Available online at www.sciencedirect.com

SCIENCE DIRECT

Journal of Sound and Vibration 265 (2003) 401–415

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Non-linear control of torsional and bending vibrations of oilwell drillstrings

S.A. Al-Hiddabi*, B. Samanta, A. Seibi

Department of Mechanical and Industrial Engineering, Sultan Qaboos University, P.O.Box 33, Al-Khod 123, Oman

Received 24 July 2001; accepted 16 July 2002

Abstract

Drillstring dynamics is highly non-linear in nature and its model can only be described by a set of non-linear differential equations. In addition to this complexity, the drillstring dynamics are not linearly controllable and thus linear control methods are not suitable for suppressing the coupled torsional and lateral vibrations of a rotating drillstring. In this paper a non-linear dynamic inversion control design method is used to suppress the lateral and the torsional vibrations of a non-linear drillstring. It was found that the designed controller is effective in suppressing the torsional vibrations and reducing the lateral vibrations significantly.

1. Introduction

Searching for oil and gas beneath the earth requires the use of rotary drilling systems. In the drilling process a rotary system creates a borehole by a rock-cutting tool, called a bit. The rotation of the bit is energized by an electric motor placed on the surface of the earth. The motor drives a large disc-shaped inertia wheel, called the rotary table, located directly above the borehole. The centre of the rotary table is connected to a drillstring, mainly consisting of drill pipe segments coupled with threaded connections. The lower part of the drillstring is connected to the bottom hole assembly, which consists of drill collars with larger stiffness than the drillpipe and the bit to crush rocks.

The drillstring, which can be viewed as a long shaft (1–8 km), drives the bit at the bottom of the borehole. Intuitively such a long rotating system, which is in continuous interaction with the formation, is subjected to severe vibrations during the drilling process. Mathematical modelling of

*Corresponding author. Tel.: +968-515301; fax: +968-513416.

E-mail address: hiddabi@squ.edu.om (S.A. Al-Hiddabi).

0022-460X/03/$ - see front matter © 2002 Elsevier Science Ltd. All rights reserved.
doi:10.1016/S0022-460X(02)01456-6

such systems are highly non-linear and fairly complex due to the drillstring dynamics which involves axial, transverse and torsional modes of coupled motion. Furthermore, impact of the drillstring with the wellbore and the stick–slip phenomena of the bit adds another dimension of complexity to the drillstring dynamics. Stick–slip oscillations are examples of limit-cycling behaviour that often occur in mechanical systems due to hysteresis, backlashes between contacting parts, dry friction between sliding parts, non-linear damping, geometrical imperfections, etc. [1].

Non-linear phenomena such as parametric resonance, bit bounce, forward and backward whirl have all been shown to occur in oil well drillstrings. The excessive vibrations caused by the above phenomena were observed to cause damage to the drilling system. Such damage is related to large cyclic stresses, which lead to fatigue, failures and abrasive wear of tubular segments, drill bits and the borehole wall. As a consequence, the drilling process becomes inefficient and costly. Thus, vibrations of the drillstrings must be studied and their effects should be controlled to a minimum for the drilling process to be optimal and economical.

In drilling deep wells, torsional vibration becomes more apparent and detrimental especially during bit sticking, which induces a high variation in the induced stress levels, thereby reducing the fatigue life of the drillstring. Several attempts have been made to study the drillstring vibrations and to overcome the difficulties encountered by field engineers [2–8]. In Ref. [9], a two-degree-of-freedom mathematical model of a drillstring is used, which only captures the torsional dynamics, to design an H_∞ controller to minimize the torsional vibration. It has been shown that the self-excited stick–slip oscillations are reduced by the application of the linear time invariant H_∞ controller. However, it is not clear how this controller will influence the lateral and axial dynamics of the drillstring since the model used captures only the linear torsional dynamics. A similar approach for controlling the torsional vibration of drillstrings is also studied in Ref. [10], but with a different control strategy called active damping technique.

In a series of papers Yigit and Christoforou [11,7,12–14] developed and studied a non-linear mathematical model of drillstrings. In Ref. [7], the coupled axial and transverse vibration problem is studied and in Ref. [12] the coupled torsional and bending vibrations problem is investigated. Recently, an attempt to minimize the effect of torsional vibrations was studied [11,14], by designing a linear quadratic regulator (LQR) based on a linearized model. It was shown that the LQR controller is effective in minimizing the torsional vibration. However, it turns out [11] that the lateral vibration is not linearly controllable via the motor torque. This is due to the nature of the coupling terms between the lateral and torsional dynamics which vanish during linearization. Thus, linear control methods cannot be used to influence lateral and axial vibrations of drillstings. However, it is important to minimize the lateral vibrations since they cause excessive bit wear and reduce the penetration rate [15]. Clearly, the linear control methods cannot influence the lateral vibrations and no previous attempts, according to our knowledge, were made to use non-linear control methods to tackle the vibration problem of drillstrings.

The motivation for this paper is the fact that the non-linear treatment of drillstrings problem is essential since by nature the problem demands the use of a control strategy which is different from linear control methods. Thus, the controller should be designed and implemented based on a coupled non-linear model in order to predict the behaviour of the whole system. In this paper, the use of non-linear control is proposed to suppress the lateral and torsional vibrations of a non-linear drillstring model. The proposed control design method is based on dynamic inversion approach [16,1].

2. Drillstring dynamics model

The equations of motion for coupled torsional and bending dynamics of drillstring can be easily derived using Newton's law or Lagrangian dynamics. In this paper the drillstring model presented in Ref. [11] is used with some modifications. The equations of motion from Ref. [11] can be stated as

$$(m + m_f)(\ddot{r} - r\dot{\theta}^2) + k(\dot{\phi})r + c_h|\mathbf{v}|\dot{r} \\ = (m + m_f)e_0[\dot{\phi}^2 \cos(\phi - \theta) + \ddot{\phi}\sin(\phi - \theta)] - F_r, \quad (1)$$

$$(m + m_f)(r\theta + 2\dot{r}\dot{\theta}) + c_h|\mathbf{v}|r\dot{\theta} \\ = (m + m_f)e_0[\dot{\phi}^2 \sin(\phi - \theta) - \ddot{\phi}\cos(\phi - \theta)] + F_\theta \quad (2)$$

$$J\ddot{\phi} + k_T(\phi - \phi_{rt}) + c_v\dot{\phi} + c_h|\mathbf{v}|\dot{r}e_0 \sin(\phi - \theta) + c_h|\mathbf{v}|r\dot{\theta}e_0 \cos(\phi - \theta) \\ = -T(\dot{\phi}) + F_\theta[R - e_0 \cos(\phi - \theta)] - F_r e_0 \sin(\phi - \theta) \quad (3)$$

$$(J_{rt} + n^2 J_m)\ddot{\phi}_{rt} + k_T(\phi_{rt} - \phi) + c_{rt}\dot{\phi}_{rt} - nT_m = 0, \quad (4)$$

where r and θ are the radial and angular displacement of the geometric centre of the drill collar, respectively, ϕ is the angle of rotation of the drill collar (bottom of the drillstring) with respect to its centre of gravity, ϕ_{rt} is the angle of rotation of the rotary table (top of the drillstring), $\mathbf{v}$ is the velocity of the geometric centre of the drill collar section, e_0 is the eccentricity of the centre of mass with respect to the geometric centre of the drill collar section, R is the radius of the drill collars, $J, m, m_f, k(\dot{\phi}), k_T, c_h$ and c_v are the equivalent mass moment of inertia, mass, added fluid mass, transverse stiffness, torsional stiffness, hydrodynamic and viscous damping coefficients. J_{rt} and J_m are the inertia of the rotary table and the drive motor, respectively.

In this paper, the dynamics of the drive system are not included and a constant weight on bit (WOB) is assumed to keep the simulation model within a reasonable complexity. However, these modelling assumptions can be relaxed without much effect on the controller design approach. This is also due to the linear coupling of drive motor dynamics with that of the rotary table.

The torque on bit (TOB) is given by

$$T(\dot{\phi}) = T_o\left[\tan\dot{\phi} + \frac{\alpha_1\dot{\phi}}{(1 + \alpha_2\dot{\phi}^2)}\right], \quad (5)$$

where T_o, α_1 and α_2 are constants. Assuming one-mode approximation for both the transverse and torsional bendings, the equivalent system parameters are given by

$$J = 2\rho I_a l_2 + (1/3)\rho I_p l_3, \quad (6)$$

$$m = \rho\pi(d_o^2 - d_i^2)l_1/8, \quad (7)$$

$$m_f = \pi\rho_f(d_i^2 + c_a d_o^2)l_1/8, \quad (8)$$

$$k(\dot{\phi}) = \frac{EI_a\pi^4}{2l_1^3} - \frac{T(\dot{\phi})\pi^3}{2l_1^2} - \frac{F_o\pi^2}{2l_1}, \tag{9}$$

$$c_h = \frac{2}{3\pi}\rho_f C_d d_o l_1, \tag{10}$$

$$c_v = \frac{\pi\mu_f v o^3 l_2}{2(d_h - d_o)}, \tag{11}$$

$$k_T = \frac{GI_p}{l_3}, \tag{12}$$

where ρ is the density of the drillstring, $I_a = \pi(d_o^4 - d_i^4)/64$ is the area moment of inertia for the collar cross-section, and d_o and d_i are the outside and inside diameters of the drill collars, respectively. The pipe polar moment of inertia is given by $I_P = \pi(\bar{d}_o^4 - \bar{d}_i^4)/32$, where $\bar{d}_o$ and $\bar{d}_i$ are the outside and inside diameters of the drill pipe, respectively.

E and G are Young's and shear modulus of the drillstring materials, ρ_f and μ_f are the density and the viscosity of the drilling mud, c_a is the added mass coefficient due to the displaced mass of the mud inside the drillstring, c_d is the drag coefficient for the hydrodynamic damping due to the mud, and d_h is the borehole diameter.

The external excitation forces F_r and F_θ are the radial and transverse contact forces, respectively, resulting from impacts of the drill collars with borehole wall. If there is no contact these forces are zero. There are several methods to represent the contact forces mathematically, among them is the penalty formulation [17] and momentum balance equation [12]. In this paper a penalty formulation has been used as described in Ref. [10]:

$$F_r = -k_w(r - c_0) - c_w\dot{r}, \tag{13}$$

$$F_\theta = -S\mu_c F_r, \tag{14}$$

where k_w, c_w represent the elastic and damping properties of the borehole wall and μ_c is the coefficient of friction between the collars and the borehole wall. The clearance c_0 is defined as $c_0 = \frac{1}{2}(d_h - d_o)$. The sign-parameter S is given by $S = \text{sign}(r\dot{\theta} + (d_o/2)\dot{\phi})$.

3. Non-linear control design

3.1. General theory of non-linear inverse dynamics

The objective of this Section is to review the techniques that can be applied to develop a non-linear controller for the drillstring system. The technique is based on the construction of a non-linear inverse dynamic controller described in Ref. [16], for a system of the form

$$\dot{x} = f(x) + g(x)u, \tag{15}$$

$$y = h(x), \tag{16}$$

where $f(x)$ and $g(x)$ are vector fields in R^n, u is the input and y is the output.

The control design process is to find an integer ρ and a state feedback

$$u = \alpha(x) + \beta(x)v, \tag{17}$$

where v is a new control variable, α and β are smooth functions defined in the neighbourhood of some point $x_0 \in R^n$ and $\beta(x_0) \neq 0$, such that the closed-loop system (15)–(17)

$$\dot{x} = f(x) + g(x)(\alpha(x) + \beta(x)v), \tag{18}$$

$$y = h(x) \tag{19}$$

has the property that the ρth order derivative of the output is given by

$$y^{\rho} = v, \quad t \in \Gamma, \tag{20}$$

where Γ is an open interval containing $t = 0$. This problem is termed as (local) *input–output feedback linearization*. The point x_0 around which the linearization is performed is called the *analysis point*.

The above idea can be implemented by successively differentiating the output $y = h(x)$ as

$$y^0 = h(x), \tag{21}$$

$$y^1 = L_f h(x), \tag{22}$$

$$\vdots$$

$$y^{\rho-1} = L_f^{\rho-1} h(x), \tag{23}$$

$$y^{\rho} = L_f^{\rho} h(x) + L_g L_f^{\rho-1} h(x) u, \tag{24}$$

where $L_f^k h(x)$ is called the *Lie derivative* of $L_f^{k-1} h(x)$ along the vector field f. Note that by choosing the control u in Eq. (24)

$$u = \frac{v - L_f^{\rho} h(x)}{L_g L_f^{\rho-1} h(x)}. \tag{25}$$

Provided that the integer ρ exits and $L_g L_f^{\rho-1} h(x) \neq 0$ in the neighbourhood of x_0, Eq. (20) can easily obtained. In this case the system has a relative degree ρ in the neighbourhood of x_0. The functions $\alpha(x)$ and $\beta(x)$ of Eq. (17) can be obtained directly from Eq. (25) as

$$\alpha(x) = -\frac{L_f^{\rho} h(x)}{L_g L_f^{\rho-1} h(x)}, \tag{26}$$

$$\beta(x) = \frac{1}{L_g L_f^{\rho-1} h(x)}. \tag{27}$$

From Eq. (20), it can be seen that the inversion-based control law (25) has the capacity in shaping the output response by simply designing the new control v to get the desired output. However, since the inversion-based control law is only based on the system's input–output dynamics, it may fail to result in a stable closed-loop system. This can happen if the controlled system is non-minimum phase [16].

3.2. Application of non-linear inverse dynamics of drillstring dynamics

In this Section the theory of non-linear inverse dynamics is applied to the problem of controlling the coupled torsional and lateral vibration of the drillstring dynamics described by Eqs. (1)–(4).

The control input for the drillstring system is the motor torque T_m and accordingly only one output can be controlled independently.

In Ref. [11] a linear controller was designed to suppress the torsional vibration of drillstrings by ignoring the lateral dynamics during the design phase. Simulations of the full closed-loop system indicates that the lateral vibration is minimized due to the suppression of the torsional vibration. Thus from the above it seems logical to choose the output from the torsional dynamics. The table speed $\dot{\phi}_{rt}$ and the bit speed $\dot{\phi}$ both seem to be candidates for being an output for the drillstring system. In this paper, both outputs are considered independently and controllers are developed for each case using the non-linear inverse theory. The effect of such a controller on the lateral and torsional vibration of the drillstring are studied using computer simulations. The main contribution of this paper is that the controller is designed without ignoring the lateral dynamics.

3.2.1. Table speed controller

In this Section a non-linear controller is designed to track a desired table speed $\dot{\phi}_{rt_d}$. This can be achieved by considering the table speed $\dot{\phi}_{rt}$ as an output for system (1)–(4). Note that by differentiating the output $y = \dot{\phi}_{rt}$ once, the input T_m appears explicitly and this is obvious from Eq. (4). Thus the relative degree of this output is one. Following the approach described in Section 3, the output is given by

$$y = h(x) = \dot{\phi}_{rt}. \tag{28}$$

Differentiating (28) once and using Eq. (4) gives

$$\dot{y} = (-1/(J_{rt} + n^2 J_m))\left[c_{rt}\dot{\phi}_{rt} + k_T(\phi_{rt} - \phi)\right] + (n/(J_{rt} + n^2 J_m))T_m. \tag{29}$$

Eq. (29) is in the form of Eq. (24), where u in this case is T_m. The inverse control according to Eq. (25) is given by

$$T_m = \frac{(J_{rt} + n^2 J_m)}{n}\left[v + \frac{1}{(J_{rt} + n^2 J_m)}(c_{rt}\dot{\phi}_{rt} + k_T(\phi_{rt} - \phi))\right]. \tag{30}$$

Note that in this case the relative degree is one ($\rho = 1$) and the input–output relations are linear and decoupled from the rest of the system. The system in normal form is given by

$$\dot{y} = v. \tag{31}$$

The new control variable v in Eq. (31) can be designed to achieve any desired table speed $\dot{\phi}_{rt_d}$. Choosing v as

$$v = \ddot{\phi}_{rt_d} - k_1(\dot{\phi}_{rt} - \dot{\phi}_{rt_d}), \tag{32}$$

guarantees asymptotic tracking of the desired table speed provided that the constant k_1 is positive. The closed-loop system defined by Eqs. (31) and (32) written in error co-ordinates is given by

$$\dot{e} = -k_1 e, \tag{33}$$

where $e = \dot{\phi}_{rt} - \dot{\phi}_{rt_d}$. Note that the original control T_m can be calculated from Eq. (30) after substituting for v from Eq. (32).

3.2.2. Bit speed controller

In this Section a non-linear controller is designed to track a desired bit speed $\dot{\phi}_d$. This can be achieved by considering the bit speed $\dot{\phi}$ as an output for system (1)–(4). Note that by differentiating the output $y = \dot{\phi}$ three times, the input T_m will appear explicitly. Thus the relative degree in this case is three. Following the approach described in Section 3,

$$y = h(x) = \dot{\phi} \tag{34}$$

differentiating (34) three times and using Eqs. (1)–(4) gives

$$\dddot{y} = f(x) + g(x)T_m, \tag{35}$$

where

$$\begin{aligned} g(x) = -\Bigg\{ & \|v\|[c_h r\dot{\theta}e_0 \cos(\theta - \phi)\dot{r} + c_h \dot{r}^2 e_0 \sin(\theta - \phi) \\ & + c_h v^2 e_0 \sin(\theta - \phi)]\frac{e_0}{J^2} k_t \sin(\theta - \phi) \\ & + \|v\|[c_h r^3 \dot{\theta}^2 e_0 \cos(\theta - \phi) + c_h v^2 r e_0 \cos(\theta - \phi) \\ & + c_h \dot{r} e_0 \sin(\theta - \phi) r^2 \dot{\theta}]\frac{e_0}{J^2 r} k_t \cos(\theta - \phi) \\ & - \left[c_v - T_o\left[\sec \dot{\phi}^2 + \frac{\alpha_1}{(1 + \alpha_2\dot{\phi}^2)} - 2\frac{\alpha_1\dot{\phi}^2\alpha_2}{(1 + \alpha_2\dot{\phi}^2)^2}\right]\right] \\ & \times \frac{k_t}{J^2} - \frac{k_t c_{rt}}{J(J_{rt} + n^2 J_m)}\Bigg\}\frac{n}{(J_{rt} + n^2 J_m)}. \end{aligned}$$

Note that $f(x)$ and $g(x)$ in Eq. (35) are complex functions of all states, and thus they include only terms of first derivative and below. The function $f(x)$ contains all the terms that are not part of the control input coefficient $g(x)$ and due to space limitation is not listed explicitly in the paper. An inverse controller that cancels the non-linear terms in Eq. (35) and tracks a desired bit speed $\dot{\phi}_d$ is designed as follows:

$$T_m = g^{-1}(x)(v - f(x)). \tag{36}$$

The new control input v is given by the following linear feedback law:

$$v = \ddddot{\phi}_d - k_1\left(\dddot{\phi} - \dddot{\phi}_d\right) - k_2\left(\ddot{\phi} - \ddot{\phi}_d\right) - k_3\left(\dot{\phi} - \dot{\phi}_d\right), \tag{37}$$

where k_1, k_2 and k_3 are constants selected so that the polynomial

$$\dddot{s} + k_1\ddot{s} + k_2\dot{s} + k_3 = 0$$

is Hurwitz.

3.2.3. Remarks on internal stability

The inversion-based control laws (30) and (37) are based on the system's input–output dynamics. Therefore, it may fail to guarantee closed-loop stability of the full system. In other words, the non-linear inversion-based control design mimics the pole-zero cancellation technique used in linear control methods. The cancelled modes in this case become unobservable and clearly if the cancellation involves unstable zeros, the resulting input–output system is stable but the observable part of the system is unstable. These types of systems are called non-minimum phase [16] and in general inversion-based control cannot be used on them. In order to check whether a nonlinear system, for a given output, is non-minimum phase or not; the stability of the

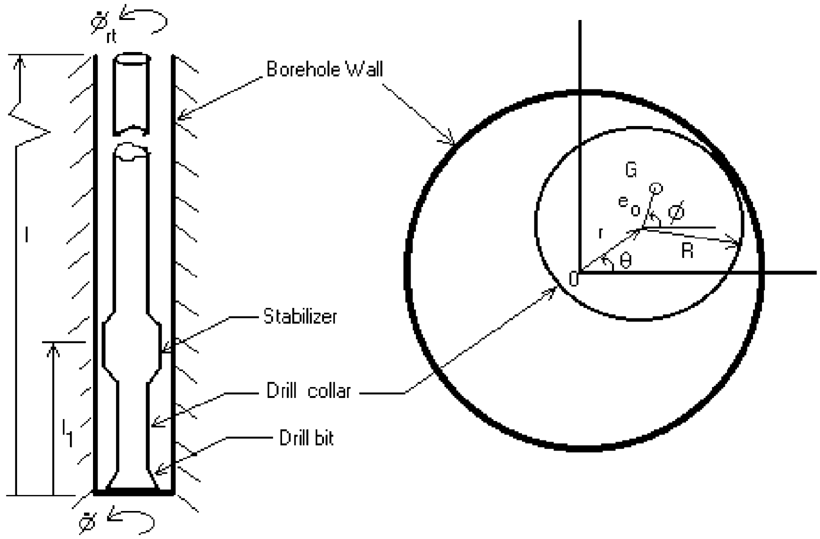

Fig. 1. System components and the geometry used for modelling.

Table 1
System parameters used in simulation

Drillstring		Drilling mud
$E = 210$ GPa		$\rho f = 1500\ \mathrm{kg/m^3}$
$\rho = 7850\ \mathrm{kg/m^3}$		$C_d = 1$
$d_o = 0.2286$		$C_a = 1.7$
$d_i = 0.0762$ m		$\mu_f = 0.2\ \mathrm{Ns/m^2}$
$E_o = 0.0127$ m		
$l_1 = 19.81$ m		Borehole
$l_2 = 200$ m		$E = 210$ Gpa
$l_3 = 2000$ m		$\rho = 7850\ \mathrm{kg/m^3}$
$\bar{d}_o = 0.127$ m		$d_h = 0.4286$ m
$\bar{d}_i = 0.095$ m		
Weight and torque on bit		
$P_0 = 100$ kN, $P_f = 50$ kN,		$T_0 = 4$ kNm
Borehole wall stiffness		
$k_w = 10^4 k$	$c_w = 338.4$ m	$\mu_c = 0.3$

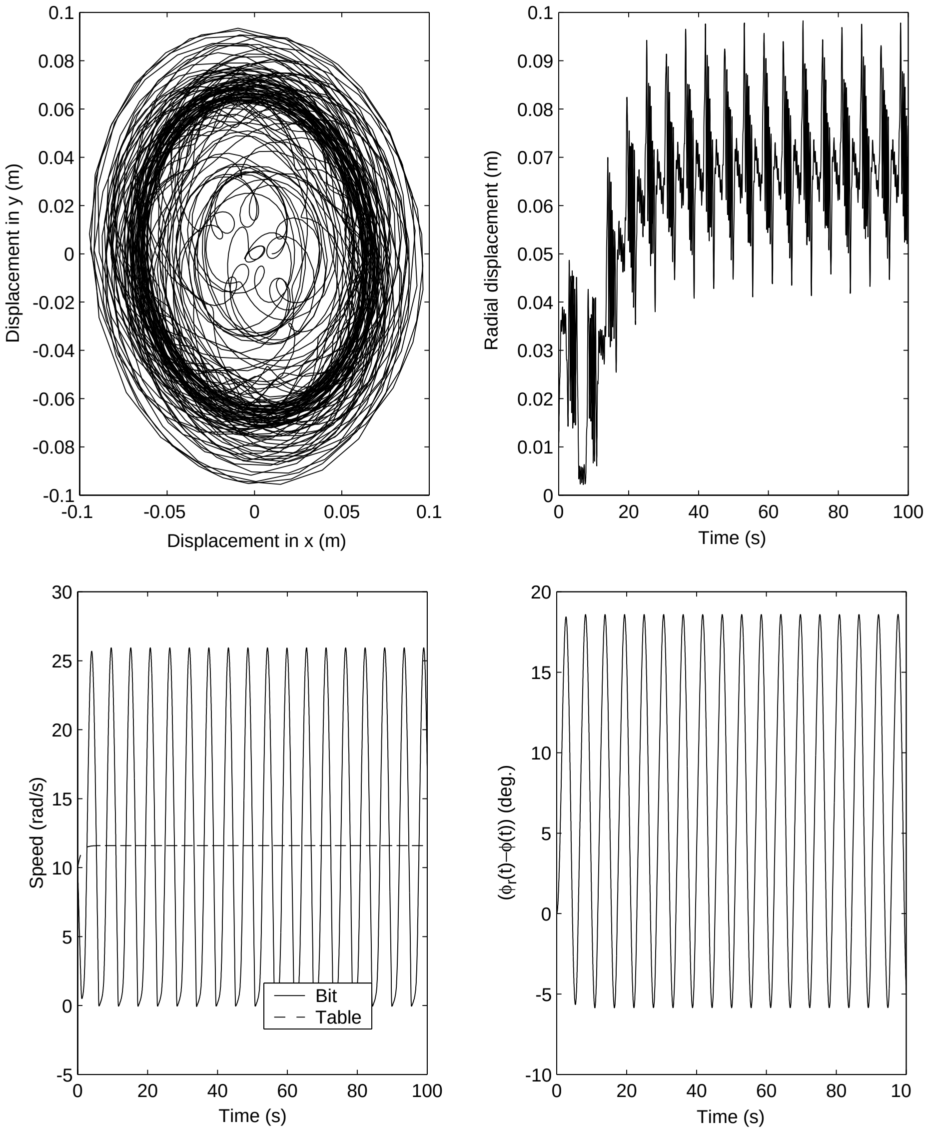

Fig. 2. Table speed control results (desired table speed = 11.6 rad/s).

zerodynamics should be studied [16]. A simple approach for investigating the stability of the resulting zerodynamics is to assume that the output variables are equal to the desired variables in the full system. Then, the stability of the remaining non-linear dynamics is checked using Jacobian linearization or any other techniques.

In this paper, the stability of the zerodynamics is not investigated theoretically due to the space limitation but numerical simulations indicate that the states of the zerodynamics are bounded. In fact the decision to use an inverse-based control design for the drillstring vibration problem is based on the cited literature which indicate that the lateral dynamics remain bounded if the

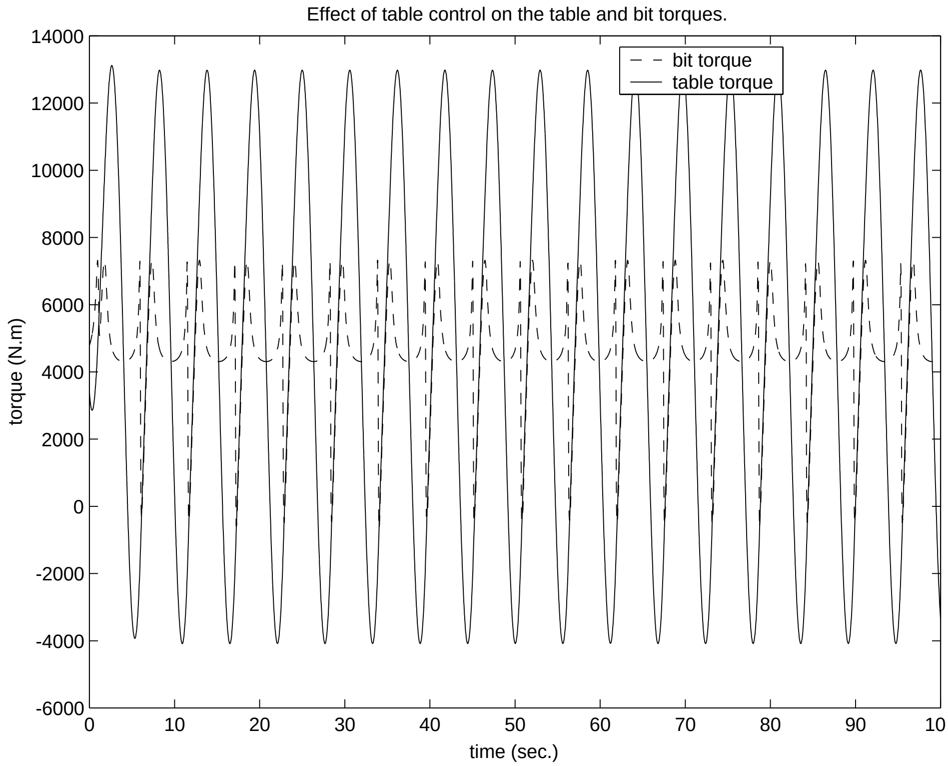

Fig. 3. Table speed control: bit and table torques.

torsional vibration is minimized [11,10]. Furthermore, the lateral vibration of drillstrings is physically constrained by the borehole boundaries and thus it cannot be unbounded.

4. Simulation results and discussion

The model of Fig. 1 has been used to simulate the dynamics of the drillstring with two control strategies: table speed control and bit speed control. The parameters used for simulations are shown in Table 1. The desired speed of the rotating table is chosen as 11.6 rad/s (110 r.p.m.) and 15.0 rad/s (145 r.p.m.) which are in the range of the common operating range of oil well drilling. These frequencies are above the system critical frequencies of torsional and whirling resonances of 1.85 and 6.14 rad/s, respectively [9]. Initially the rotary table and the bit are assumed to have same rotational speed of 10 rad/s when the bit is off bottom.

4.1. Table speed control

A controller for rotary table speed has been designed using the procedure of Section 3.2.1. Results of simulations with this controller are presented in Fig. 2, for a desired table speed of

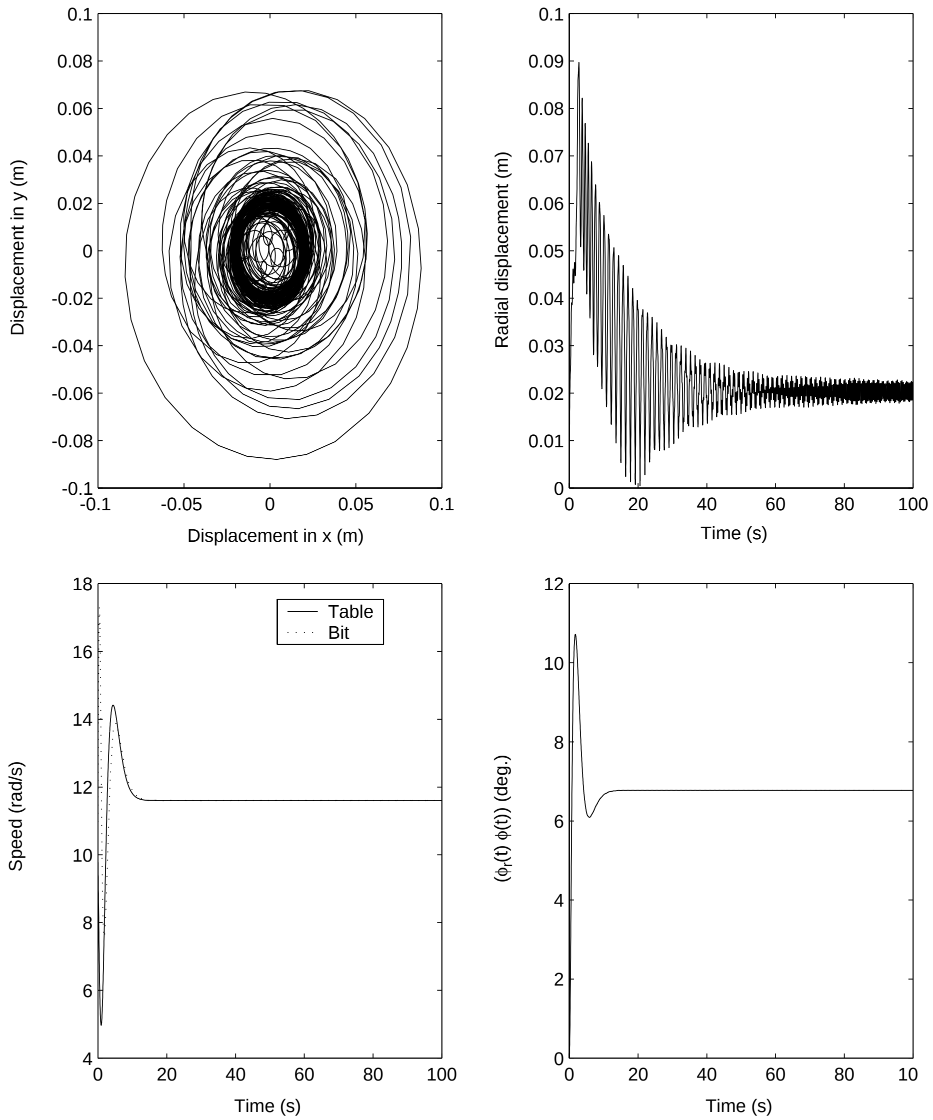

Fig. 4. Bit speed control (desired bit speed = 11.6 rad/s).

11.6 rad/s. Fig. 2 shows the trajectory of the drill collar geometric centre, the radial displacement of the bit (drill collar), time variations of bit and table and the corresponding twist in the drillstring over the period of the simulation. The corresponding torques of the table and the bit are shown in Fig. 3.

Though the table speed is maintained at its desired level, the controller is not very effective in suppressing the motions of the bit and the drill collar in the presence of stick–slip phenomenon as is evident from the results. The lower effectiveness of the table speed controller may be attributed to the quite high inertia of the rotary table making it almost an open-loop controller.

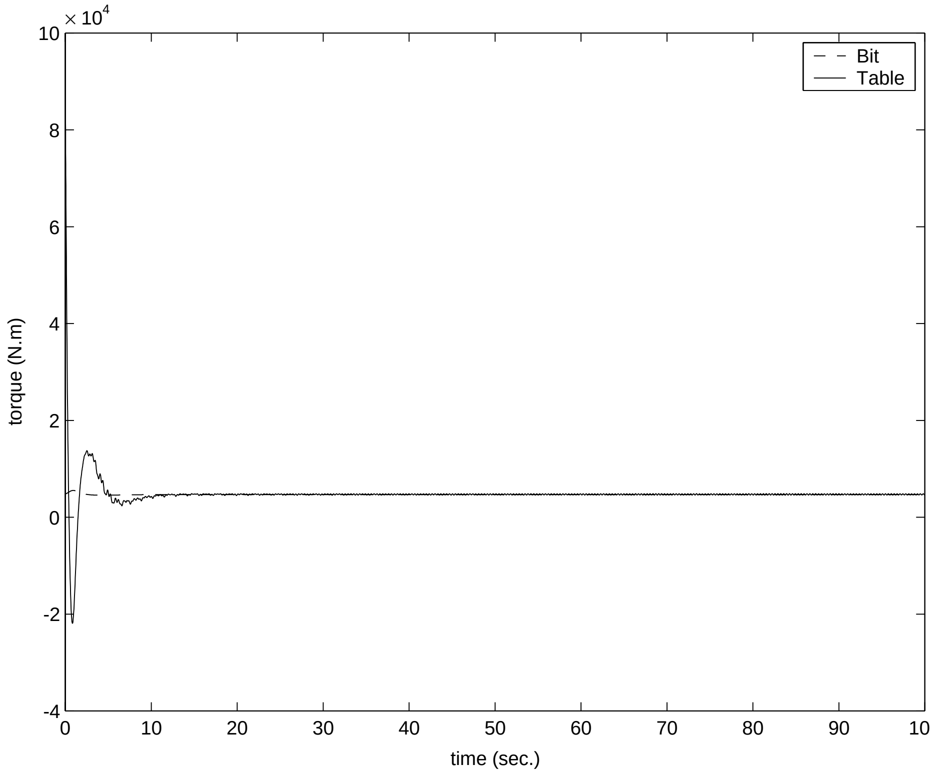

Fig. 5. Bit speed control (11.6 rad/s): bit and table torques.

4.2. Bit speed control

The simulations have been repeated with a bit speed controller for a desired bit speed of 11.6 rad/s. The controller has been designed using the procedure of Section 3.2.2. Results are presented in Figs. 4 and 5 which show the effectiveness of the controller in maintaining both bit and table speed at the desired level whilst eliminating the stick–slip phenomenon. The lateral displacement of the drill bit is substantially reduced compared to that with table speed control Fig. 2. Figs. 6 and 7 show the results with bit speed control with a desired bit speed of 15.0 rad/s. The effectiveness of the controller is evident from these results. The proposed bit speed controller is able to control torsional motion of the bit with a performance similar to that of some linear controllers presented in Ref. [11]. The reduction in lateral motion is due to consideration of the coupling between the torsional and lateral dynamics in designing the non-linear controller and the low amplitudes of torsional motion. Analysis of the robustness of the proposed controller has been studied with respect to some of the system parameters and the operating speed. It has been found that the controller works well even with deviations from nominal design conditions. The implementation of the proposed controller is also feasible in digital form with full state measurement or estimation using an observer. These are subjects of future study.

For implementation of the proposed non-linear what?, the second derivative of the bit speed is necessary. It is difficult, if not impossible, to measure the second derivative of the bit speed. An

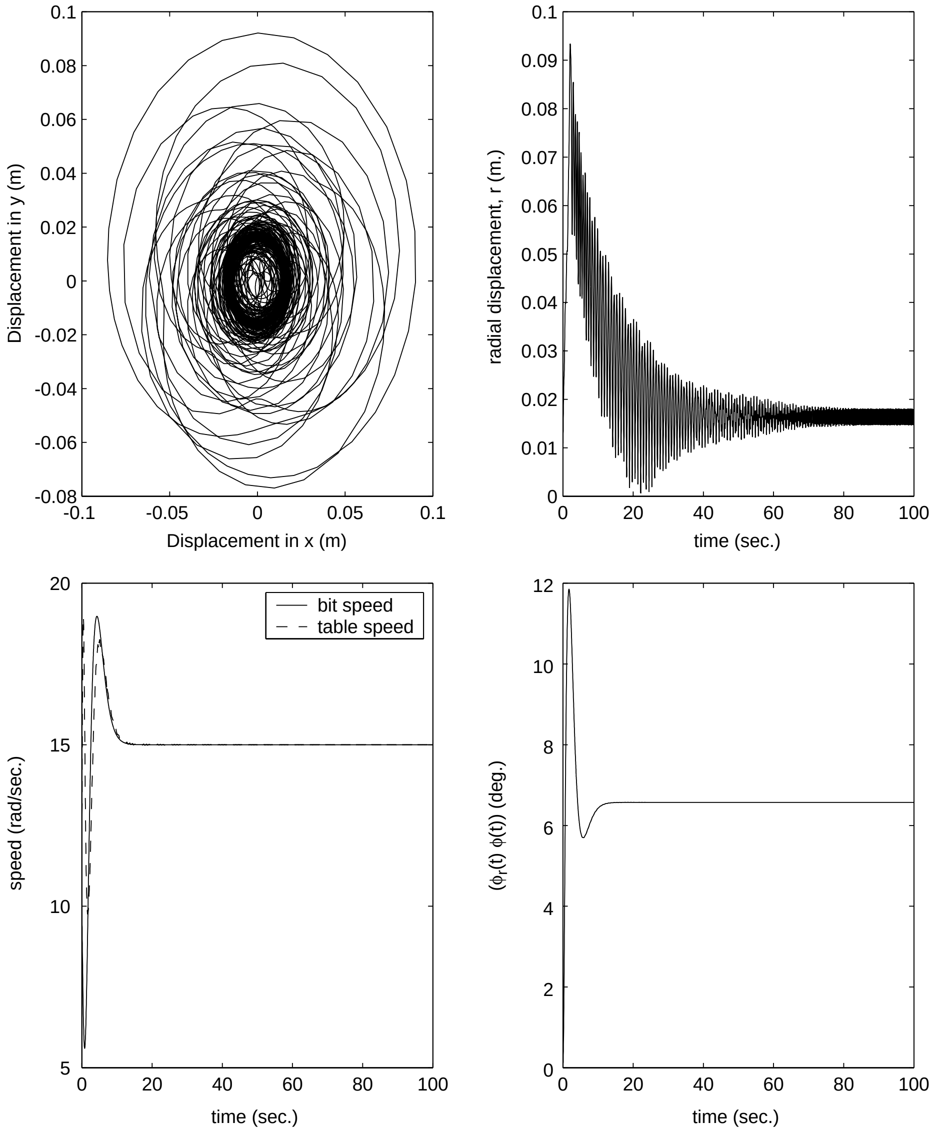

Fig. 6. Bit speed control (desired bit speed = 15 rad/s).

alternative to direct measurement may be to estimate it using the dynamic model of the drill string given in Eq. (3). However, the potential issues in implementation taking into account the effects of modelling errors and uncertainties of parameters need to be studied further.

5. Conclusions

In this paper, a non-linear controller for minimizing torsional and lateral vibrations of a non-linear drillstring dynamics model is studied. Dynamic inversion is first used to design a controller

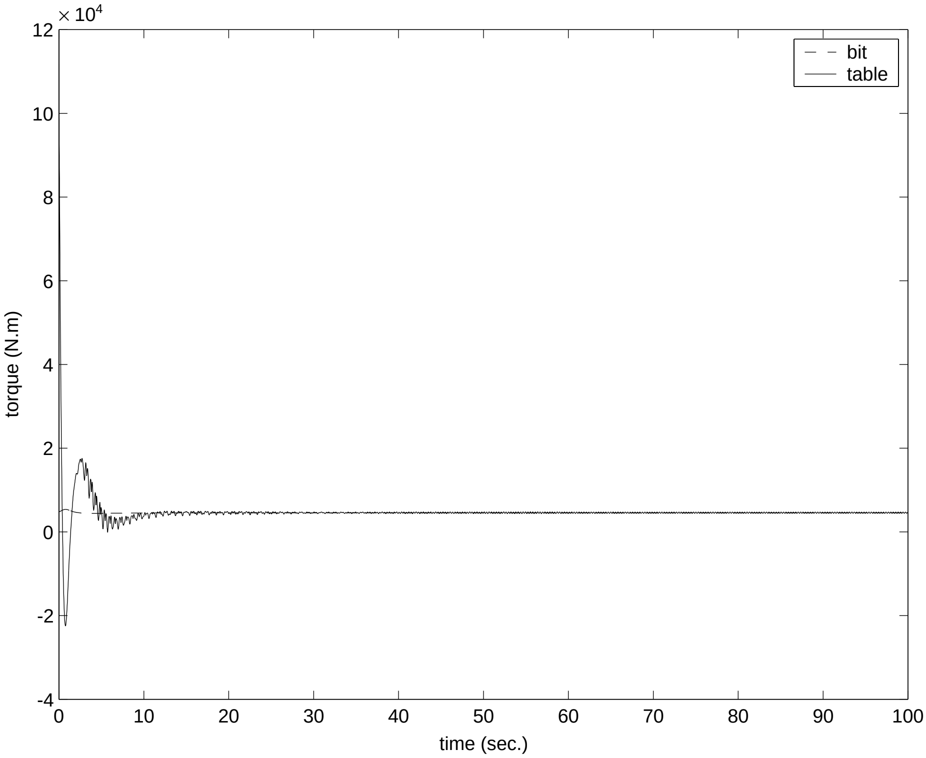

Fig. 7. Bit speed control (15 rad/s): bit and table torques.

for tracking a desired table speed. It has been found that controlling the table speed gives rise to large torsional and lateral vibrations of the drillstring.

Dynamics inversion is then used to design a non-linear controller for the bit to track a desired bit speed. Results showed that this controller completely eliminates the torsional vibrations and reduces effectively the lateral vibrations. This controller is examined to track two different bit speeds 11.6 and 15 rad/s. It has been found that the tracking error is zero in both cases. Numerical simulations showed that this controller is robust to variations of the parameters from their nominal values.

References

[1] H.K. Khalil, Nonlinear System, 2nd Edition, Prentice-Hall, Upper Saddle River, NJ 07458, 1996.

[2] B.V. Derjaguin, V.E. Push, D.M. Tolstoi, A theory of stick-slip sliding of solids, Proc. Conf. Lub. Wear, Inst. Mech. Eng. 1957, pp. 257–268.

[3] C.A. Zamudio, J.L. Tlusty, D.W. Dareing, Self-excited vibrations in drill-strings SPE 16661, 62nd Technical Conference, 1987, pp. 117–125.

[4] G.W. Halsey, A. Kyllingstad, A. Kylling, Torque feedback used to cure slip-stick motion, SPE Drilling Conference, Houston, 1988, SPE 18049, pp. 277–282.

[5] J.D. Jansen, L. Van Densteen, Active damping of self-excited torsional vibrations in oil well drill-strings, Journal of Sound and Vibration 179 (4) (1995) 647–668.

[6] A. Berlioz, J. Der Hagopian, R. Dufour, E. Draoui, Dynamic behavior of drill-string: experimental investigation of lateral instabilities, Transaction of the American Society of Mechanical Engineers, Journal of Vibration and Acoustics 118 (1996) 292–298.
[7] A.S. Yigit, A.P. Christofourou, Coupled axial and transverse vibrations of oil well drillstrings, Journal of Sound and Vibration 195 (4) (1999) 617–627.
[8] A. Baryshnikoy, A. Calderoni, A. Ligrone, P. Ferrara, A new to the analysis of drillstring fatigue behavior, SPE Drilling and Completion, June 1997, pp. 77–84.
[9] A.F.A. Serrarens, M.J.G. Van De Molengraft, J.J. Kok, L. Van Den Steen, H_∞, Control for suppressing stick-slip in oil well drillstrings, IEEE Control System Magazine 18 (2) (1998) 19–30.
[10] J.D. Jansen, Nonlinear dynamics of oilwell drillstrings, Ph.D. Thesis, Delft University Press, 1993.
[11] A.S. Yigit, A.P. Christofourou, Coupled torsional and bending vibrations of actively controlled drillstrings, Journal of Sound and Vibration 234 (1) (2000) 67–83.
[12] A.S. Yigit, A.P. Christofourou, Coupled torsional and bending vibrations of drillstrings subject to impact with friction, Journal of Sound and Vibration 215 (1) (1998) 167–181.
[13] A.P. Christofourou, A.S. Yigit, Dynamic modelling of rotating drillstrings with borehole interactions, Journal of Sound and Vibration 206 (2) (1997) 243–260.
[14] A.P. Christofourou, A.S. Yigit, Active control of stick-slip vibrations: the role of fully coupled dynamics, 12th SPE Middle East Oil Show and Conference, Bahrain, 2001.
[15] A. Kyllingstad, G.W. Halsey, A study of slip/stick motion of the bit, Society of Petroleum Engineers 3 (1998) 369–373.
[16] A. Isidori, Nonlinear Control System, 3rd Edition, Springer, New York, 1985.
[17] J.D. Jansen, Whirl and chaotic motion of stabilized drill collars, SPE Drilling Engineers, June 1992, pp. 107–114.