{2n}(\omega_{2n}^2 - p^2)]^{-1},$$

$$b_n = \sin(0.5n\pi) = (-1)^{0.5(n-1)}, \quad n = 1, 3, 5, \ldots \; .$$

The steady state forced vibrations of the system are (21)

$$w_1(x,t) = \sin(pt) \sum_{(n)} A_{1n} \sin(k_n x),$$
$$w_2(x,t) = \sin(pt) \sum_{(n)} A_{2n} \sin(k_n x), \quad n = 1, 3, 5, \ldots \; . \tag{25}$$

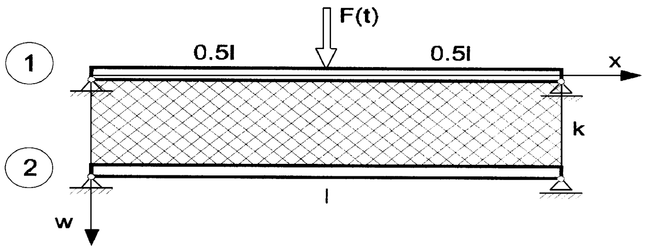

Fig. 4. An elastically connected complex double-beam system subjected to concentrated harmonic force.

The analysis of the steady state vibration amplitudes A_{in} (24) leads to the fundamental conditions

(a) *Condition of resonance*: $p = \omega_{in},\ i = 1, 2,\ n = 1, 3, 5, \ldots,$
(b) *Condition of dynamic vibration absorption*:

$$p^2 = p_n^2 = \Omega_{22n}^2 = (K_2 k_n^4 + k)m_2^{-1} = [K_2 l^{-3}(n\pi)^4 + K]M_2^{-1},$$
$$A_{1n} = 0, \quad A_{2n} = -2b_n F K^{-1}, \quad n = 1, 3, 5, \ldots \ . \tag{26}$$

Case 2: *Moving concentrated forces*. First, the general case of a moving concentrated force arbitrarily varying in time is presented. The first beam is traversed by a point force which moves with a constant velocity v along a beam from the left support ($x = 0$) to the right support ($x = l$) (see Fig. 5). The exciting loading of a double-beam system is $f_1(x, t) = F(t)\delta(x - vt)$, $f_2(x, t) = 0$, $0 < t < T$, $T = lv^{-1}$, where $F(t)$ is the concentrated force being an arbitrary function of time; v is the constant velocity of a moving force; $\delta(x)$ is the Dirac delta function; T is the time of load traverse over the beam.

On the basis of relations (10) and (12), the time functions (10) for a moving concentrated force take the form

$$S_{in}(t) = b_{in}\omega_{in}^{-1} \int_0^t F(s)\sin(p_n s)\sin[\omega_{in}(t - s)]\mathrm{d}s, \tag{27}$$

where

$$b_{1n} = 2a_{2n}[(a_{2n} - a_{1n})M_1]^{-1}, \quad b_{2n} = 2a_{1n}[(a_{1n} - a_{2n})M_1]^{-1},$$
$$p_n = k_n v = l^{-1}n\pi v = n\pi T^{-1}, \quad i = 1, 2, \quad n = 1, 2, 3, \ldots \ .$$

Case 2.1: *Moving constant force.* The classical problem of a moving constant concentrated force $F(t) = F$ at constant velocity [1,5–7,27,35,37] is discussed. The exciting loading of a double-beam system is $f_1(x, t) = F\delta(x - vt)$, $f_2(x, t) = 0$, $0 < t < T$, $T = lv^{-1}$, where F is the magnitude of a constant force. After calculation of the time functions (27), the forced vibrations of beams (11) are described by the expressions

$$w_1(x, t) = \sum_{n=1}^{\infty} \sin(k_n x)\left[A_{1n}\sin(p_n t) + \sum_{i=1}^{2} B_{in}\sin(\omega_{in} t)\right],$$
$$w_2(x, t) = \sum_{n=1}^{\infty} \sin(k_n x)\left[A_{2n}\sin(p_n t) + \sum_{i=1}^{2} a_{in}B_{in}\sin(\omega_{in} t)\right], \tag{28}$$

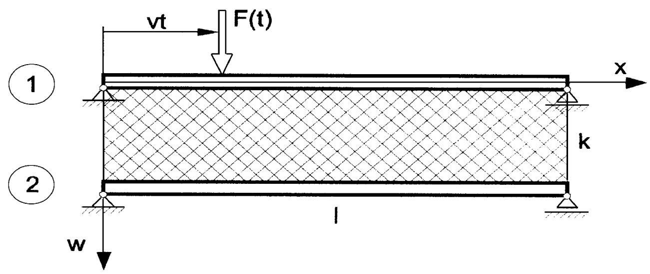

Fig. 5. An elastically connected complex double-beam system subjected to a moving concentrated force.

where

$$\begin{aligned} A_{1n} &= 2FM_1^{-1}(\Omega_{22n}^2 - p_n^2)[(\omega_{1n}^2 - p_n^2)(\omega_{2n}^2 - p_n^2)]^{-1}, \\ A_{2n} &= 2FK^{-1}\Omega_{120}^4[(\omega_{1n}^2 - p_n^2)(\omega_{2n}^2 - p_n^2)]^{-1}, \\ B_{1n} &= 2a_{2n}FM_1^{-1}p_n[(a_{1n} - a_{2n})\omega_{1n}(\omega_{1n}^2 - p_n^2)]^{-1}, \\ B_{2n} &= 2a_{1n}FM_1^{-1}p_n[(a_{2n} - a_{1n})\omega_{2n}(\omega_{2n}^2 - p_n^2)]^{-1}, \\ p_n &= k_n v = l^{-1}n\pi v = n\pi T^{-1}, \quad T = lv^{-1}, \quad 0<t<T. \end{aligned} \tag{29}$$

Case 2.2: *Moving harmonic force.* The second interesting problem of a moving concentrated force is the uniform motion of a harmonic force $F(t) = F\sin(pt)$ [1,5,7,36,43]. The exciting loading of a double-beam system is now the following: $f_1(x,t) = F\sin(pt)\delta(x - vt)$, $f_2(x,t) = 0$, $0<t<T$, $T = lv^{-1}$, where F and p are the amplitude and frequency of the exciting harmonic force, respectively.

The forced vibrations of a two-beam system are expressed by the relations

$$\begin{aligned} w_1(x,t) &= \sum_{n=1}^{\infty} \sin(k_n x)\left[A_{1n}\sin(p_n t)\sin(pt) + B_{1n}\cos(p_n t)\cos(pt) + \sum_{i=1}^{2} C_{in}\sin(\omega_{in} t)\right], \\ w_2(x,t) &= \sum_{n=1}^{\infty} \sin(k_n x)\left[A_{2n}\sin(p_n t)\sin(pt) + B_{2n}\cos(p_n t)\cos(pt) + \sum_{i=1}^{2} a_{in}C_{in}\sin(\omega_{in} t)\right], \end{aligned} \tag{30}$$

where

$$\begin{aligned} A_{1n} &= 2FM_1^{-1}(a_{1n}m_{1n}n_{1n}u_{2n} - a_{2n}m_{2n}n_{2n}u_{1n})[(a_{1n} - a_{2n})m_{1n}m_{2n}n_{1n}n_{2n}]^{-1}, \\ A_{2n} &= 2FM_2^{-1}(m_{1n}n_{1n}u_{2n} - m_{2n}n_{2n}u_{1n})[(a_{2n} - a_{1n})m_{1n}m_{2n}n_{1n}n_{2n}]^{-1}, \\ B_{1n} &= 4FM_1^{-1}pp_n(a_{1n}m_{1n}n_{1n} - a_{2n}m_{2n}n_{2n})[(a_{2n} - a_{1n})m_{1n}m_{2n}n_{1n}n_{2n}]^{-1}, \\ B_{2n} &= 4FM_2^{-1}pp_n(m_{1n}n_{1n} - m_{2n}n_{2n})[(a_{1n} - a_{2n})m_{1n}m_{2n}n_{1n}n_{2n}]^{-1}, \\ C_{1n} &= 4a_{2n}FM_1^{-1}m_{2n}n_{2n}pp_n[(a_{2n} - a_{1n})m_{1n}m_{2n}n_{1n}n_{2n}]^{-1}, \\ C_{2n} &= 4a_{1n}FM_2^{-1}m_{1n}n_{1n}pp_n[(a_{1n} - a_{2n})m_{1n}m_{2n}n_{1n}n_{2n}]^{-1}, \end{aligned}$$

$$m_{in} = \omega_{in}^2 - (p_n - p)^2, \quad n_{in} = \omega_{in}^2 - (p_n + p)^2, \quad u_{in} = \omega_{in}^2 - p_n^2 - p^2,$$

$$i = 1,2, \quad n = 1,2,3,\ldots, \quad p_n = k_n v = l^{-1}n\pi v = n\pi T^{-1}, \quad T = lv^{-1}, \quad 0<t<T.$$

4. Numerical example

To illustrate theoretical considerations presented, the forced vibrations of a double-beam system depicted in Fig. 3 due to an harmonic uniformly distributed load (see Case 1.1) are discussed in detail. The exciting loading of the system is $f_1(x,t) = f\sin(pt)$, $f_2(x,t) = 0$, where f and p are the amplitude and frequency of exciting harmonic load.

For simplicity of the analysis, it is assumed that both beams are geometrically and physically identical, then the values of the parameters from reference [11] can be used in the numerical

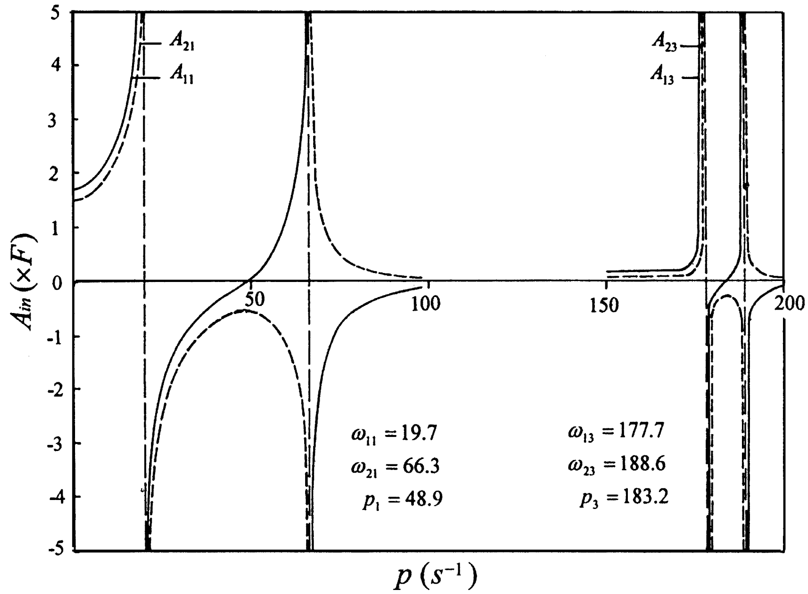

Fig. 6. The resonant diagram of the steady state forced harmonic vibrations of an elastically connected complex double-beam system subjected to uniform distributed harmonic load.

calculations:

$$E = E_i = 1 \times 10^{10}\ \mathrm{N\,m^{-2}}, \quad F = F_i = 5 \times 10^{-2}\ \mathrm{m^2}, \quad J = J_i = 4 \times 10^{-4}\ \mathrm{m^4}, \quad i = 1,2,$$
$$K_0 = K_i = E_i J_i = 4 \times 10^6\ \mathrm{N\,m^2}, \quad K = kl, \quad k = 2 \times 10^5\ \mathrm{N\,m^{-2}}, \quad l = 10\ \mathrm{m},$$
$$M = M_i = m_i l = 1 \times 10^3\ \mathrm{kg}, \quad m = m_i = \rho_i F_i = 1 \times 10^2\ \mathrm{kg\,m^{-1}}, \quad \rho = \rho_i = 2 \times 10^3\ \mathrm{kg\,m^{-3}}.$$

The steady state harmonic responses of the system are described by relations (21)

$$w_1(x,t) = \sin(pt) \sum_{(n)} A_{1n} \sin(k_n x), \quad w_2(x,t) = \sin(pt) \sum_{(n)} A_{2n} \sin(k_n x), \quad n = 1,3,5,\ldots,$$

where

$$A_{1n} = 4F(Mn\pi)^{-1}(\Omega_{22n}^2 - p^2)[(\omega_{1n}^2 - p^2)(\omega_{2n}^2 - p^2)]^{-1},$$
$$A_{2n} = 4F(Kn\pi)^{-1}\Omega_{120}^4[(\omega_{1n}^2 - p^2)(\omega_{2n}^2 - p^2)]^{-1},$$
$$\Omega_{22n}^2 = (K_0 k_n^4 + k)m^{-1} = [K_0 l^{-3}(n\pi)^4 + K]M^{-1} = 0.5(\omega_{1n}^2 + \omega_{2n}^2),$$
$$\Omega_{120}^4 = k^2 m^{-2} = K^2 M^{-2}, \quad \omega_{1n}^2 = K_0 k_n^4 m^{-1}, \quad \omega_{2n}^2 = (K_0 k_n^4 + 2k)m^{-1},$$
$$F = fl, \quad i = 1,2, \quad K = kl, \quad k_n = l^{-1} n\pi, \quad n = 1,3,5,\ldots\ .$$

These solutions are expressed only by the symmetric mode shapes of vibration [11] because of the symmetry of the applied load. Performing calculations of component amplitudes A_{1n} and A_{2n} versus the exciting frequency p, the resonant diagram can be built. The resonant diagram shown in Fig. 6 characterizes the progress of steady state forced harmonic vibrations of the system, and comprises the first two resonance curves. The full lines 11, 13 represent the amplitudes of the first beam vibration components A_{11}, A_{13}, and the broken lines 21, 23 describe the amplitudes of the second beam vibration components A_{21}, A_{23}. The resonances take place when the excitation frequency of harmonic load is equal to the one of the natural frequencies of the system $p = \omega_{11}, \omega_{21}, \omega_{13}, \omega_{23}$, then the corresponding amplitudes $A_{11}, A_{21}, A_{13}, A_{23}$ tend to infinity. The frequencies p_1, p_3 denote the tuned exciting frequencies for which the dynamic vibration absorption is realized, and the amplitudes A_{11}, A_{13} are suppressed. These frequencies are evaluated from the condition of dynamic vibration absorption (22)

$$p_n^2 = \Omega_{22n}^2 = [K_0 l^{-3}(n\pi)^4 + K]M^{-1} = 0.5(\omega_{1n}^2 + \omega_{2n}^2),$$

which leads to the beam amplitudes

$$A_{1n} = 0, \quad A_{2n} = -4F(Kn\pi)^{-1}, \quad n = 1, 3, 5, \dots\ .$$

The dynamic absorption phenomenon is of great practical importance and can be applied to reduce forced harmonic vibrations of elastically connected double-beam systems.

5. Conclusions

The undamped forced transverse vibrations of an elastically connected complex simply supported double-beam system have been considered. The modal expansion method is applied to receive the dynamic response of beams caused by arbitrarily distributed continuous loads. General solutions obtained are used to determine forced vibrations for several interesting cases of stationary exciting harmonic loadings and moving concentrated forces. Analyzing responses due to exciting harmonic forces the conditions of resonance and dynamic vibration absorption are formulated. Tuning parameters found can be employed to optimum design of a dynamic absorber of the beam type. The beam-type dynamic absorber is a new concept of a continuous dynamic vibration absorber (CDVA), which can be used to suppress excessive vibrations of corresponding beam systems. It is well known that DVAs are of great practical importance in engineering applications [3,4,7,9,45–47], and among them CDVAs play considerable role. Many recent studies have been devoted to beam-type [15,23,28–31,33–36, 38–41], string-type [35,41,43], membrane-type [35,48], plate-type [35,49], and shell-type [50] continuous dynamic vibration absorbers.

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