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Letter to the Editor

Low order continuous-time filters for approximation of the ISO 2631-1 human vibration sensitivity weightings

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1. Introduction

Human health and comfort under whole-body vibration depend on the level, frequency, direction, location, and duration of the acceleration. The international standard ISO 2631-1 [1] specifies a method of evaluation of the effect of exposure to vibration on humans by weighting the root-mean square (r.m.s.) acceleration with human vibration sensitivity curves. There are six frequency-weighting curves given as magnitudes W_i at the one-third octave frequencies f_i:

$$W(\omega)|_{\omega=2\pi f_i} = W_i, \quad i = 1, 2, 3, \dots, N. \tag{1}$$

In order to use these weightings in design, it is sometimes necessary to make a good approximation of these curves using stable rational transfer functions of the form

$$W(s) = \frac{b_m s^m + \cdots b_1 s + b_0}{s^n + \cdots a_1 s + a_0}. \tag{2}$$

Unlike previous versions of the standard published in 1974 and 1985, the recent ISO 2631-1 (1997) standard gives high order s-plane equations to describe the weighting curves. However, low order filter approximations are still preferred in practical applications, especially in controller design.

Muller et al. [2] approximate the ISO 2631-1 (1974) weighting W_k (for vertical acceleration) by a second order filter given by

$$W(s) = \frac{50s + 500}{s^2 + 50s + 1200}. \tag{3}$$

This shape filter has been cited by many researchers in passive and active vehicle suspension design (e.g., Refs. [3–7]), even though the ISO weighting curves have been updated several times since 1974. Other second order filter approximations have also been used by Kim and Yoon [8] and Wang and Wilson [9]. As reported by Lewis and Griffin [10], the weighted r.m.s. accelerations

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can differ by more than 20% among the ISO 2631-1 (1985) and ISO 2631-1 (1997) standards. It is therefore necessary to design good low order filters to approximate the ISO 2631-1 weighting curves.

2. Method and results

A least-pth approximation in the frequency domain is usually desired, where $p = 2$ for a least-square approximation and $p \to \infty$ for a minimax (Chebyshev) approximation. The problem then is to determine the coefficients a_k and b_k such that

$$\sum_{i=1}^{N} ||W(s_i)| - W_i|^p, \quad \text{where } s_i = 2\pi \mathrm{j} f_i \tag{4}$$

is minimized. This is a highly non-linear problem, and no analytical solution can be obtained even for the first order least-square approximation [11]. Fortunately, in the field of digital signal processing, a great effort has been devoted to advanced filter design in the z-domain, and many standard codes are available. Therefore, we use the method of bilinear transformation [12] to design the ISO 2631-1 filters.

Step 1: Map the specifications of the weighting curves in continuous-time frequency ω_i onto the unit circle (z-domain) via the bilinear transformation

$$\Omega_i = \frac{2}{T}\tan^{-1}\left(\frac{T\omega_i}{2}\right) = \frac{2}{T}\tan^{-1}(T\pi f_i), \tag{5}$$

where Ω_i is the discrete-time frequency and T is a constant.

Step 2: Design the optimal least-pth digital filter $W_D(z)$ to minimize

$$\sum_{i=1}^{N} ||W_D(z_i)| - W_i|^p. \tag{6}$$

where $z_i = \mathrm{e}^{\mathrm{j}\Omega_i}$ and

$$W_D(z) = \frac{\bar{b}_n z^{-n} + \cdots \bar{b}_1 z^{-1} + \bar{b}_0}{z^{-n} + \cdots \bar{a}_1 z^{-1} + \bar{a}_0}. \tag{7}$$

The *Matlab Filter Design Toolbox* [13] provides the function `iirlpnorm`, which computes the IIR digital filter that best approximates (in the least-pth sense) the desired frequency response within single or multiple bands.

Step 3: Obtain the s-domain filter via the bilinear transformation

$$W(s) = W_D(z)|_{z=(1+Ts/2)/(1-Ts/2)}. \tag{8}$$

This filter is not an optimal least-pth *analog* filter, but it is a very good approximation to the ideal continuous-time frequency response. We call such filters "quasi-least-pth continuous-time filters." Setting $p \geqslant 16$ [14] yields a good approximation to the minimax (Chebyshev) design.

The following are the second through fifth order quasi-least-square ($p = 2$) filters obtained by taking $T = 0.5$ to approximate the ISO 2631-1 (1997) frequency-weighting curve W_k for vertical acceleration:

$$W_k^{(2)}(s) = \frac{86.51s + 546.1}{s^2 + 82.17s + 1892},$$

$$W_k^{(3)}(s) = \frac{80.03s^2 + 989.0s + 0.02108}{s^3 + 78.92s^2 + 2412s + 5614},$$

$$W_k^{(4)}(s) = \frac{81.89s^3 + 796.6s^2 + 1937s + 0.1446}{s^4 + 80.00s^3 + 2264s^2 + 7172s + 21196},$$

$$W_k^{(5)}(s) = \frac{87.72s^4 + 1138s^3 + 11336s^2 + 5453s + 5509}{s^5 + 92.6854s^4 + 2549.83s^3 + 25969s^2 + 81057s + 79783}.$$

The magnitude frequency responses are shown in Fig. 1.

The second through fourth order filters similarly obtained for the weighting curve W_d for horizontal acceleration are

$$W_d^{(2)}(s) = \frac{13.55s}{s^2 + 12.90s + 47.16},$$

$$W_d^{(3)}(s) = \frac{14.55s^2 + 6.026s + 7.725}{s^3 + 15.02s^2 + 51.63s + 47.61},$$

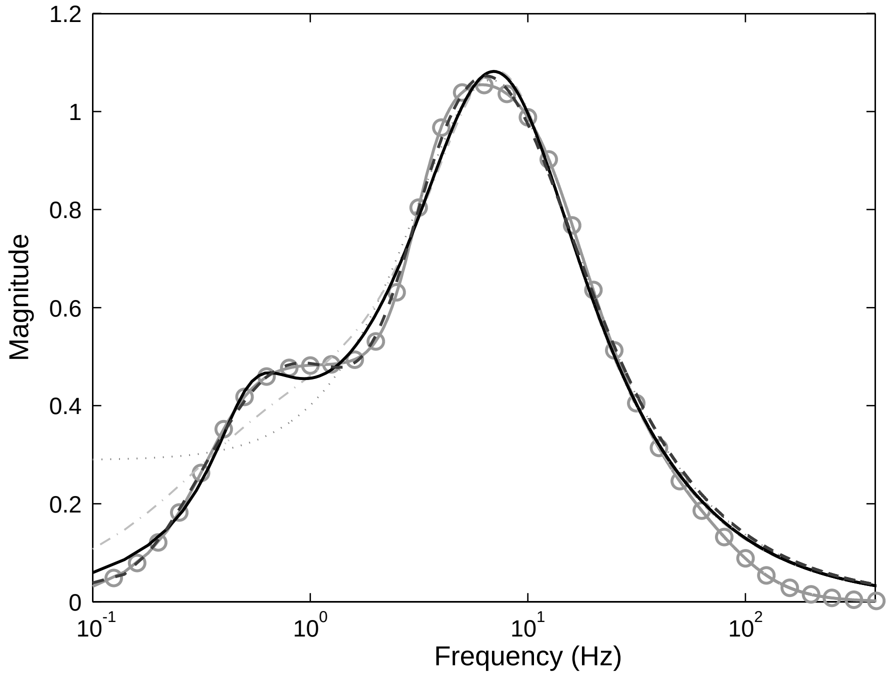

Fig. 1. ISO2631-1 frequency-weighting curve W_k (circles) and quasi-least-square filter approximations: second order (dot), third order (dash), fourth order (solid), fifth order (dash–dot).

$$W_d^{(4)}(s) = \frac{12.66s^3 + 163.7s^2 + 60.64s + 12.79}{s^4 + 23.77s^3 + 236.1s^2 + 692.8s + 983.4},$$

and their magnitude frequency responses are shown in Fig. 2.

The third principal frequency-weighting curve W_f is used for motion sickness. The second through fifth order quasi-least-square filter approximations are

$$W_f^{(2)}(s) = \frac{0.8892s}{s^2 + 0.8263s + 1.163},$$

$$W_f^{(3)}(s) = \frac{0.05726s^3 + 3.876s}{s^3 + 4.263s^2 + 4.777s + 4.396},$$

$$W_f^{(4)}(s) = \frac{0.02633s^4 + 0.0238s^3 + 2.286s^2 + 0.2335s + 0.02902}{s^4 + 2.527s^3 + 4.584s^2 + 2.993s + 1.373},$$

$$W_f^{(5)}(s) = \frac{0.1457s^4 + 0.2331s^3 + 13.75s^2 + 1.705s + 0.3596}{s^5 + 7.757s^4 + 19.06s^3 + 28.37s^2 + 18.52s + 7.230},$$

and their magnitude frequency responses are shown in Fig. 3.

Low order quasi-least-square filter approximations for the three additional frequency weightings can be similarly obtained.

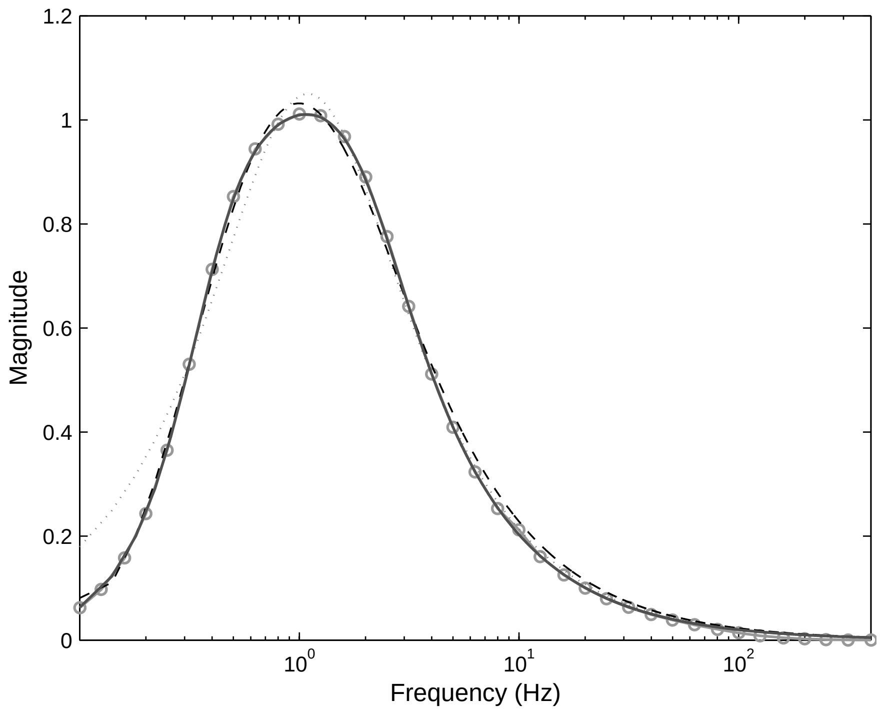

Fig. 2. ISO2631-1 frequency-weighting curve W_d (circles) and quasi-least-square filter approximations: second order (dot), third order (dash), fourth order (solid).

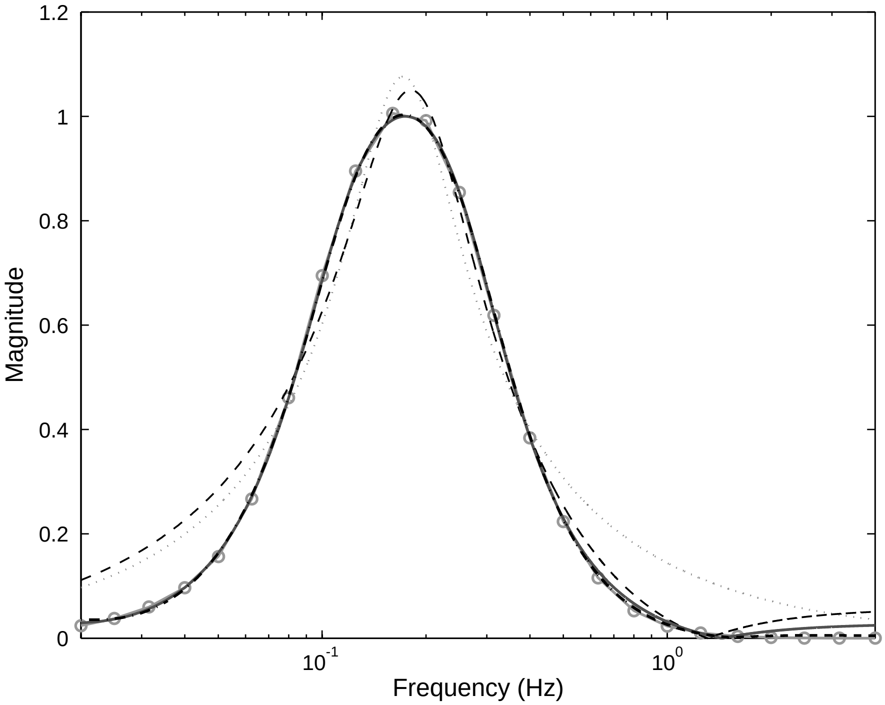

Fig. 3. ISO2631-1 frequency weighting curve W_f (circles) and quasi-least-square filter approximations: second order (dot), third order (dash), fourth order (solid), fifth order (dash–dot).

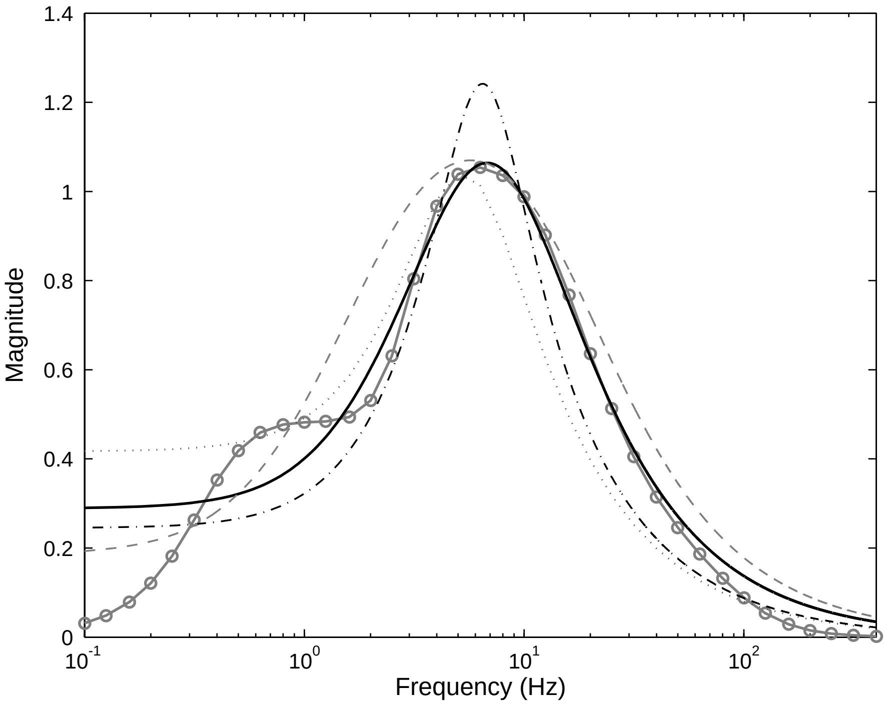

Fig. 4. Second order filter approximations: Muller's design [2] (dot), Hankel model reduction (dash), balance model reduction (dash–dot), quasi-least-square (solid) ISO2631-1 weighting curve W_k (circles).

3. Discussion

Unlike previous versions of the standard, that published in 1997 provides a high order s-plane description. So a low order approximation could also be obtained by model reduction. The most commonly used methods are the balanced and Hankel model reductions. Standard codes to perform these reductions are also available, such as the functions `balmr` and `ohkapp` in *Matlab Robust Control Toolbox* and the function `modred` in *Matlab Control System Toolbox* [13].

Figs. 4 and 5 show the magnitude frequency responses of second and fourth order shape filters obtained by balanced and Hankel model reductions of the ISO 2631-1 (1997) weighting $W_k(s)$, in comparison with the quasi-least-square filter approximations and Muller's filter (3). The r.m.s. errors at the one-third octave frequencies are compared in Table 1.

We can see that the quasi-least-square shape filters are much better than Muller's filter and those obtained by either model reduction technique.

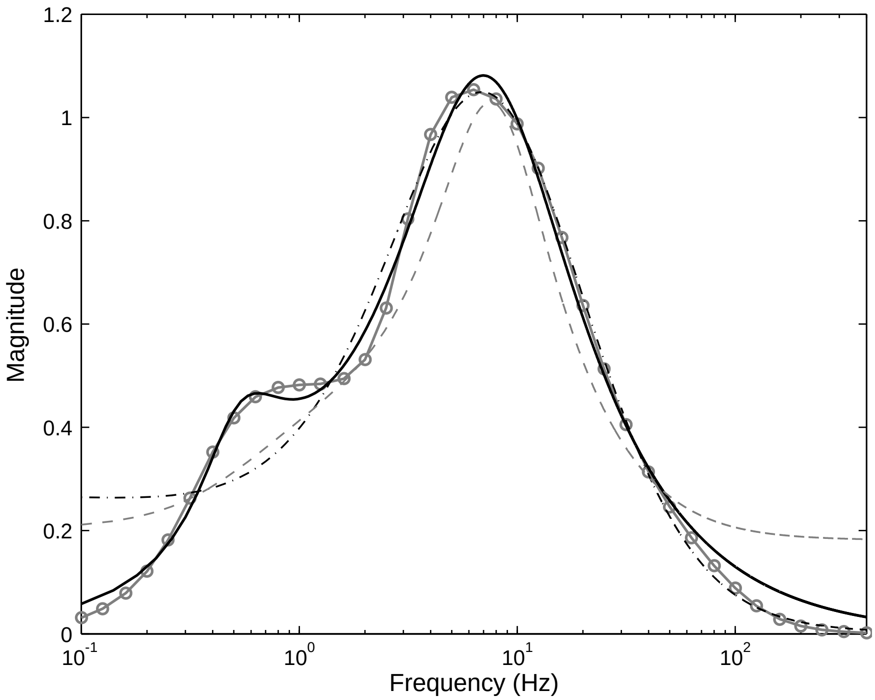

Fig. 5. Fourth order filter approximations: Hankel model reduction (dashed), balance model reduction (dash–dot), quasi-least-square (solid) , ISO2631-1 weighting curve W_k (circles).

Table 1
Comparison of r.m.s. errors at the one-third octave frequencies

	Quasi-least-square	Balanced model reduction	Hankel model reduction	Muller's design [2]
Second order	0.0895	0.1150	0.1141	0.1641
Fourth order	0.0299	0.0822	0.1123	—

4. Conclusion

In this letter, we propose a simple procedure for design of low order quasi-least-pth continuous-time filters that approximate the ISO 2631-1 frequency-weighting curves. We use the bilinear transformation to map the magnitude-frequency specifications into the z-domain, design the optimal digital filters using standard codes, and thence obtain the corresponding quasi-least-pth s-plane filters. The method is very easy to implement, and the obtained filters are a better match to the ISO 2631-1 (1997) frequency weightings than those obtained by model reduction techniques and those widely cited in the literature. The low order filters presented herein can be used in optimization of vehicle suspensions and other applications.

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