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Letter to the Editor

Sound radiation from a baffled rectangular plate under a variable line constraint

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1. Introduction

Sound radiation from plates in baffles, and shells is a well-studied subject. More recently, researchers are trying to optimize these structures for minimizing sound radiation [1]. A few typical parameters which are optimized include material tailoring [2], location and number of point masses attached to the structure [3], dynamic compliance [4], and damping layer placements [5]. These techniques include numerical methods such as BEM and FEM.

One of the situations faced in a typical industry is quietening of a radiating panel set into vibration by various forces. The engineer is often with the challenge of performing a quick study and coming up with a solution which does not involve extensive computations, expensive packages and time. There needs to be a quick method to study the influence of a line support under various orientations and find the optimum angle. This by no means downplays the importance of FEM/BEM techniques which are very much required for complex problems, and specially in the design stage. However, in an industry scenario, a quick insight may become important, for which an approximate closed-form approach is attractive. Thus, the salient feature of this paper is the simplicity with which a constraint can be implemented analytically, which affords quick physical insights and helps make a quick decision.

A point-force-driven simply supported rectangular plate set in an infinite baffle is subjected to a line constraint. The line constraint is approximated by attaching infinitely stiff springs along the intended line (shown in Fig. 1). Receptance method [6–8] is used to arrive at the new natural frequencies and mode shapes. The method has the advantage that the new mode shapes (and natural frequencies) are expressible in terms of the original mode shapes (and natural frequencies) and the strategy is easily programmable in a computational package like MATLAB.

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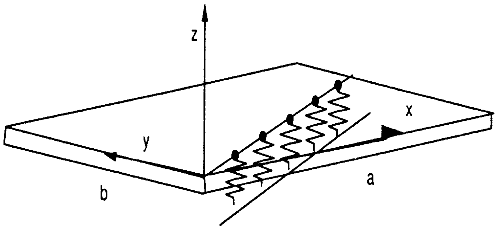

Fig. 1. Five springs attached to a plate along a straight line.

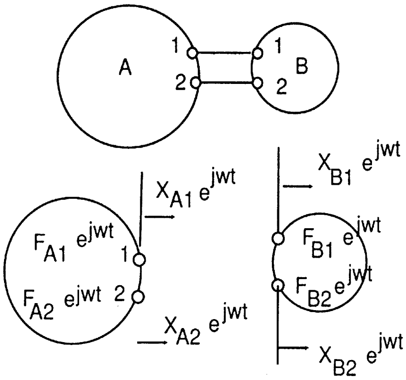

Fig. 2. Two structures A and B, connected at two points 1 and 2.

2. Theory

The receptance method is well developed and a detailed description for plates can be found in Ref. [7]. The method is described in brief here. Given the two structures connected at the two points shown in Fig. 2, the displacement and force relationships for structure A are given by

$$\begin{bmatrix} X_{A1} \\ X_{A2} \end{bmatrix} = \begin{bmatrix} \alpha_{11}\alpha_{12} \\ \alpha_{21}\alpha_{22} \end{bmatrix} \begin{bmatrix} F_{A1} \\ F_{A2} \end{bmatrix}, \tag{1}$$

where X_{A1} and X_{A2} are the displacements at locations A1 and A2, respectively. F_{A1} and F_{A2} are forces at the same locations applied to structure A. Similarly, the equations for structure B are given by

$$\begin{bmatrix} X_{B1} \\ X_{B2} \end{bmatrix} = \begin{bmatrix} \beta_{11}\beta_{12} \\ \beta_{21}\beta_{22} \end{bmatrix} \begin{bmatrix} F_{B1} \\ F_{B2} \end{bmatrix}. \tag{2}$$

Thus, α_{ij} and β_{ij}, $i,j = 1,2$ are the drive point and cross receptances, which have the units of displacement per unit force. α_{11} is the displacement at point 1 due to a unit force at point 1, and α_{12} is the displacement at point 1 due to a unit force at point 2. When two such systems are joined together, the forces F_A and F_B become internal forces and they have to add to zero, and the displacements have to be equal. Thus,

$$\{F_A\} = -\{F_B\} \tag{3}$$

and

$$\{X_A\} = \{X_B\}, \tag{4}$$

where the curly brackets indicate force and displacement vectors. By combining the two equations the following expressions is obtained:

$$(\alpha_{11} + \beta_{11})F_{A1} + (\alpha_{12} + \beta_{12})F_{A2} = 0,$$
$$(\alpha_{21} + \beta_{21})F_{A1} + (\alpha_{22} + \beta_{22})F_{A2} = 0. \tag{5}$$

In general,

$$[[\alpha] + [\beta]]\{F_A\} = 0. \tag{6}$$

$F_{A1} = F_{A2} = 0$ being the trivial solution, the non-trivial solution is found by setting the determinant

$$\begin{vmatrix} \alpha_{11} + \beta_{11} & \alpha_{12} + \beta_{12} \\ \alpha_{21} + \beta_{21} & \alpha_{22} + \beta_{22} \end{vmatrix} = 0. \tag{7}$$

Thus, one needs to know the α's and the β's of the two structures. In our problem, a simply supported plate is connected to springs. So let the α's belong to the plate and the β's to the springs. The 2×2 receptance matrix $[\alpha]$ for the plate is derived below.

2.1. Receptance matrix for a simply supported plate

The response at a point x, y of a simply supported plate due to a harmonic point force at x_p, y_p is given by Ref. [7]:

$$W(x, y, t) = \frac{1}{\rho h N_{mn}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(m\pi x_p/a)\sin(n\pi y_p/b)F\mathrm{e}^{\mathrm{j}\omega t}}{\omega_{mn}^2 - \omega^2} U_{mn}, \tag{8}$$

where F is the force amplitude, a the x length of the plate, b the y length of the plate, ρ the plate density, h the thickness of the plate, ω_{mn} the (m, n)th natural frequency in rad/s, and ω is the driving frequency. The mode shapes of a simply supported plate are given by

$$U_{mn} = \sin(m\pi x/a)\sin(n\pi y/b) \tag{9}$$

and

$$N_{mn} = \int_0^a \int_0^b U_{mn}^2 \,\mathrm{d}x\,\mathrm{d}y, \tag{10}$$

which is $ab/4$ for a simply supported plate.

If λ is the equivalent viscous damping factor, the modal damping coefficient is given by

$$\xi_{mn} = \frac{\lambda}{2\rho h \omega_{mn}} \tag{11}$$

and the damped response is given by

$$W(x,y,t) = \frac{1}{\rho h N_{mn}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(m\pi x_p/a)\sin(n\pi y_p/b) F e^{j\omega t}}{\omega_{mn}^2 - \omega^2 + 2j\xi_{mn}\omega_{mn}\omega} U_{mn}. \tag{12}$$

Thus, α_{11} is given by (using Eq. (10) for N_{mn})

$$\begin{aligned}\alpha_{11} &= \frac{X_1}{F_1 e^{j\omega t}} \\ &= \frac{4}{\rho hab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(m\pi x_1/a)\sin(n\pi y_1/b) F e^{j\omega t}}{\omega_{mn}^2 - \omega^2} \sin(m\pi x_1/a)\sin(n\pi y_1/b),\end{aligned} \tag{13}$$

and α_{12} by

$$\begin{aligned}\alpha_{12} &= \frac{X_1}{F_2 e^{j\omega t}} \\ &= \frac{4}{\rho hab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(m\pi x_2/a)\sin(n\pi y_2/b) F e^{j\omega t}}{\omega_{mn}^2 - \omega^2} \sin(m\pi x_1/a)\sin(n\pi y_1/b).\end{aligned} \tag{14}$$

Similarly, the other α's can be found.

2.2. The receptance for a spring

The receptance for a spring is obtained from the equation for spring dynamics:

$$X_B = \frac{F_B e^{j\omega t}}{K_B}. \tag{15}$$

Thus,

$$\beta = \frac{X_B}{F_B e^{j\omega t}} = \frac{1}{K_B}. \tag{16}$$

For the case in this paper where the plate is connected to discrete springs, the cross receptances for the springs are zero, since the force on one spring does not cause the other spring to respond.

2.3. Receptance for plate attached to springs

For a plate connected to two springs, by using Eq. (7), the receptance matrix is given by

$$\begin{vmatrix} \alpha_{11} + \dfrac{1}{K_{B1}} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} + \dfrac{1}{K_{B2}} \end{vmatrix} = 0. \tag{17}$$

The structure of the matrix now can be extended to the case where N springs are attached along a line (as in Fig. 1). The stiffness of the springs is now set to infinity creating a "line" support. The determinant of the $N \times N$ receptance matrix set to zero gives the new natural frequencies of the plate under this line constraint. The new mode shapes of the plate can be determined from the point response expression of the original plate [7,8]. For the case of a single spring, Eq. (8) gives the new mode shape, when ω the excitation frequency is set to the new natural frequency and x_p, y_p are the spring attachment co-ordinates. For an $N \times N$ matrix, there will be N roots which are the new resonances. A similar method as for the single spring works, except that here, since the plate is constrained at N points, it will experience point forces at those locations. The magnitudes of these point forces are given by the elements of the eigenvector corresponding to the zero eigenvalue of the receptance matrix evaluated at the new natural frequency ω_k. Thus, the new kth mode shape is given by substituting ω_k for ω in Eq. (8) with an additional summation term as follows:

$$U_k(x, y) = \frac{4}{\rho hab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sum_{i=1}^{N} \sin(m\pi x_i/a)\sin(n\pi y_i/b)F_{ik}}{\omega_{mn}^2 - \omega_k^2} U_{mn}, \tag{18}$$

where F_{ik} is the ith element of the eigenvector (of the zero eigenvalue) corresponding to the kth new natural frequency and x_i, y_i are the location of the ith spring.

The response of the constrained plate to a point force can again be calculated as

$$W(x, y, t) = \frac{1}{\rho h} \sum_{k=1}^{\infty} \frac{1}{N_k} \frac{U_k(x_i, y_i)F e^{j\omega t}}{\omega_k^2 - \omega^2 + 2j\xi_k\omega_k\omega} U_k(x, y), \tag{19}$$

where F is the force amplitude at location x_i, y_i, ω_k the kth natural frequency in rad/s, ξ_k the modal damping coefficient (calculated as in Eq. (11)) and ω is the driving frequency. U_k is the new mode shape given by Eq. (18). And

$$N_k = \int_0^a \int_0^b U_k^2 \, dx \, dy. \tag{20}$$

2.4. Radiated sound power

The radiated sound power expression for a rectangular plate set in a baffle was derived by Ref. [5]. The power is calculated by an integral of sound intensity over the plate surface area. The whole expression turns out to be in terms of plate velocities. Only the final expression is given below:

$$W_p = \frac{\rho_a \omega}{4\pi} \int_0^a v_n^*(x', y') \int_0^b v_n(x, y) \frac{\sin(k\,|\sqrt{x'^2 + y'^2} - \sqrt{x^2 + y^2}|)}{|\sqrt{x'^2 + y'^2} - \sqrt{x^2 + y^2}|} \, dx \, dy \, dx' \, dy', \tag{21}$$

where $v_n(x, y)$ is the velocity at point (x, y) on the plate and $v_n(x', y')$ is the velocity at point (x', y') on the plate. ρ_a is the density of air, k is the wavenumber.

3. Procedure

A plate with dimensions and properties given in Table 1 is attached with five springs along a line starting with an angle of 20° to x-axis and going through the origin (see Fig. 1).

The angle is varied in increments of 5° till 70°. The point force is located at $x = 0.0326$, $y = 0.056$. The excitation frequency is 1000 rad/s which is an off-resonance frequency of the original plate. The new resonances, mode shapes and velocity response are computed by using

Table 1

E (N/m^2)	ρ (kg/m^3)	a (m)	b (m)	h (m)	μ	λ
20.7e10	7850	0.326	0.56	0.002	0.3	0.01

Table 2

Constraint angle (deg)	Method	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
20	FEM	70.398	136.55	205.75	237.4	273.96
	Receptance	70.574	137.37	206.29	239.24	275.66
25	FEM	72.65	143.38	208.2	246.92	286.38
	Receptance	72.78	143.84	208.46	248.36	287.35
60	FEM	156.45	177.76	282.34	320.7	409.15
	Receptance	156.63	178.25	282.61	316.06	401.24
65	FEM	145.53	208.31	260.24	363.66	368.35
	Receptance	145.78	208.46	260.43	361.79	367.01
70	FEM	129.70	197.53	275.88	337.72	354.66
	Receptance	130.09	197.93	275.91	338.46	356.37

All values in Hz.

Eq. (17), (18) and (19), respectively. Eq. (21) is then used to compute sound power at each of the angles. Three-point Gaussian quadrature is used to compute power after dividing the plate into 10×10 equal segments. A few resonances of the new modes for different constraint angles are given in Table 2.

4. Results and discussion

A few modes along with the line constraint are shown in Fig. 3. The first, second and third rows are for constraints at 20°, 40° and 70°, respectively. And Fig. 4 shows response of the second mode at the 70° constraint angle (1243 rad/s). The point-marked line is the response along the line constraint. And the other lines are responses at other constant "y" locations. The constraint is at an angle to the x-axis hence it stops short of the full length of the plate. As is evident, the mode shape has no response at the spring locations and low response along the line constraint. The springs are located close enough that radiation to the sinusoidal response along the line should mostly cancel. It is important to note that the new resonances should be computed accurately. Else the mode shapes will be in error. Table 2 shows the comparison between the ANSYS computed resonances and the current method for a few constraint angles. In ANSYS, the vertical displacement for the nodes along the line constraint was set to zero. Every time the constraint angle was changed, the element aspect ratios were changed accordingly so that nodes would lie on the constraint.

Fig. 5 shows the computed sound power for the different constraint angles. The power is maximum at the constraint angle of 35°. The second mode at this angle is at 1020 rad/s which

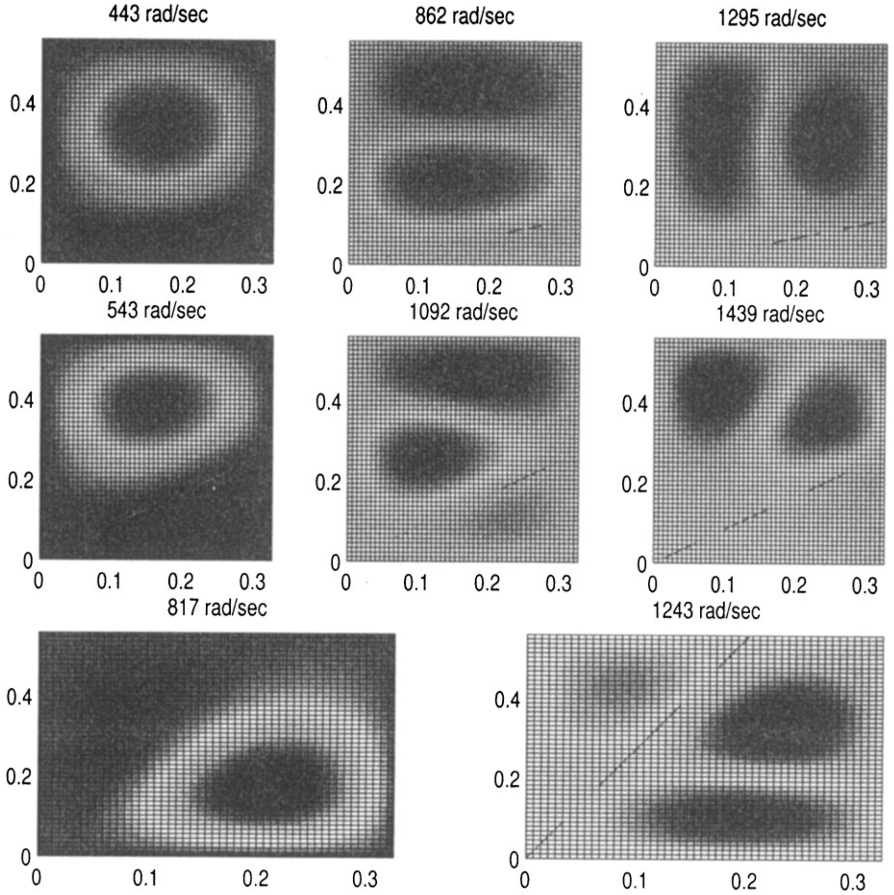

Fig. 3. New modes for three constraint angles: 1st row, 20°; 2nd row, 40°; 3rd row, 70° to the x-axis.

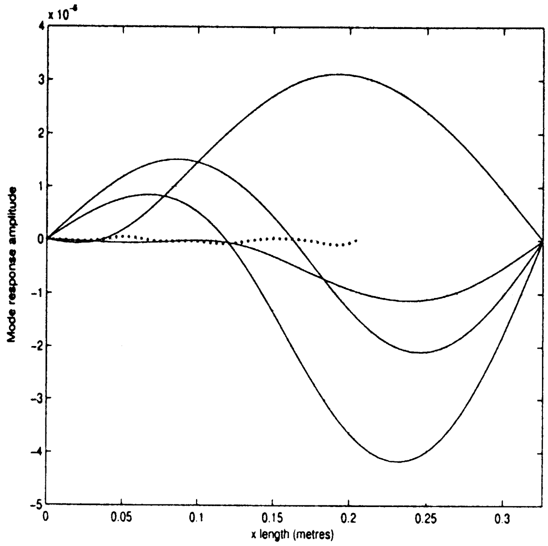

Fig. 4. Response along the constraint and at other points for the 1243 rad/s mode at 70° constraint angle.

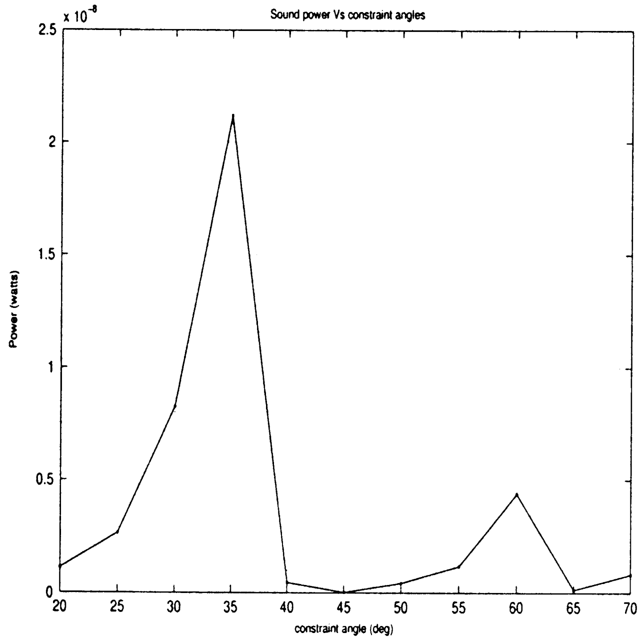

Fig. 5. Sound power as a function of constraint angle for a point-excited simply supported plate in a baffle. Frequency = 1000 rad/s.

results in a high response. However, at 30°, 40° and 60°, there are resonant modes with frequencies 956, 1092 and 984 rad/s, respectively. The resonances at 956 and 1092 are quite away from 1000. The last one (984 rad/s) is only 16 rad/s away from the driving frequency. Yet it radiates less. The reason for low radiation is that at this angle, the constraint is exactly along the plate diagonal and hence divides into two halves. The mode has the two halves out of phase, like a dipole. Since 1000 rad/s is below critical frequency, the two halves exactly cancel. Whereas at 35°, the mode has unequal out-of-phase areas which do not cancel. If the problem is posed as a sound minimization problem, the primary mechanism would be distancing the new resonances from the driving frequency as is the case for 20° and 70°, except for cases like the 984 rad/s resonance.

5. Conclusion

In conclusion, a line constraint was approximated on a simply supported plate using infinitely stiff springs attached along a line and sound power calculated for a point-driven case. The angle of the constraint was varied and the minimum radiation angle found. It was shown to be a valid approximation without use of extensive FEM/BEM programs. The programs were written in MATLAB. This is a quick and useful method for engineers in an industrial scenario where time is a constraint sometimes.

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